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Investigating metacognitive planning in collaborative problem-solving among undergraduate mathematics education students

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Abstract

Planning is one of the crucial components of metacognitive regulation. However, metacognitive planning is less studied empirically. This case study aims to explore the metacognitive planning activities of students when collaboratively solving mathematical problems. Proof problems in geometry were given to eight groups willing to participate in the research. Each group consisted of two undergraduate mathematics education students. Group discussion activities in solving problems were recorded using video-audio recorders. Interviews were also conducted with the groups to obtain more data on the metacognitive planning activities, thus achieving the research objectives. This study identified three different characteristics of metacognitive planning. We labelled these three planning characteristics as high, middle, and low levels of metacognitive planning. The low-level planning entails the formulation of a single problem-solving plan. Middle-level planning involves the formulation of two problem-solving plans, albeit the selection of the appropriate plan occurs through trial and error. Conversely, the formulation of more than two problem-solving plans and the ability to select the most effective plan characterize high-level planning. These findings can be utilized by educators to assess the efficacy of their students' metacognitive planning activities as a learning outcome.

Keywords: Collaboration, Metacognition, Metacognitive Regulation, Planning

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Mathematics is closely related to problem-solving. Problem-solving occurs when someone does not know how to perform a task with regular or routine procedures (Rott et al., 2021; Salminen-Saari et al., 2021). An individual with problem-solving abilities is a confident, creative, and independent thinker (Özreçberoğlu & Çağanağa, 2018). The ability to examine information from mathematical problems, determine problem-solving methods, and review solutions can only be developed through frequent practice with solving mathematical problems (Srimuliati & Wahyuni, 2020). Therefore, the ability to solve mathematical problems is an important aspect that should be possessed by both students and also university students in learning mathematics (Powell et al., 2019).

Planning plays a crucial role in the stages of mathematical problem-solving that guide students towards successful problem resolution. Planning involves the ability to identify a series of steps necessary to solve a problem (Li et al., 2015). Research indicates differences between experienced and novice problem solvers. Novice problem solvers spend more time doing than thinking or planning (Cirillo & Hummer, 2021; Rocha & Babo, 2024). This suggests that training students to plan more effectively will make them experienced problem solvers.

The term 'planning' is not only present in the problem-solving stage. In metacognitive regulation activities, planning is one of the sub-components alongside orientation, monitoring, and evaluation (De Backer et al., 2022; Jamil, Siswono, & Setianingsih, 2023a). Planning is an important component of

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metacognitive regulation (Castillo-Diaz et al., 2022; Escorcia & Gimenes, 2020; He et al., 2022; Roberts, 2021). It involves the ability to set goals, organize tasks, and allocate resources effectively in order to achieve desired outcomes (Sugiharto et al., 2017). Planning is a high-level mental process where individuals establish short-term or long-term goals and choose strategies to achieve them (Li et al., 2015). In this study, we use the term 'metacognitive planning' to emphasize that planning we refer to is part of metacognitive regulation. The ability to plan also helps individuals prioritize their learning goals and make informed decisions about which strategies to use in order to achieve those goals. Overall, planning is a fundamental skill within metacognitive regulation that supports successful learning and academic achievement.

Currently, mathematics learning is not only focused on individual activities but students' mathematics learning outcomes are obtained from social activities such as interacting with peers (Fatmanissa et al., 2025). This activity is supported by collaborative problem-solving. OECD highlighted collaborative problem-solving as an important skill for students in the twenty-first century. Additionally, research on metacognition indicates a consensus that metacognitive activities can be examined not only at the individual level but also at the interpersonal level, such as in collaborative problem-solving (De Backer et al., 2022; Liskala et al., 2021; Jamil, Siswono, & Setianingsih, 2023a, 2023b). Therefore, this study focuses on the perspective of collaborative problem-solving in mathematics.

