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Exploration Pedagogical Content Knowledge of Undergraduate Mathematics Education Students in Teaching

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Abstract

This study explores the algebra content knowledge and teaching practices of undergraduate student's mathematics in their seventh semester at a state university in East Nusa Tenggara, Indonesia. Using a case study approach, the research focuses on students' conceptual and procedural understanding of algebra and how this knowledge translates into classroom teaching. Three instruments were used: (i) Teaching Content Algebra Problems (TCAP), assessing students' ability to explain different solution approaches; (ii) Mathematical Ability Test (MAT) covering fundamental topics such as numbers, sets, algebra, and linear equations; and (iii) the Teacher Knowledge for Teaching Quality (TKQ) observation tool to evaluate instructional practices. Validated by three associate professors, all instruments showed strong content validity. A total of 46 students completed the TCAP and MAT, and three were selected for interviews and classroom observation based on their unique written responses. TCAP responses were analyzed using the Meaning-Based Approach (MBA) and Direct Translation Approach (DTA) framework. The analysis revealed that S1 was categorized as MBA-j, demonstrating conceptual reasoning and the ability to justify mathematical procedures and contextual interpretations. In contrast, S2 and S3 were categorized as DTA-lc, indicating that although they applied relevant procedures, they experienced difficulties in connecting mathematical solutions to the contextual meaning of the problems. Classroom observations further showed that S1 implemented more meaningful and context-rich algebra instruction, whereas S2 and S3 demonstrated limited contextual integration in their teaching practices. These findings highlight the importance of integrating deep content knowledge with pedagogical strategies to support the development of conceptual, procedural, and contextual understanding in mathematics teacher education programs.

Keywords: Pedagogical Content Knowledge, Mathematics Education, Teaching, Undergraduate Students

Algebra content knowledge is a fundamental component of mathematics education, particularly in preparing undergraduate students with the skills required to teach mathematical concepts effectively. Algebra not only serves as a core element of the mathematics curriculum across educational levels but also plays a crucial role in developing advanced mathematical thinking, such as modeling, generalization, and the analysis of abstract structures (Hu, 2022). Therefore, mastering algebra content knowledge is essential in equipping prospective teachers to face the challenges of real classroom instruction. This knowledge involves understanding the curriculum and integrating algebraic concepts into teaching practices aligned with students' grade levels (Hill, 2008; Sahidin et al., 2019; Shulman, 1986; Ünver et al., 2020).

However, content knowledge alone is insufficient. Effective instruction also depends on prospective teachers' pedagogical knowledge specifically, their ability to teach algebra in ways that are meaningful and accessible to students. This includes the ability to explain multiple solution strategies and to design instruction that aligns with students' characteristics and learning needs (Ball & Bass, 2003). This type of knowledge is part of Pedagogical Content Knowledge (PCK), which emphasizes the

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integration of subject matter understanding with appropriate teaching strategies. PCK includes three key components: Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Curriculum (KC) (An et al., 2004; Grossman, 1990). KCS refers to understanding mathematics content in relation to students' ways of thinking. KCT involves knowledge of instructional strategies and how to support student understanding. KC pertains to knowledge of curriculum goals and the effective use of instructional resources.

Despite its importance, several studies have indicated that undergraduate mathematics education students often struggle to translate their algebra knowledge into effective instructional practice. Previous studies have reported that undergraduate mathematics education students' algebra problem-solving abilities remain at a moderate level (Güler & Çelik, 2021). A large-scale study involving 1,367 pre-service mathematics teachers in Turkey found that participants performed lowest on algebra content, which had negative implications for their teaching competence (Güler & Çelik, 2018; Güler & Çelik, 2021) . In addition, prospective teachers often encounter difficulties in solving and teaching contextual algebra problems because of insufficient conceptual understanding, limited pedagogical strategies, and challenges in adapting mathematical content to diverse student characteristics (Widjaja, 2013). These findings suggest that many prospective teachers possess procedural competence but struggle to transform algebraic knowledge into meaningful instructional practices that support students' conceptual learning.

Previous studies have extensively investigated the Pedagogical Content Knowledge of undergraduate mathematics education students and pre-service mathematics teachers across various instructional contexts. The findings consistently indicate that although prospective teachers generally demonstrate adequate mathematical content knowledge, many experience difficulties in integrating that knowledge with appropriate pedagogical approaches during instruction. Specifically, they often struggle to identify students' misconceptions, select appropriate mathematical representations, design meaningful learning activities, facilitate mathematical reasoning, and adapt instructional strategies to diverse classroom situations (Depaepe, 2013; Krepf & König, 2022). Furthermore, studies focusing on algebra instruction reveal that prospective mathematics teachers frequently emphasize procedural fluency rather than conceptual understanding, making it difficult for them to explain abstract algebraic ideas, connect multiple representations, and respond effectively to students' mathematical thinking (An et al., 2004; Ball, 2008). Collectively, these studies demonstrate that prospective teachers continue to experience difficulties in integrating multiple dimensions of PCK during mathematics instruction.

