

Modeling of Multiple Statistical Distributions for Extreme Rainfall Data Using Maximum Likelihood Estimation Methods and Bayesian Methods

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ABSTRACT

The city of Pekanbaru has rapidly developed into a metropolitan hub, facing challenges such as floods and haze caused by extreme rainfall events. This study proposes a novel combination of Generalized Extreme Value (GEV), Generalized Logistic (GLO), and Generalized Pareto (GP) distributions, utilizing Bayesian Markov Chain Monte Carlo (MCMC) and Maximum Likelihood Estimation (MLE) methods, to model annual extreme rainfall data for the period 2010–2024. Rainfall data were sourced from NASA/POWER. Model performance was evaluated using Relative Root Mean Square Error (RRMSE), Relative Absolute Square Error (RASE), and Probability Plot Correlation Coefficient (PPCC). The Bayesian method yielded superior performance with RRMSE = 0.3166, RASE = 0.2682, and PPCC = 0.00485 for the GEV distribution, outperforming MLE. The novelty lies in applying this methodological combination to Pekanbaru's rainfall dataset for the first time, providing valuable insights for flood mitigation, drainage planning, and urban water resource management.

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1. INTRODUCTION

The city of Pekanbaru as the capital of Riau Province, has developed rapidly into a metropolitan city that plays a role as an economic, transportation, and industrial center on the island of Sumatra. This growth was further accelerated by various development programs, including the metropolitan planning program "Pekansikawan," which involved the surrounding area [1]. However, these developments also pose significant challenges, such as flooding from heavy rainfall and haze from forest fires in the surrounding regions [2]-[4].

Flooding conditions are frequent in several sub-districts, including Rumbai, Tenayan Raya, and Tampan [2], which are exacerbated by clogged drainage systems and siltation of the Siak River [5][6]. Meanwhile, the haze that occurs every dry season not only disrupts local community activities but also affects neighboring countries such as Singapore and Malaysia [7]. Therefore, accurate information on extreme rainfall is needed to support effective disaster mitigation. A critical tool for providing information on extreme rainfall is the distribution of opportunities [8].

There are several studies on the distribution of opportunities that are often applied to extreme rainfall data across various regions. As in the study [9] using the distribution of Normal, Log-Normal two parameters, Log-Normal three parameters, Gumbel, Gamma 2 parameters, Pearson III, and Log-Pearson III. This study concludes that the Pearson III distribution is an appropriate opportunity distribution model for extreme rainfall data. Furthermore, the study [10] used the generalized extreme value distribution (GEV), Generalized Pareto (GP), Generalized Logistic (GLO), Pearson Type III (PE3), and Gumbel distributions to determine the probability distribution models for extreme rainfall data. The results of this study indicate that GLO, GP, and GEV are appropriate distributions for modeling extreme rainfall data. Some of the methods that are often used to estimate the parameters of the opportunity distribution are the L-Moment Method, the Maximum Likelihood Estimation Method (MLE), and the Bayesian Markov Chain Monte Carlo Method (MCMC) [11]. According

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to [12], the MLE method is superior for determining the parameters of the opportunity distribution [13]. However, in the study [14], it was reported that the MLE method performed poorly with small sample sizes. Pekanbaru, the capital of Riau Province in Indonesia, has experienced rapid urbanization and expanding impervious surfaces, increasing vulnerability to extreme rainfall and urban flooding. Accurate probabilistic modeling of rainfall extremes is essential for designing drainage systems, flood mitigation measures, and resilient urban planning. To overcome this problem, the Bayesian MCMC method can be used to determine the parameters of the opportunity distribution.