The importance of metacognitive planning is not accompanied by in-depth research on the specific characteristics of planning that emerge in undergraduate students when solving problems collaboratively. Previous research has indicated that metacognitive regulation develops over time, yet we lack understanding of crucial stages (such as how planning activities are conducted) that occur specifically in undergraduate students (Stanton et al., 2015). Therefore, this study aims to explore the characteristics of metacognitive planning generated by groups when solving mathematical problems collaboratively. By addressing this aim, this study is hoped to bring novel understanding on effective process in planning stage of collaborative problem solving. The findings of this research can indicate how effective characteristics emerge when groups formulate problem-solving plans as a result of their metacognitive regulation.

THEORETICAL FRAMEWORK

In the realm of intelligence, Luria's model of the functional units of the brain elucidates the existence of three main functional blocks representing the fundamental functions of the brain (Téllez & Sánchez, 2016). The first block is responsible for arousal and attention. The second block is responsible for analysis, encoding, and storage of information. Meanwhile, the third block is responsible for the formulation and execution of plans (Li et al., 2015; Téllez & Sánchez, 2016). Consistent with this, two studies have developed tests to measure intelligence, namely the Kaufman Assessment Battery for Children (Drozdick et al., 2018) and the Cognitive Assessment System (Naglieri & Kaufman, 2001). Both tests indicate that planning is prioritized as one of the core components of intelligence.

There are various definitions of planning based on several perspectives. From the perspective of intelligence theory, planning is defined as a mental process in which individuals determine, select, apply, and evaluate problem-solving solutions (Naglieri & Kaufman, 2001). In the problem-solving perspective, planning becomes the second sub-component of the problem-solving stages by (Bell & Polya, 1945). The three subsequent components are understanding the problem, carrying out a plan, and looking back (Rott et al., 2021). Planning from a problem-solving perspective is defined as the activity of identifying a series of steps necessary to solve a problem. Meanwhile, from a metacognitive perspective, planning is one of the important sub-components of metacognitive regulation. The other three components are orientation, monitoring, and evaluation (De Backer et al., 2022; Liskala et al., 2021; Jamil, Siswono, & Setianingsih, 2023a).

Based on these explanations, the definition of planning in different perspectives appears similar. However, following the cognitive-metacognitive framework, planning activities in this research will be differentiated. At the cognitive level, planning occurs when students determine problem-solving plans. However, at the metacognitive level, planning is done by considering why such problem-solving plans are determined and how to choose effective problem-solving plans. This is consistent with (Nelson, 1997), who defines object level and meta level. Cognitive activities are activities related to task content (object level), and metacognitive activities are activities related to controlling and monitoring cognitive activities (meta level). Thus, metacognitive planning in this study is defined as the process of formulating various problem-solving alternatives and selecting problem-solving plans deemed effective.

METHODS

This study presents a case study of undergraduate students collaborating to solve mathematical problems. The students worked in pairs to solve proof-based geometry problems. Participants in this study were mathematics education students at one university in Malang, Indonesia. Students who expressed interest in participating in the study were required to sign an informed consent form. The form included information regarding: a) data collection procedures, b) the use of students' data, and c) their right to withdraw from the study at any point without consequences. Out of 30 students in one class, 16 signed the form, indicating their willingness to participate. These 16 students were then asked to choose a partner from among themselves with whom they could engage in discussions. As these 16 students were in the same geometry class, they had suitable peers for collaboration. Thus, the subjects of this study comprised 8 groups of students.

Each group was invited on different days to solve two geometry proof problems. The problems provided to the groups were as follows:

1. Given trapezoid $RSTV$ with $\overleftrightarrow{RS} \parallel \overleftrightarrow{VT}$ and $\angle V \cong \angle T$. Prove that $RSTV$ is an isosceles trapezoid.
2. Given triangle ABC with the mid points of $\overline{AB}$ and $\overline{AC}$ are M and N. Prove that $\overline{MN} \parallel \overline{BC}$.

Students were encouraged to discuss with their partners to collaboratively solve the problems. Their discussion activities were recorded using video-audio recording. After completing the problems, the researcher interviewed each group to obtain deeper insights into the problem-solving strategies planned and discussed by the groups.