Although previous research has substantially advanced understanding of prospective teachers' PCK, several important gaps remain. First, most existing studies have primarily focused on measuring the level or individual components of PCK through questionnaires, written assessments, lesson-plan analysis, or classroom observations. Such approaches provide valuable information regarding the extent of prospective teachers' PCK but offer limited insight into how the different components of PCK interact during instructional practice. Second, relatively little empirical evidence explains how undergraduate mathematics education students dynamically integrate algebra content knowledge, knowledge of students' thinking, instructional strategies, and pedagogical reasoning while making instructional decisions during teaching. Third, only a limited number of studies have specifically examined this integration within authentic algebra teaching practices at the junior secondary school level, where students frequently experience conceptual difficulties and misconceptions. Consequently, current knowledge remains limited regarding how prospective teachers enact PCK while teaching algebra in real classroom contexts. This limitation makes it difficult to understand how prospective

teachers transform their mathematical knowledge into effective instructional decisions during authentic algebra teaching.

Addressing these research gaps, the present study explores the Pedagogical Content Knowledge of undergraduate mathematics education students during algebra instruction through an in-depth analysis of their teaching practices. Unlike previous studies that primarily assessed the level or separate components of PCK, this study investigates how prospective teachers dynamically integrate algebra content knowledge, knowledge of students' thinking, instructional strategies, and pedagogical reasoning while planning, implementing, and evaluating algebra lessons. By focusing on the interaction among these PCK components within authentic teaching contexts, this study provides a more comprehensive understanding of how prospective mathematics teachers enact professional knowledge during instruction. This integrated perspective provides new empirical evidence regarding how the components of PCK interact simultaneously during authentic algebra instruction, rather than being examined as isolated constructs.

The findings of this study are expected to contribute both theoretically and practically. Theoretically, this study contributes to the existing PCK framework by providing empirical evidence of how KCS, KCT, and KC interact dynamically during algebra instruction, thereby providing empirical evidence for understanding the enactment of PCK in authentic teaching contexts. Practically, the findings are expected to inform the design of teacher education curricula, microteaching activities, and teaching practicum programs by identifying aspects of algebra-specific PCK that require further development. The findings may also serve as a reference for designing algebra-specific professional development activities aimed at improving prospective teachers' instructional decision-making. These contributions may support teacher educators in designing learning experiences that better prepare prospective mathematics teachers to connect mathematical understanding with effective instructional practice.

Based on these considerations, this study aims to explore the Pedagogical Content Knowledge of undergraduate mathematics education students in teaching algebra at the junior secondary school level. Specifically, the study investigates how prospective teachers integrate algebra content knowledge, knowledge of students' thinking, instructional strategies, and pedagogical reasoning during teaching. The findings are expected to identify critical aspects of PCK that require further development and provide recommendations for strengthening teacher preparation programs to better integrate mathematical understanding with effective instructional practice in algebra.

METHODS

This study employs a case study approach to observe the characteristics of student response sheets and teaching practices, with the aim of exploring algebra content knowledge and teaching skills of undergraduate students. The participants in this study were seventh-semester students from the Mathematics Education program at a state university in East Nusa Tenggara, Indonesia, who were engaging in the School Field Experience activity. Based on the results of initial observations, two classes were selected as the subjects of this study: Class A (20 female students) and Class B (3 male students and 20 female students). In both classes, the majority of the students demonstrated active, interactive, and responsive behavior, and were able to perform teaching tasks effectively during the microteaching activities. This study focuses on two main aspects: algebra content knowledge and teaching ability. The instruments used in this study include: (i) Teaching Content Algebra Problems

(TCAP), which consists of three open-ended questions to measure students' knowledge in explaining different approaches to solving algebra problems (see Appendix); (ii) Mathematical Ability Test (MAT), which includes five open-ended questions to assess students' knowledge on topics such as numbers, sets, algebra, and linear equations; (iii) Teacher Knowledge for Quality Observation (TKQ), which includes 23 items related to teaching practices (Rowland, 2005).

All three instruments were validated and deemed appropriate for use, with minor revisions made to the wording of the questions by three associate professors using a 5-point Likert scale. The validation results indicated that both instruments were valid, with average scores of 4.6 for TCAP and 4.0 for MAT (maximum score: 5.0). Based on suggestions and feedback from the three validators, 46 students completed the TCAP and MAT, each within 60 minutes, under the direct supervision of the researcher and with assistance from one Mathematics Education lecturer. The Meaning-Based Approach (MBA) and Direct Translation Approach (DTA) (Pape, 2004) were used to analyze the characteristics of the students' response sheets. Meanwhile, the TCAP and MAT results were obtained from each individual item. A total of 46 students received TCAP and MAT scores. These scores were then tested using the Pearson Product Moment (Adamson & Prion, 2013) with $\alpha = 5\%$ and SPSS version 26.0. The results of the empirical validity test showed the following: (a) The average difficulty level of the TCAP and MAT questions was 0.69;(b) The average discrimination index for TCAP was 0.58 and for MAT was 0.60; (c) The empirical validity test scores for TCAP were 0.48 and for MAT were 0.53; (d) The reliability test scores for TCAP were 1.000 and for MAT were 0.99. These results indicate that the instruments were valid, reliable, with easy difficulty levels, and exhibited good discrimination ability for each item.