Based on this background, this study aims to determine the distribution and the best parameter estimation method for modeling extreme rainfall data in the city of Pekanbaru during the 2010-2024 period. The distributions used include Generalized Extreme Value (GEV), Generalized Logistic (GLO), and Generalized Pareto with the Maximum Likelihood (MLE) and Bayesian Markov Chain Monte Carlo (MCMC) parameter estimation methods. This study compares three candidate extreme-value distributions (GEV, GLO, GP) and two estimation paradigms (frequentist MLE and Bayesian MCMC) on 2010–2024 NASA/POWER-derived annual maxima. We demonstrate the added value of Bayesian inference (improved uncertainty quantification and, in our analysis, a modestly better fit) and provide practical recommendations for Pekanbaru stakeholders. This research is expected to develop an appropriate distribution model to predict extreme rainfall, thereby helping to design more effective mitigation measures and generate synthetic data for the future.

2. THEORETICAL FOUNDATION

2.1 Extreme Rainfall

Rainfall is part of the hydrological cycle and the main form of precipitation in the tropics [15]. Rainfall is the amount of rainwater that falls on the surface of the ground in a given period [16], measured in millimeters (mm) using an ombrometer as per WMO standards [17]. The ombrometer is installed in an open area and measures with an accuracy of 0.1 mm [18]. EVT provides asymptotic models for block maxima (GEV) and for threshold exceedances (GP). The block maxima approach used here extracts the annual maximum daily rainfall and models it with the GEV family, which nests the Gumbel, Fréchet, and Weibull distributions via the shape parameter ξ [19].

2.2 Extreme Value Theory (EVT)

Extreme events play an essential role across fields such as climatology, hydrology, economics, insurance, and finance. Extreme Value Theory (EVT) is a statistical theory used to study the behavior of distribution tails, thus allowing the determination of the probability of extreme values. This theory is often applied to analyze major events in natural phenomena, such as rainfall, floods, and air pollution. In EVT, there are two primary methods, namely block maxima and peaks over threshold (POT) [20]. In this study, extreme values were identified using the block maxima method, which selects the maximum value from each block of data, defined by a specific period (e.g., monthly or yearly). The maximum value of each block is then used as the primary data for further analysis [21].

2.3 Statistical Distribution

The cumulative distribution function and opportunity density function of each GEV, GLO, and GP distribution are presented in Table 1, where y they are the variables studied, γ denotes the scale parameter, δ is the location parameter, and ε denotes the shape parameter [22][23][10]. We select the GEV for block maxima (standard EVT model), the GP for threshold exceedances (POT framework), and the GLO (Generalized Logistic) as an alternative, flexible three-parameter distribution that often fits heavy-tailed or skewed precipitation series in tropical climates. Choice is motivated by prior studies showing these families' applicability to rainfall extremes and by their complementary tail behaviors.

Table 1. Cumulative Distribution Function and Probability Density Function of Each GEV, GLO, and GP

Distribution

Distribution	Cumulative Distribution Function	Probability Density Function
GEV	$F(y) = \exp \left[- \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}} \right], \varepsilon \neq 0$	$f(y) = \frac{1}{\gamma} \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1-\varepsilon}{\varepsilon}} \exp \left[- \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}} \right], \varepsilon \neq 0$
GLO	$F(y) = \exp \left(- \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}} \right), 1 + \varepsilon \frac{y-\delta}{\gamma} > 0$	$f(y) = \frac{1}{\gamma} \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}-1} \exp \left(- \left(1 + \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}} \right)$
GP	$F(y) = 1 - \left(1 - \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1}{\varepsilon}}, \varepsilon \neq 0$	$f(y) = \frac{1}{\gamma} \left(1 - \varepsilon \frac{y-\delta}{\gamma} \right)^{\frac{1-\varepsilon}{\varepsilon}}, \varepsilon \neq 0$

2.6 Maximum Likelihood Estimation (MLE) Method

The stages of parameter estimation using the MLE method include:

1. Defining the Likelihood Function;
2. Perform a natural logarithm on the likelihood function, which is then called the log-likelihood function;
3. Maximize the log-likelihood function by lowering the function for each parameter.