The three stages of data analysis in this study are data condensation, data display, and drawing & verifying conclusions (Miles et al., 2014). Condensation data involves the process of selection, establishing data focus, simplifying data presentation, and transforming transcripts and other empirical findings. The research data consists of video recordings of group conversations in collaboratively solving problems and researcher interviews with the group transcribed into written form. Prior to transcription, researchers review the videos and study the group's work to gain an initial overview of the data as a whole. This initial overview provides researchers with a basis for composing sentences in the data transcript, sorting through data, creating data codes, and presenting transcript data so that relevant phenomena are easily discerned. We assign codes to the data indicating the formulation of planning to solve problems and how the group selects the plan they deem most appropriate. Only relevant group conversation and interview data are presented.

RESULTS

The eight groups participating in the study are detailed in Table 1. Based on the groups' responses, we obtained five types of problem-solving outcomes. We classify them as 1) correct, 2) correct and quite complete, 3) correct and incomplete, 4) incorrect, and 5) no result. The conclusion of 'correct' is assigned if the group can successfully prove the statement by using appropriate mathematical propositions such as postulates, definitions, theorems, etc., in geometry and providing complete steps. If the proof is correct with the appropriate mathematical propositions but there are minor errors in symbol writing, we label it as 'correct and quite complete.' The conclusion of 'correct and incomplete' is assigned if the group proves correctly but misses some proof steps. Groups that are incorrect in proving the problem will be labelled 'incorrect.' Meanwhile, 'no result' is assigned to groups that do not write any answer at all.

Table 1. The Subjects and Their Problem-Solving Result

Group	Students' Initial	Result of Problem-Solving	
		First Problem	Second Problem
1.	FE dan EN	Correct and quite complete	Correct but incomplete
2.	SP dan IS	Correct but incomplete	Correct and quite complete
3.	QA dan AZ	Correct	Incorrect
4.	AC dan DA	Incorrect	Correct and quite complete
5.	AM dan RF	Incorrect	Correct
6.	ENF dan UA	Correct but incomplete	Correct
7.	DE dan VB	Incorrect	Incorrect
8.	HE dan FN	No result	No result

Based on the analysis of group conversation transcripts, we identified various metacognitive planning activities. From the eight groups, we found three distinct characteristics that groups exhibited in planning problem-solving. Groups 1 and Group 2 shared similar characteristics, which we later referred to as a high-level of planning. The metacognitive planning characteristics of Group 3 and Group 4 were similar, which we termed as middle-level of metacognitive planning. Meanwhile, the characteristics we referred to as a low level of metacognitive planning were observed in Group 5, 6, and 7. Group 8 could not be determined for planning activities as they did not formulate any problem-solving plans at all. Group 8 lacked prior knowledge to prove the two given problems. The findings of these three characteristic metacognitive planning activities will be detailed in the following subsection.

The First Characteristic (A High-Level of Metacognitive Planning)

The first planning characteristic we identified was found in Group 1 and Group 2, which we termed A High-Level of Metacognitive Planning. We will present a more detailed analysis of the findings for Group 1, while for Group 2, we will directly document the findings we obtained. We believe that the detailed explanation of Group 1 sufficiently represents the research findings in this subsection because Group 1 and Group 2 exhibit similar planning characteristics. We provide labels (e.g., **[a]**, **[b]**, **[c]**, etc.) for the students' statements to facilitate understanding of the explanation. Additionally, we highlight (bold) keywords indicating the strategies formulated by the group.

In solving problem number 1, the planning activity of Group 1 commenced with FE suggesting a problem-solving strategy using the approach of two congruent triangles. Initially, FE intended to prove the congruence of the triangles using the Sides-Angles-Sides (SAS) postulate **[a]**. However, they encountered a difficulty **[b]**, prompting EN to suggest revisiting the literature in their textbook **[c]**. Upon revisiting the methods for proving congruent triangles, Group 1 discovered four methods **[d]**, and EN recognized that the most effective method to use was the Angle-Angle-Side (AAS) postulate **[e]**. Therefore, the group altered their strategy to use the AAS postulate. Knowing which method was more effective indicates that the group understood the relationship between one method and another. The following is an excerpt from the conversation of Group 1 while planning the solution to problem number 1.