Based on the analysis of the characteristics of the responses from each TCAP response sheet, the MBA and DTA methods were used. The MBA and DTA categories include: Direct Translation Approach-proficient (DTA-p), Direct Translation Approach-non-proficient (DTA-np), Direct Translation Approach-limited context (DTA-lc), Meaning-Based Approach-full context (MBA-fc) & Meaning-Based Approach-justification (MBA-j) (Pape, 2004). These categories are explained in the Table 1 below:

Table 1. Indicators of Knowledge of Content and Teaching

Knowledge of Content and Teaching
Relevance between knowledge and information, concepts, rules, and procedures for solving problems using different approaches: • Students logically connect and prove three out of four pieces of knowledge to solve the problem: (1) Knowledge of information, (2) Knowledge of information and concepts, (3) Knowledge of information, concepts, and rules, (4) Knowledge of information, concepts, rules, and procedures. Choosing knowledge to determine different approaches • Students determine and prove knowledge to solve the problem: (1) Knowledge of the final outcome, (2) Knowledge of conclusions. Using knowledge to plan and review steps to determine different approaches • Students use at least three out of five types of knowledge: (1) Information, (2) Concepts, (3) Rules, (4) Procedures, (5) Conclusions.

Based on Table 1, the indicators of knowledge of content and teaching are grouped in the following Table 2.

Table 2. Grouping of Knowledge of Content and Teaching

Grouping				
MBA-j	**MBA-fc**	**DTA-lc**	**DTA-np**	**DTA-p**
Relevant and logical Contextually appropriate Effective, systematic, and comprehensive	Relevant and partly logical Contextually appropriate Effective, systematic, and comprehensive	Relevant and illogical Not contextually appropriate Effective, systematic, and partial review	Not relevant and illogical Not contextually appropriate Ineffective, unsystematic, and no comprehensive review	Not relevant and illogical Not contextually appropriate Ineffective, unsystematic, and no comprehensive review

Next, three students were selected based on the following criteria: (a) having unique answers, (b) answers from TCAP falling at least into one of the following categories: MBA-j, MBA-fc, DTA-lc, DTA-np, or DTA-p, (c) willingness to participate in interviews outside of class sessions and PLP activities, and (d) willingness to be observed during PLP activities in class. Based on the analysis of the characteristics and criteria, three students were chosen for interviews. The interviews were structured based on the uniqueness of the students' answers on the TCAP response sheets, with the aim of deepening the understanding of their algebra content knowledge. Additionally, classroom observations were conducted to assess the students' teaching abilities. The interview and observation processes were carried out in a relaxed and comfortable atmosphere to support the authenticity of the data collected. This study employed the method triangulation technique to ensure the validity of the data, especially in linking written responses, interview results, and students' teaching practices.

RESULTS AND DISCUSSION

Results of TCAP Responses

Written responses to the TCAP were analyzed using the MBA and DTA approaches. The analysis focused on the characteristics of each subject's written answers.

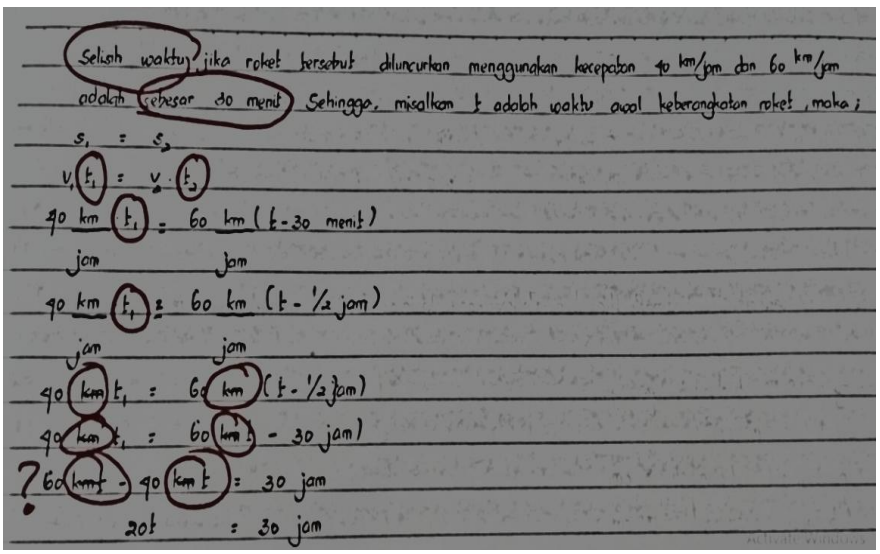


Figure 1. TCAP Written Response Sheet of S1 for Question Number 1

The following is an interview between the researcher and S1.