MLE provides point estimates via log-likelihood maximization and is computationally efficient. Bayesian MCMC incorporates prior information (if available), yields complete posterior distributions for parameters and return levels, and better represents estimation uncertainty—particularly important for short records or heavy-tailed data.

2.7 Metode Bayesian Markov Chain Monte Carlo (MCMC)

The Bayesian method was first introduced by Reverend Thomas Bayes in 1763, a statistical approach that combines preliminary information with sample data through a conditional probability theorem to produce a posterior distribution [24]. This method helps analyze extreme data and make predictions, though it is often criticized for the difficulty of evaluating complex integrals in posterior distributions [25]. To overcome these challenges, simulation techniques such as the Markov Chain Monte Carlo (MCMC) method are used, which consists of two main approaches: Gibbs Sampling and Metropolis-Hastings. Gibbs Sampling simulates conditional distributions in multivariate models, whereas Metropolis-Hastings allows sampling from distributions that are difficult to simulate directly [26]. Both approaches support parameter estimation and posterior inference in complex data analysis, though their success depends on an effective, stable simulation process. Figure 1–3 below illustrates typical PDF shapes for the GEV, GLO (approximation), and GP distributions, with example parameter values to aid interpretation.

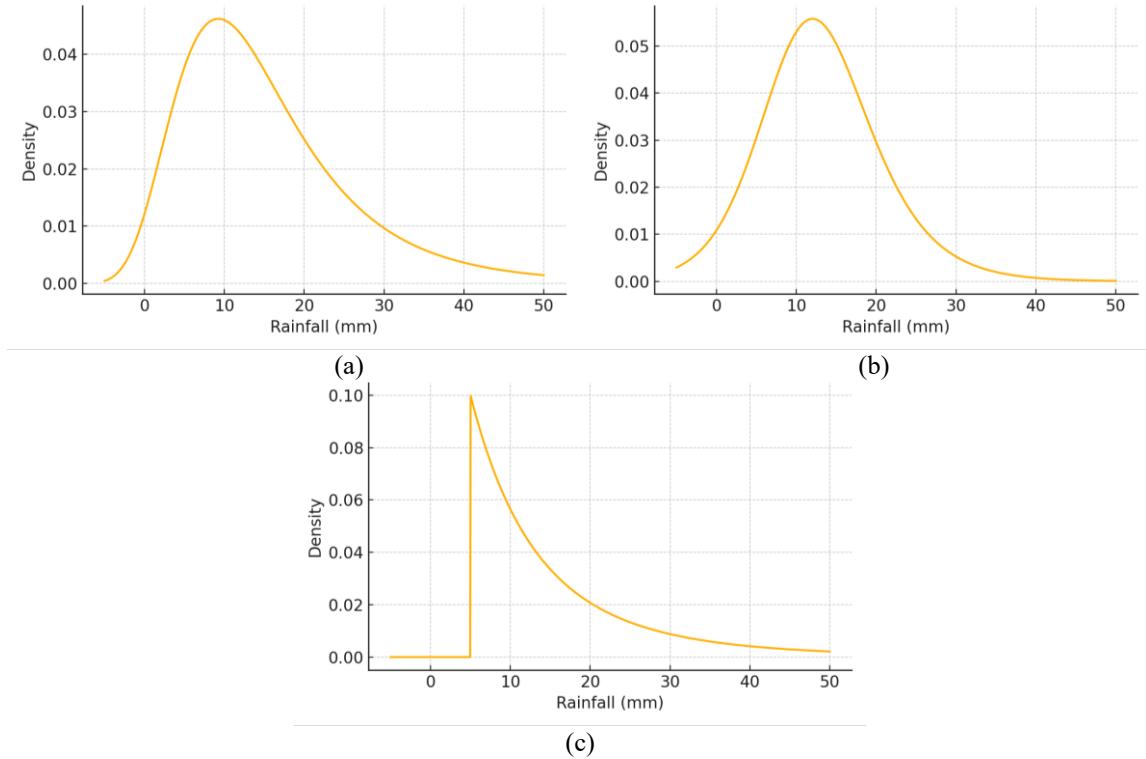


Figure 1. Illustrative PDF (a) GEV, (b) GLO, and (c) GP

2.8 Model Testing

In this study, three tests were used to model rainfall data in the city of Pekanbaru, namely the Relative Absolute Square Error Test (RASE), RASEThe Relative Root Mean Square Error Test (RRMSE), and the Probability Plot Correlation Coefficient (CC). These three model tests are based on quantile values and frequency distributions. Here is a summary of the three tests [27][14]:

$$RRMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n \left(\frac{y_{i:n} - \hat{Q}(F_i)}{y_{i:n}} \right)^2} \quad (1)$$

$$RASE = \frac{1}{n} \sum_{i=1}^n \left| \frac{y_{i:n} - \hat{Q}(F_i)}{y_{i:n}} \right| \quad (2)$$

$$PPCC = \frac{\sum_{i=1}^n (y_{i:n} - \bar{y})(\hat{Q}(F_i) - \bar{Q}(F_i))}{\sqrt{\sum_{i=1}^n (y_{i:n} - \bar{y})^2} \sqrt{\sum_{i=1}^n (\hat{Q}(F_i) - \bar{Q}(F_i))^2}} \quad (3)$$

with