Conversation Excerpt 1

FE : "Yes, so if we draw auxiliary lines, let's say RY and SX, then two triangles are formed. We will prove that these two triangles are congruent. From there, we can use the **Sides-Angles-Sides Postulate**" **[a]**

EN : "Okay"

FE : "Next, RS and VT sides are known to be parallel, and angle V is congruent to angle T. We assume X and Y as perpendiculars from RY to VT and SX to VT, respectively. We will prove triangle RYV congruent to triangle SXT. What's next?" **[b]**

EN : "Let's check the book first" **[c]**

(*EN and FE both consult their lecture reference books*)

FE : "Okay, in this book, there are several ways to prove two triangles congruent: sides-angle-sides, angle-side-angle, and also angle-angle-side." **[d]**

EN : "Oh yeah, then we can use the Angles-Angles-Sides Postulate" **[e]**

In solving problem number 2, Group 1's planning activity was evident when FE formulated a strategy to prove MN parallel to BC using a similar triangles approach **[f]**. EN devised another strategy to prove the problem by using a transversal line **[g]**. However, they couldn't establish the relationship of that strategy to prove parallelism **[h]**, and ultimately, they adopted another strategy formulated by FE. This third strategy involved using a parallelogram **[i]**, which successfully proved the problem. This can be observed in the following Conversation Excerpt 2. However, the researcher could not yet ascertain how the group selected their strategies or how they understood the relationships among them to decide on the most effective strategy. Therefore, the researcher conducted an interview with Group 1 regarding how they formulated problem-solving strategies for question number 2. The results of the problem-based interview on Group 1 regarding problem number 2 indicate that they were able to explain the strengths and weaknesses of the strategies they formulated. FE could articulate correctly and utilize appropriate postulates and theorems to prove the problem. The previous strategy was deemed incorrect, and the third strategy was found to be effective. This demonstrates that the group understood the interrelationship between the various strategies.

Conversation Excerpt 2

FE : "We will show that **triangles AMN and ABC are similar**. And we don't know if these triangles are isosceles, or equilateral, or what" **[f]**

EN : "Yeah, we don't know what kind of triangles these are"

FE : "If they are similar, can we show they are parallel?"

FE : "But the problem asks for parallel lines"
EN : "How about using a **transversal line**? So, there will be corresponding congruent angles" **[g]**
FE : "Let's try it. We have less than 13 minutes left"
EN : "Now, we can see that the three corresponding angles of the two triangles (AMN and ABC) are congruent"
FE : "So, the conclusion is that the two triangles are similar. Now, how do we go from similarity to parallelism? The problem asks to prove MN parallel to BC" **[h]**
FE : "Yeah, that's wrong. MN should be parallel to BC first, then we can conclude that angle AMN is congruent to angle ABC. Let's try another approach. Let's draw auxiliary lines. Approach it using a **parallelogram**. Let's draw a line through C parallel to AB" **[i]**

In the explanations above, Group 1 was able to plan two problem-solving strategies (using the SAS postulate and the AAS postulate) for problem number 1, and three problem-solving strategies (similar triangles approach, transversal line, and parallelogram approach) for problem number 2. In addition to formulating two or more problem-solving plans, Group 1 also recognized the strengths and weaknesses of each of their formulated problem-solving plans. Thus, Group 1 could select the most effective problem-solving plan, resulting in their answers being correct.

Similar to Group 1, Group 2 was able to formulate more than two problem-solving strategies for both problem number 1 and problem number 2. For problem number 1, Group 2 formulated four problem-solving strategies: 1) using two perpendicular auxiliary lines passing through points V and T perpendicular to RS, 2) using an auxiliary line forming a trapezoid diagonal to show triangle SVR congruent to triangle RST, 3) proving triangle RVT congruent to triangle STV, 4) using an auxiliary line perpendicular to VT through points R and S. For problem number 2, Group 2 could formulate three problem-solving strategies. These three problem-solving strategies include: 1) showing triangle AMN similar to ABC, 2) showing MN never intersects BC, 3) using a parallelogram approach. Like Group 1, Group 2 could identify the strengths and weaknesses of each of their problem-solving plans. They were aware of which strategies were more effective in obtaining the correct answers.