R : Is it possible to calculate distance using the time difference between the rockets?
S1 : Yes, I calculated the distance using the time difference because both rockets were launched simultaneously.
R : Can different units and variables be operated on together?
S1 : Units can be operated on to determine the quantity used. Meanwhile, $t_1 = t$ because it represents the arrival time of the rocket
R : Could you explain (while pointing at "$(t_0 + \frac{1}{6})$") and "$(t_0 - \frac{2}{6})$")?
S1 : t_0 is the initial time, while the rest is derived from the rocket arriving 10 minutes late. The 10 minutes need to be converted into hours, so $\frac{10}{60} = \frac{1}{6}$ hours. Since it was 10 minutes late, the operation used is subtraction.

S1 explained and logically justified their understanding of information, concepts, rules, and procedures. S1 stated that the symbol *t* was introduced based on the information provided in the problem. The problem indicated that "the arrival times of the two rockets at the mesosphere layer were 10 minutes and 20 minutes respectively, resulting in a 30-minute time difference." However, S1 was unable to explain the information and concept of time, as they assumed both rockets were launched simultaneously.

S1 explained that the symbol *s* represented distance, derived from the formula $s = v \times t$, where v stands for velocity and t for time. S1 was able to differentiate between the concepts of distance, speed, and time in context: "The length of the path from city A to city B is called distance, measured in kilometers (km); time refers to the duration of the journey, for example 50 minutes or 1 hour; whereas speed refers to how fast the distance is covered, typically measured in km/h."

S1 explained that dividing units in mathematical operations is essential to obtain answers with correct units. They gave an example as evidence: "If the distance is 10 meters and the time is 2 seconds, then the speed is $\frac{10}{2} = 5$. The result is just the number 5, which without units gives no information about speed. However, if $v = \frac{10}{2} = 5$ m/s, this represents the unit of speed and conveys meaning."

S1 was able to explain that $t_1 = t$ refers to time and therefore can be seen as two variables that may have different values. The subscript 1 in t_1 indicates the distinction between the arrival times of the rockets. Based on the written response and the interview confirmation, S1 consistently explained and logically justified concepts, rules, and procedures, categorized as MBA-j.

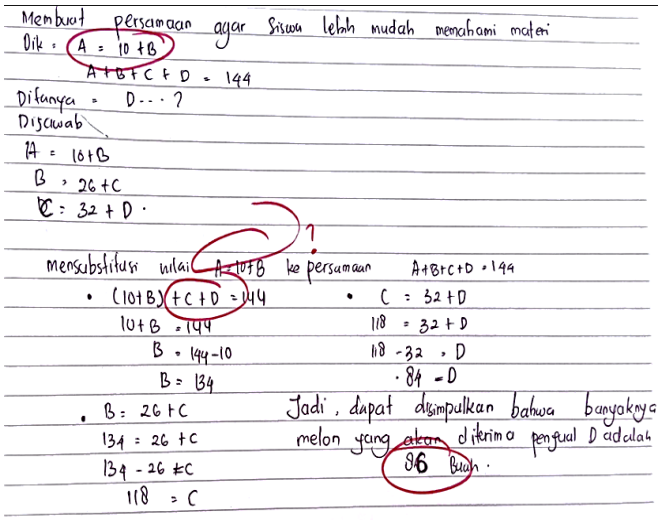

Membuat persamaan agar Siswa lebih mudah memahami materi
Dik : A = 10 + B
A + B + C + D = 144
Ditanya = D ... ?
Dijawab
A = 10 + B
B = 26 + C
C = 32 + D
mensubstitusi nilai A = 10 + B ke persamaan A + B + C + D = 144
(10 + B) + C + D = 144
10 + B = 144
B = 144 - 10
B = 134
B = 26 + C
134 = 26 + C
134 - 26 = C
118 = C
C = 32 + D
118 = 32 + D
118 - 32 = D
84 = D
Jadi, dapat disimpulkan bahwa banyaknya melon yang akan diterima pengual D adalah 86 Buah.

Figure 2. TCAP Written Response Sheet of S2 for Question Number 2

The following is an interview between the researcher and S2.

R : What do you understand from question number 2?
S2 : Seller A received 10 more melons than seller B, 26 more than seller C, and 32 more than seller D.
R : Can you show where that information is mentioned?
S2 : I wrote it in the answer section, not in the "known information" part of the question.
R : Why are you so confident that substituting A = 10 + B into A + B + C + D = 144 gives 134?
S2 : This result comes from 144 - 10 = 134.
R : Then what about the values of C and D from A + B + C + D = 144?
S2 : Those will be found separately. Since the value of B has been obtained, we just substitute it into B = 26 + C and C = 32 + D

S2 explained and demonstrated the connections between informational knowledge, concepts, rules, and procedures. S2 described the relationship between informational knowledge and conceptual understanding. Although S2 understood the overall information, only part of it was written down: "Seller A received 10 more melons than seller B," which was then modeled as "A = 10 + B." S2 applied modeling knowledge by translating the real-world problem into an algebraic equation. S2 connected informational knowledge, concepts, and rules, and used the substitution rule to find the value of D from the equation A + B + C + D = 144.