$$\bar{Q}(F_i) = \frac{1}{n} \sum_{i=1}^n \hat{Q}(F_i) \quad (5)$$

Where is the formula for the position of the Gringorton plot, which is stated as follows:

$$F_i = \frac{i-0.44}{n+0.12}, i = 1, 2, \dots, n \quad (6)$$

The sample order value of the random $\hat{Q}(F_i)$ is defined as the estimated quantile of order i . The RRMSE and RASE statistics aim to measure the extent of the difference between the quantiles estimated from the distribution and those observed in the data. The smallest values of the RRMSE and RASE statistics indicate that the tested model is the best. Meanwhile, the PPCC statistic is used to measure the correlation between the value and the observation y_i .

3. Research Methodology

The data taken is rainfall data from 2010 to 2024 in the city of Pekanbaru, sourced from the official website of the National Aeronautics and Space Administration (NASA) (<https://power.larc.nasa.gov/data-access-viewer/>). Daily rainfall (2010–2024) was obtained from NASA/POWER (surface meteorology and solar energy) and preprocessed to remove missing values and apply quality-control checks. Annual maxima were extracted for each calendar year to form the block-maxima series used for GEV fitting. In this study, R software assistance was used to simplify calculations. The following are the stages in this study

3.1. Data sources and preprocessing

Daily rainfall (2010–2024) was obtained from NASA/POWER (surface meteorology and solar energy) and pre-processed to remove missing values and apply quality-control checks. Annual maxima were extracted for each calendar year to form the block-maxima series used for GEV fitting.

3.2. Fitting strategy

We fit candidate distributions using (a) MLE via numerical optimization, and (b) Bayesian MCMC using Metropolis-Hastings within Gibbs, where applicable. For Bayesian runs we used 50,000 iterations and discarded the first 10,000 as burn-in. Convergence diagnostics included trace plots, Gelman-Rubin statistics (\hat{R}), and effective sample sizes.

3.3. Model selection and diagnostic metrics

Model skill was compared using Relative Root Mean Square Error (RRMSE), Relative Absolute Square Error (RASE), and Probability Plot Correlation Coefficient (PPCC), which measure quantile fit and the correlation between theoretical and empirical quantiles. Return levels (e.g., 10-, 25-, 50-, 100-year) were computed from fitted parameter sets and posterior samples (for the Bayesian method) to quantify uncertainty