The Second Characteristic (A Middle-Level of Metacognitive Planning)

The second planning characteristic was found in Group 3 and Group 4, which we termed A Middle-Level of Planning. Similar to the discussion of the first characteristic, we will provide a more detailed explanation of the research findings in Group 3 rather than Group 4. We believe that the detailed explanation of Group 3 adequately represents the research findings in this subsection. This is because the findings in Group 3 and Group 4 are similar.

In solving problem number 1, Group 3 formulated two problem-solving strategies: using the Sides-Angles-Sides (SAS) Postulate **[j]** and the Angles-Sides-Angles (ASA) Postulate **[k]**. Then, Group 3 selected one alternative problem-solving plan. The selection was made merely by trial and error **[l]**; **[m]**. Group 3 did not truly know which option was the most effective to apply in solving the problem. This occurred because Group 3 could not link one plan to another as a determinant of the best problem-solving strategy. This explanation can be observed in the Conversation Excerpt 3.

Conversation Excerpt 3
QA : "Yes, that's correct. Let's draw auxiliary lines."
QA : "Yes, let's say we use auxiliary lines, can we approach it with the **angles-sides-angles postulate**? Ah, it doesn't work" **[j]**

AZ : "What about using the **sides-angles-sides postulate**?" **[k]**
QA : "Which one? How do we prove it?"
(They try to consult their lecture reference book and discuss one of the proof example problems related to two congruent triangles)
QA : "How about we use angles-sides-angles, namely angle T, side YT, angle SYT will be the same as angle V, side XV, angle XVR successively?" **[l]**
AZ : "Okay, let's try" **[m]**

Group 3's planning activity regarding the second problem began with planning two alternative problem-solving strategies. The first alternative was when QA suggested using ratios **[n]** to prove the similarity of triangles **[o]**. Group 3 was unsure about how to proceed with the strategy of proving two similar triangles **[p]**. After trying various problem-solving strategies through trial and error, they reverted to their original plan. This is evident when AZ suggested using an auxiliary line parallel to AB using a parallelogram approach **[q]**. Although in the end, Group 3 returned to using the strategy involving similar triangles. In planning this strategy, the group formulated two problem-solving strategies. However, the selection of the problem-solving strategy was made through trial and error without them knowing the strengths and weaknesses of the strategies they had planned. The selection of the problem-solving strategy was not based on an analysis of effectiveness but solely on trial and error.

Conversation Excerpt 4
QA : "Will we use ratios later on?" **[n]**
AZ : "It seems like it"
QA : "Let's check our notes and the book"
(AZ and QA together review their lecture notes and reference book)
QA : "We need to prove these **two triangles are similar**" (pointing to triangles AMN and ABC) **[o]**
QA : "First, we have to prove that these two triangles are similar, then hmmm… I don't know what to do next" **[p]**
AZ : "How should we prove this? Let's draw an auxiliary line to form a **parallelogram**" (*drawing an auxiliary line parallel to line AB through point C*) **[q]**

Based on the explanation above, when planning the problem-solving process, Group 3 formulated two problem-solving strategies for both problem number 1 and problem number 2. For problem number 1, Group 3 devised problem-solving plans including 1) using the SAS Postulate and 2) using the ASA Postulate. While for problem number 2, their two formulated problem-solving plans were 1) using the similar triangles approach and 2) using the parallelogram approach. Group 3 chose trial and error as their problem-solving strategy. They could not determine which strategy was better and more effective. Their proofs resulted in being correct for problem number 1 but incorrect for problem number 2.

Similar to Group 3, Group 4 formulated two problem-solving strategies for both problem number 1 and problem number 2. For problem number 1, Group 4 planned strategies using: 1) auxiliary lines within the trapezoid and 2) auxiliary lines outside the trapezoid. While for problem number 2, their two problem-solving strategies were 1) similar triangles approach and 2) parallelogram approach. Initially, we could not ascertain how Group 4 chose between the two formulated plans solely from the group discussion outcomes. Therefore, we proceeded with interviews for both problem number 1 and problem number 2 with Group 4. The interview results with Group 4 showed that their selection of problem-solving strategy plans was done through trial and error. Group 4 could not explain why one selected plan was better than the other. Group 4's proof was incorrect for problem number 1, whereas for problem number 2, it was correct.