However, S2's explanation of the procedure using their knowledge was not logical. S2 substituted A = 10 + B into the equation A + B + C + D = 144, but the result was B = 134, which ignored variables C and D. S2 stated that the values of C and D would be found separately. This indicates that S2 did not fully apply their knowledge to obtain a result consistent with the context of the problem.

S2 used their knowledge to make a systematic and effective plan, but in the process, only partially checked the steps taken. Based on the written answer and the interview confirmation, S2 was consistently relevant and partially logical but did not solve the problem according to the contextual connections, and only reviewed part of the problem-solving steps. Therefore, S2 is categorized as DTA-Ic.

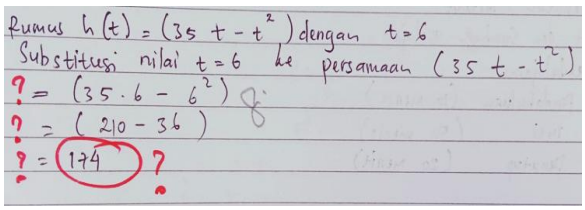


Figure 3. TCAP Written Response Sheet of S3 for Question Number 3

The following is an interview between the researcher and S3.

R : What is the goal of solving this problem?
S3 : To find the height of the rocket at the 6th second after launch height of the rocket launched at the 6 second.
R : Does the final result indicate the height of the rocket?
S3 : It indirectly indicates the rocket's height. Since the question is asking for that specific value, I thought that writing the number would already represent the rocket's height.

Based on Figure 3, there appears to be no apparent error in S3's written response. However, a closer analysis reveals a disconnect between the response and the contextual demand of the task, which specifically requires determining the height of the rocket at the 6th second after launch. S3 utilized relevant knowledge of the problem, including the given equation and the time variable, and proceeded by applying the substitution rule. In executing the solution procedure, S3 selected appropriate numerical operations and algebraic rules to arrive at a final result.

Nevertheless, S3 failed to interpret the result in relation to the context of the problem. The student merely computed a numerical value without ascribing contextual meaning to it. This indicates a

lack of integrated conceptual understanding, as S3 was unable to process the given information meaningfully or to interpret the mathematical solution as a representation of a real-world quantity. Drawing on both the written work and supporting interview data, S3's reasoning appeared consistently relevant and partially logical. However, the response did not fully address the contextual demands of the problem and only partially involved evaluation of the problem-solving steps. Therefore, S3 is categorized as DTA-lc.

Teaching Observation Results

This section presents each case from the individual subjects. It begins with a brief description of each subject's teaching, accompanied by images captured during the observation. The researcher then provides an analysis for each subject based on the TKQ observation sheet.

Case of S1

In the case of S1, the subject developed a lesson plan focused on the subtopic of applying algebraic operations in everyday life, with an allocated time of 2 × 40 minutes (one session). The researcher identified several key components in the lesson design as follows: (a) providing motivation and stating the learning objectives through the use of a contextual problem; (b) implementing a Problem-Based Learning (PBL) approach to support instruction, beginning with the presentation of a problem scenario; (c) dividing students into small groups and assigning them the task of constructing algebraic models based on the given problem; (d) allowing students time for group discussion and problem solving, while occasionally offering guidance to groups experiencing difficulties; (e) having one student from each group present their solution, followed by feedback from other groups; (f) conducting a class-wide discussion of the solutions; and (g) assessing students based on their presentation performance and a written test. To introduce the topic and motivate students, S1 posed the following contextual illustration: *"Have you ever gone shopping at a store? Suppose you buy 3 packs of instant noodles and 2 bottles of a drink. You don't yet know the exact prices, but you want to estimate the total cost. Let's say the price of noodles is x rupiahs, and the drink is y rupiahs. Then your total spending can be written as 3x + 2y."* Following this introduction, S1 proceeded with the main instructional activity. Students were divided into small groups and given a worksheet containing illustrative problems. One such illustration is shown in Figure 4 below.



You have just returned from a vacation and brought home x boxes of milkfish (bandeng presto) and y packs of shrimp crackers as souvenirs. You gave 2 boxes of milkfish to your neighbors and 3 packs of shrimp crackers to your friend.

a. Express the remaining number of boxes of milkfish as an algebraic expression.

b. Express the remaining number of packs of shrimp crackers as an algebraic expression.

c. Express the total number of remaining souvenirs as an algebraic expression.