4. RESULTS AND DISCUSSION

[Table 2](#) shows the results of parameter estimation, using the Maximum Likelihood Estimation Method and the Bayesian Markov Chain Monte Carlo Method (MCMC). In the Bayesian Method, 50,000 iterations were performed, with the initial 10,000 iterations ignored as part of the Burn-in process. Meanwhile, the results of the best model testing for each method are presented in [Table 3](#). [Figures 2 and 3](#) show the Bayesian Method MCMC analysis on the GEV and GLO distributions to estimate the parameter (δ, γ) . With trace plots in the first line that depict the movement of parameter values, and posterior plots in the second line that represent the posterior distribution. Parameter estimates from MLE and Bayesian MCMC are reported in [Table 2](#) (see attached full tables). Bayesian posteriors were generally well-behaved for location and scale parameters; shape parameters showed wider uncertainty—familiar in extreme-value estimation with short samples. Trace plots and $\hat{R} < 1.1$ for key parameters indicate adequate convergence.

Table 2. Parameter estimation results and from the MLE and Bayesian MCMC Methods $\delta, \gamma, \varepsilon$

Distribution	MLE			Bayesian MCMC		
	$\hat{\delta}$	$\hat{\gamma}$	$\hat{\varepsilon}$	$\hat{\delta}$	$\hat{\gamma}$	$\hat{\varepsilon}$
GEV	0,71552130	0,16864017	0,04743377		0,7593862	0,1494039
GLO	0,7484139	0,1239730	-0,4912021		0,8636545	0,1078012
GP	0,8036287	0,3481993	0,6521079			

Table 3. Parameter estimation results and from the MLE and Bayesian MCMC Methods $\delta, \gamma, \varepsilon$

Distribution	MLE			Bayesian MCMC		
	<i>RRMSE</i>	<i>RASE</i>	<i>PPCC</i>	<i>RRMSE</i>	<i>RASE</i>	<i>PPCC</i>
GEV	0,3617348	0,3056398	0,0085125		0,3166069	0,2682355
GLO	0,9377225	0,5575972	0,0932706		0,8987533	0,5304467
GP	0,5096301	0,3764322	0,0417302			0,0937314

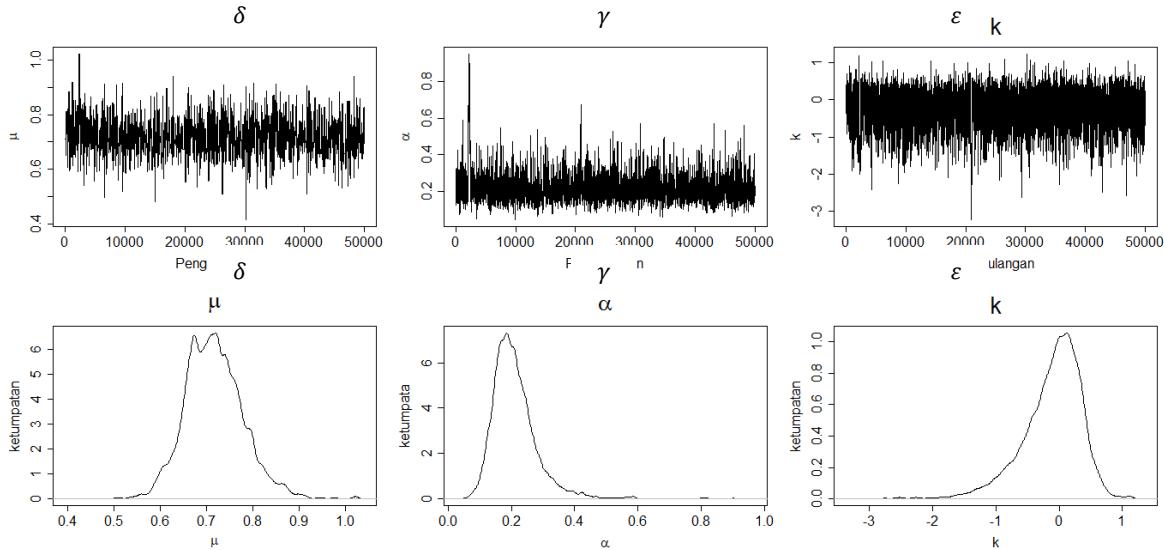


Figure 2. Graph of Repetition and Posterior Density of GEV Distribution Parameters for Extreme Rainfall Data in Pekanbaru City in 2010-2024