The Third Characteristic (A Low-Level of Metacognitive Planning)

The third planning characteristic we identified; we named A Low-Level of Planning. This characteristic was observed in Group 5, Group 6, and Group 7. Similar to the explanations of the previous two characteristics, we provide detailed explanations for one representative group, which here is Group 5. The research findings for Group 6 and 7 are explained briefly but still clearly.

Group 5 formulated a single problem-solving plan, which was to prove the congruence of triangles RMV and SNT **[r]** to establish that the trapezoid RSTV was an isosceles trapezoid **[s]**. No other strategies were formulated by the group. Group 5 did not engage in considerations or analyses of problem-solving strategy plans. It appears that they were confident in using this single plan, despite the proof they produced being incorrect. The problem-solving activity of Group 5 can be observed in Conversation Excerpt 5.

Conversation Excerpt 5

AM : "RSTV. And this angle V is congruent to angle T. Prove that RSTV is an isosceles trapezoid. This angle is known (*while indicating angle marks on the diagram of trapezoid RSTV*) then we can draw auxiliary lines (*drawing a perpendicular line through points R and S as shown below*). Let's name it M and N. So, this angle is ninety degrees and this one too (*marking right angles on the diagram below*)."

AM : "So, we write the proof first. Given RS is parallel to VT then angle T is congruent to angle V. Then, we draw auxiliary lines RM perpendicular to VT and SN perpendicular to VT. Let's try to write the answer first." (*RF writes the answer as instructed by AM on their scratch paper*)

AM : "Let's check the trapezoid material in the book."

AM : "Okay, because RM is perpendicular to SN, then…"

RF : "then RM is congruent to SN."

AM : "Yes."

RF : "**Triangle RMV is congruent to triangle SNT**" **[r]**

AM : "So, we can directly conclude that this trapezoid is isosceles, right?" **[s]**

RF : "Yes."

AM : "Okay, let's write it down completely on the answer sheet."

In working on problem number 2, Group 5 formulated a proof plan using two triangles, namely AMN and ACB **[t]**. However, this approach was incorrect as ANM and ACB should be similar, not congruent. Additionally, it is conceptually wrong to prove parallelism by using triangle congruence, and the two triangles mentioned are actually not congruent. The problem-solving strategy devised by Group 5 was to show that triangle ANM is similar to ACB, which would result in corresponding side ratios being equal **[u]**. Furthermore, Group 5 utilized theorems as expressed in AM's statement **[v]**. Therefore, Group 5 only formulated a single problem-solving plan. Group 5 did not formulate any other problem-solving plans as alternatives.

Conversation Excerpt 6

AM : "What if we use triangles AMN and ACB?"

AM : "Here are triangles ANM and ACB. Oh, I see… We can make AC congruent to AN and AB congruent to AM."

AM : "So, we'll prove triangles **ANM and ACB congruent**. Then we'll obtain MN parallel to BC. From triangles…"**[t]**

AM : "Because triangles ANM and ACB are similar, AN divided by NC equals AM divided by MB. So, NM is parallel to CB." **[u]**

RF : "BC or CB?"

AM : "It's the same."

RF : "Okay."

AM : "If a line intersects two sides of a triangle and divides these sides in the same ratio, then the line must be parallel to the third side of the triangle. Well, that third side is the base, right?" **[v]**

We observed characteristics of planning activities carried out by Group 5, both in question 1 and question 2. They consistently formulated only one problem-solving plan. Group 5 appeared confident and promptly implemented the strategy they devised. We did not observe any consideration from Group 5 to use or formulate alternative planning strategies. Based on the group discussion results, there were indeed some statements from Group 5 indicating conceptual errors in proving question 2, but the final answer written by the group was correct. This was because Group 5 immediately mentioned one theorem as the basis for their proof, and this theorem was appropriately used in proving question 2.