Figure 4. Illustration provided by S1 to the students

Following the distribution of the worksheet, S1 allowed students to engage in collaborative discussions within their groups and provided support to those experiencing difficulties. During the group work, S1 was actively involved in monitoring student progress and offering guidance when needed. Furthermore, S1 created space for students to present their group's solutions, while encouraging peer feedback by inviting students from other groups to respond and engage critically with the presentations.

Case of S2

S2 designed a lesson plan to introduce the elements of algebraic expressions, including variables, coefficients, constants, and terms. S2 employed a question-and-answer model, complemented by group discussions. The key points for instruction developed by S2 were as follows: (a) providing real-life examples that can be represented algebraically; (b) posing questions to stimulate students' thinking about symbols and unknown numbers; (c) writing several examples of algebraic expressions on the board (e.g., $2y + 5$, $a^2 - 4b + 7$); (d) asking students to observe the examples and present their observations; (e) explaining the concepts of variables, constants, coefficients, and terms; and (f) assigning practice exercises as a form of assessment.



Figure 5. Classroom Question-and-Answer Session in Case of S2

Based on Figure 5, S2 began the lesson by presenting a real-life scenario: "If the price of one apple is 'x' rupiah, what is the price of 3 apples?" S2 then allowed time for students to respond to the question, which led to a discussion of variables, constants, coefficients, and terms. In the meantime, S2 posed a guiding question: "($\oplus$ - 7 = 3)." The lesson continued in the question-and-answer format until the session before the practice exercises. However, S2 did not administer a written test for assessment, citing that the mathematics class time had already ended.

Case of S3

In this case, S3 designed a lesson plan to teach the application of algebra in solving single-variable linear equations, using a scientific approach with a problem-solving method. S3 identified the key elements of the lesson as follows: (a) providing a brief review of linear equations to motivate students; (b) presenting a problem via a PowerPoint slide; (c) allowing students to solve the problem independently; (d) giving students the opportunity to explain their solution approach; (e) revisiting the solution by offering other students a chance to ask questions; and (f) assigning homework to be collected during the next session as an assessment for the students. Specifically, S3 aimed to link algebra with single-variable linear equations. S3 presented a real-life problem to the students: *"A car departs from city A to city B at an average speed of 60 km/h. After covering a certain distance, the car*

stops for a 30-minute break. The car then resumes its journey at the same speed and arrives in city B after a total travel time of 3 hours (including the break). What is the distance traveled by the car from city A to city B?" This problem is illustrated in Figure 6 below.

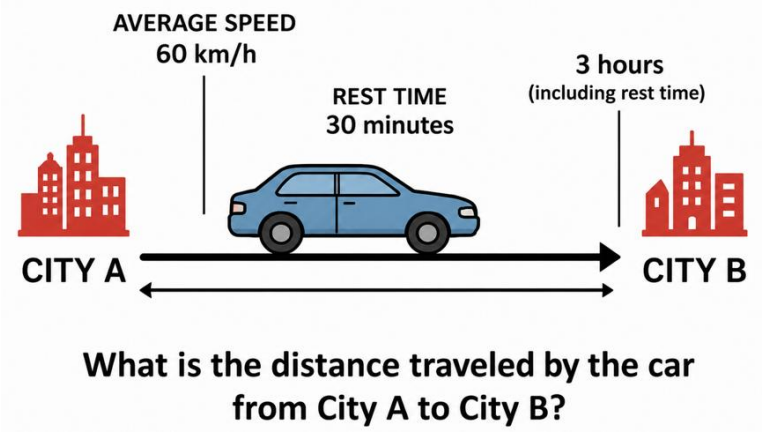


Figure 6. S3 presents the problem illustration to the students

S3 then guided the students to let the speed of the car be represented by *t*, and instructed them to convert the 30-minute break into 0.5 hours. S3 further directed the students to manipulate the equation for the journey: t - 0.5 = 3, leading to t = 3 - 0.5, which simplifies to t = 2.5. Next, S3 instructed the students to calculate the distance by multiplying the speed and time, yielding a result of 150 km.

Teaching Evaluation for Each Undergraduate Student

To evaluate the teaching practices of the three cases (S1, S2, and S3), several key criteria from the learning evaluation based on the TKQ observation sheet were used, including: planning, implementation, approach/method, student engagement, and assessment, as follows: S1 chose the theme of applying algebraic operations in everyday life, and the teaching was conducted using the Problem-Based Learning (PBL) method. S1 began the lesson with a concrete and contextually relevant motivation (shopping experience). S1 systematically applied the Problem-Based Learning approach (orientation, group division, discussion, presentation). S1 fostered student interaction through structured discussions and presentations, ensuring active participation. S1 performed comprehensive assessment through both presentations and a written test. S2 selected the theme of introducing the elements of algebraic expressions (variables, coefficients, constants, terms) using the question-and-answer method and group discussions. S2 initiated the lesson with a real-world context (apple pricing). S2 provided students with space to analyze and observe algebraic expressions, followed by explanations after student exploration, encouraging meaningful construction of knowledge. However, S2 did not conduct formal assessments, relying only on discussion and observation. S2 did not clearly demonstrate a thorough reinforcement of the concepts. S3 focused on the theme of applying algebra in single-variable linear equations using the scientific model – problem-solving method. S3 chose a realistic and analytical context for the problem (car journey). S3 encouraged students to solve the problem independently. Assessment was carried out through homework submissions. However, S3 made a logical error in solving the problem (the time should be subtracted from the total time, not from the break time, as reflected in the equation *t - 0.5 = 3*). S3 did not emphasize student interaction, as no discussion or collaboration was observed. In summary, the teaching evaluation for S1, S2, and S3 is shown in the Table 3 below.