Across candidate families, no single distribution dominates in all metrics. However, the Bayesian GEV fit produced lower RRMSE and RASE values, and a PPCC closer to 1, compared to MLE-based fits within the same family, supporting the use of Bayesian inference for more robust quantification of uncertainty and slightly better empirical quantile matching. Practical interpretation: smaller RRMSE/RASE implies closer agreement in return level estimation, which directly impacts design margins for drainage infrastructure

Table 3 shows that none of the parameter estimation methods is clearly superior for modeling extreme rainfall data in Pekanbaru City, due to differences in the best-fit distributions from the RRMSE and RASE tests relative to the PPCC. Nevertheless, each distribution can model extreme rainfall data in Pekanbaru City. On the other hand, the Bayesian Method is the appropriate method for modeling extreme rainfall data in Pekanbaru City, because the best distribution test results for RRMSE and RASE have lower values than those of the MLE Method, and the PPCC values are close to 1. GEV is recommended for block maxima approaches (annual extremes), GP for threshold exceedances (POT) when sufficient exceedances are available, and GLO as a practical alternative when data show marked skewness not well captured by GEV/GP. Model choice should be informed by diagnostics, return-level needs, and stakeholder risk tolerance

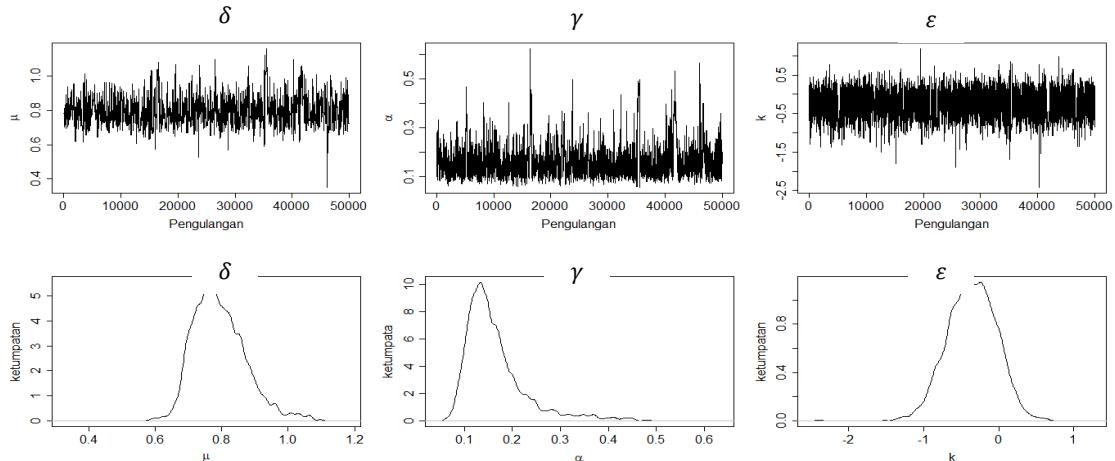


Figure 3. Graph of Repetition and Posterior Density of GLO Distribution Parameters for Extreme Rainfall Data in Pekanbaru City in 2010-2024

5. CONCLUSION

Bayesian MCMC produced a modestly better fit (lower RRMSE/RASE and PPCC closer to 1 for GEV) and substantially improved uncertainty quantification for return-level estimates that inform flood-mitigation planning. Limitations include reliance on satellite-derived rainfall (no rain-gauge cross-validation in this version), relatively short record (15 years) for extreme estimation, and the approximation used for GLO. Future work should incorporate station data, spatial/temporal non-stationarity, and hierarchical Bayesian models.

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