Tabel 3. Teaching Evaluation for Each Undergraduate Student

TKQ Aspect	S1	S2	S3
Learning Context	Relevant and strong	Relevant	Relevant and applicable
Method/Model	Complete and effective PBL	Question and answer, less exploratory	Scientific, but minimal discussion
Student Activity	High (discussion & presentation)	Moderately active	Limited (more individual)
Assessment	Comprehensive	Incomplete	Present, but not reflective
Critical Engagement with the Material	High	Moderate	High, but with errors

Discussion

Students' mathematical knowledge and skills are influenced by the knowledge and skills of their teachers (Ekawati et al., 2022). Therefore, it is crucial to assess both the content knowledge and pedagogical knowledge of prospective teachers, particularly undergraduate students, in the context of teaching algebra. This highlights the importance of exploring the algebraic content and teaching practices of undergraduate students. Two key aspects are essential in this assessment: the ability to explain problem-solving processes and the ability to teach algebra effectively.

The analysis of written responses and teaching observations of the three subjects revealed variations in the level of mathematical knowledge and the implementation of classroom instruction. Regarding algebraic content knowledge, the students demonstrated different degrees of conceptual and procedural understanding, as reflected in their responses. In Problem 1, the students were able to describe the contextual and mathematical relationships among concepts such as distance, speed, and time. In Problem 2, they exhibited relevant knowledge in identifying and translating problem information into algebraic form. However, they also showed procedural reasoning limitations. Errors in substitution and the omission of certain information (values C and D) indicated that students had not fully connected procedural steps to the problem context in a logical manner. This suggests that their conceptual imagination of symbols remains limited (Akgün, 2010). Meanwhile, in Problem 3, although the numerical answer was correct, it was not meaningfully connected to the context of the problem. Students failed to interpret the solution meaningfully. This was evident in the TCAP results, where students were unable to explain the rocket's height in relation to a given time. These findings are consistent with previous studies indicating that both teachers and prospective teachers often possess incomplete content knowledge (Thanheiser et al., 2010). Therefore, it is essential for teachers and future educators to have a detailed understanding of the mathematics content they intend to teach (Baumert et al, 2010).

The teaching practices of undergraduate students were analyzed based on classroom observation using the TKQ rubric. The analysis revealed that students generally possessed knowledge of different instructional models aligned with the lesson themes (Ünver et al., 2020). In the case of Subject 1 (S1), the student consistently implemented a Problem-Based Learning (PBL) approach, allowing students to explore, discuss, and build understanding through group activities. This demonstrated a strong connection between conceptual understanding (as indicated in the TCAP results) and instructional implementation, as seen in the student's ability to guide learners in constructing algebraic models from real-life situations (Calor et al., 2020; Ünver et al., 2020). This aligns with previous findings that students are unlikely to develop adequate conceptual knowledge without being exposed to multiple representations of a concept (Moru, 2006). Consistent with the

recommendations, multiple representations should be employed to support mathematical thinking and learning (Julie et al., 2020). S1 not only emphasized procedures but also promoted meaning-making of mathematical symbols and units. The assessment strategies used were comprehensive (including presentations and written tests), reflecting a holistic instructional approach.

In the case of Subject 2 (S2), the student employed a questioning and discussion-based approach focused on exploring basic algebraic forms. This underscores the importance of teacher direction in classroom discussions, which plays a critical role in shaping students' learning opportunities as experienced in the classroom (Herbert & Bragg, 2021; Jacobs & Empson, 2016). Students were given opportunities to observe and describe components of algebraic expressions; however, conceptual reinforcement was not maximized due to the absence of formal assessment. This is consistent with the TCAP results, which indicated that while connections among pieces of information could be established, the ability to reflect on and verify procedures remained limited.

In the case of Subject 3 (S3), the student implemented a scientific approach involving realistic problem-solving. However, the implementation lacked sufficient emphasis on student discussion and collaboration. Additionally, there were logical errors in presenting problem solutions (e.g., subtracting break time from total time). S3 appeared to rely heavily on mechanical procedures without reinforcing conceptual meaning, consistent with the TCAP findings.

CONCLUSION

The findings of this study affirm that undergraduate students' content and pedagogical knowledge in algebra significantly influence the quality of students' mathematical understanding and learning experiences. The content and pedagogical knowledge demonstrated by the undergraduate participants varied, reflecting different levels of conceptual and procedural mastery in both problem-solving and instructional practice. Analysis of written responses and teaching observations indicated that strong conceptual understanding contributed to meaningful and contextualized teaching, as exemplified by Subject 1 (S1). In contrast, limitations in procedural reasoning and contextual interpretation, as observed in Subjects 2 (S2) and 3 (S3), hindered the effective communication of algebraic meaning to students.

These findings underscore the importance of professional development for prospective teachers, which should include the deepening of algebra content knowledge alongside the enhancement of pedagogical skills for representing mathematical concepts in varied and contextualized ways. Such an approach is essential for fostering students' conceptual, procedural, and applied mathematical understanding.

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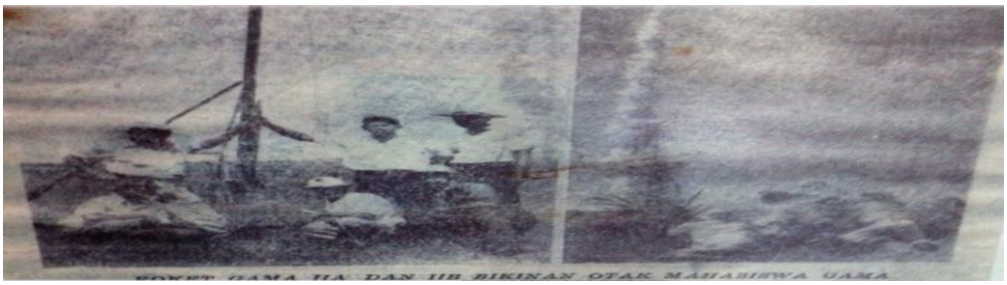
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Appendix: Teaching Content Algebra Problems (TCAP)

Problem 01

In 1963, Indonesia launched its first two rockets from Sanden Beach, Bantul, Special Region of Yogyakarta. These rockets, named Gama II A and Gama II B, were designed and built by students of Universitas Gadjah Mada (UGM). Each rocket had a thrust of 454 kg of solid fuel and was shaped as illustrated below.



Sumber: http://pengabdian.ugm.ac.id/2020/05/12/gama-roket-pertama-milik-bangsa/(12/7/2022 22:00)

A rocket is launched from Earth toward the Moon and is scheduled to enter the mesosphere (the third atmospheric layer above Earth) at 07:00 a.m. If the rocket is launched at a speed of 40 km/h, it will enter the mesosphere 10 minutes later than 07:00 a.m. If it is launched at a speed of 60 km/h, it will enter the mesosphere 20 minutes earlier than 07:00 a.m. What is the distance between Earth and the mesosphere?

Question:

How would you provide feedback in the form of step-by-step problem-solving guidance for the problem above?

Problem 02

Melon-Harvesting Robot was developed by IPB University in collaboration with PT XL Axiata and Universitas Dian Nuswantoro. The robot is equipped with image-processing technology to identify the ripeness level of melons.



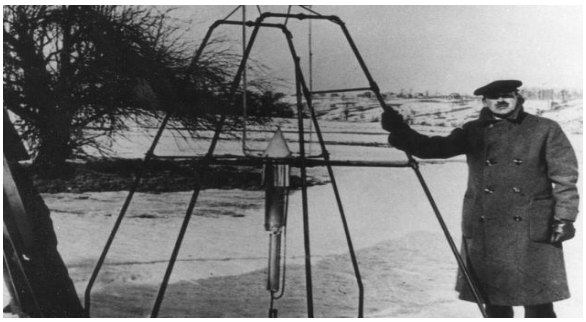
Sumber : https://edukasi.sindonews.com/read/538144/211/pakar-robot-ipb-ciptakan-robot-cerdas-pemanen-buah-melon-ini-penampakannya-1631376549 (13/7/2022 12:41)

A farmer uses the Melon-Harvesting Robot to harvest Alisha F1 melons. Alisha F1 is a melon variety that is resistant to pathogenic microorganisms. The robot harvests 144 melons, which are then distributed to four vendors: A, B, C, and D. Vendor A receives: 10 more melons than Vendor B; 26 more melons than Vendor C, and 32 more melons than Vendor D. Determine the number of melons received by Vendor D.

Question: Explain the problem-solving process above in a way that helps students understand it more easily**.**

Problems 03

Robert Hutchings Goddard was an American scientist who is recognized as the pioneer of modern rocketry. He worked as a professor of engineering at Clark University. On March 16, 1926, he successfully launched the world's first gasoline-powered rocket, as illustrated below.



Sumber: https:// https://nl.wikipedia.org/wiki/Robert_Hutchings_Goddard (13/7/2022 13:23)

A rocket launched by NASA from Earth toward the Moon has a height that is modeled by the formula h (t) = (35t - t^2) *m,* where (t) represents time in seconds. Determine the height of the rocket at the 6th second.

Question: Explain the step-by-step solution process for the problem above in a way that helps students understand it easily.