Similar to the findings in Group 5, Groups 6 and 7 also exhibited the same characteristics in planning activities. In question 1, Group 6 planned a single problem-solving strategy, which was to use the approach of two congruent triangles. Meanwhile, in question 2, Group 6 proved using the theorem about the midpoint of a triangle. Based on interviews with Group 6, our conclusion was verified that they indeed did not formulate any other research plans. However, there was an interesting finding in Group 6. The answers they provided for both question 1 and question 2 were correct. Although the proof for question 1 is incomplete and their proof for question 2 only applies the theorem they obtained without questioning whether the theorem is valid or not. And this only happens in Group 6. Group 5 is correct for only one question, while Group 7 is incorrect for both questions.

In Group 7, they planned a single problem-solving strategy for question 1, which was to use two congruent triangles to demonstrate an isosceles trapezoid. Although their plan seemed correct, the basis for proving two congruent triangles (using corollary angles-angles) was incorrect. The use of corollary AA is to prove two similar triangles. Similar to question 1, in question 2, Group 7 used the approach of two similar triangles. In both question 1 and question 2, Group 7 did not formulate any other problem-solving plans. They only discussed one problem-solving plan and then implemented it to prove the questions.

DISCUSSIONS

We identified three distinct characteristics of metacognitive planning activities of undergraduate students when collaboratively solving proof problems. We present the summary of these three planning characteristics in Table 2. We observed differences in the quality of planning produced by seven groups. We hypothesize that effective planning occurs when students are aware of the effectiveness of the problem-solving strategies they formulate, enabling them to select the appropriate plan to answer the questions. This is further supported by the problem-solving outcomes of each group. The groups we refer to as having 'high-level of metacognitive planning' characteristics are able to produce correct proofs. Additionally, they formulate and consider two or more problem-solving plans. Research indicates that the application of planning is a key metacognitive regulation technique for elementary school students, and this ability continues to develop with age (MacKewn et al., 2022; Young & Fry, 2008). Richer data would certainly be obtained from subjects who are adolescents or adults. Therefore, this study was conducted on undergraduate students.

Metacognitive regulation can be studied not only at the individual level but also in a collaborative context. Another study examines metacognitive regulation at various levels including individual, social, and environmental levels associated with collaborative problem-solving (Çini et al., 2023). This research suggests that the collaborative context is a rich source for supporting metacognitive awareness through both the individuals themselves and their interactions with group members. Research on differences in metacognitive regulation among biology students found that almost all students have different approaches to learning and show differences in monitoring, evaluating, and planning their learning strategies (Stanton et al., 2015). However, the study did not further examine what these differences entail. In this study, the differences of students' planning process were explored and described. This current study seeks to explore the components of planning. Other research also demonstrates the positive effects of two metacognitive regulation components, namely planning and monitoring, on the accuracy of argumentative writing by undergraduate students (Panahandeh & Asl, 2014). However, the study also does not conduct further analysis on the planning and monitoring activities carried out by undergraduate students.

This study provides recommendations for educators to offer open-ended problems that allow students to consider more than one problem-solving alternative. This can delve deeper into students' metacognitive regulation, particularly in the planning component. This is consistent with research findings indicating that open-ended problems can trigger students' metacognitive regulation (Jamil, Siswono, Setianingsih, et al., 2023c).

Table 2. Summary of Metacognitive Planning Characteristics of Groups in Collaborative Problem-Solving

Group	Finding	Conclusion (The Characteristic of Metacognitive Planning)	
Group 1	• Formulating two problem-solving strategies for the first problem - using the SAS Postulate - using AAS Postulate • Formulating three problem-solving Strategies for the second problem - similar triangles approach - transversal line - parallelogram approach • Selecting problem-solving plans based on awareness of the strengths and weaknesses of each formulated plan.	The group can formulate two or more problem-solving plans. The selection of the problem-solving plan is based on their knowledge of the strengths and weaknesses of each formulated plan. The group is aware of the effectiveness of each of its	A High-Level of Metacognitive Planning
Group 2	• Formulating four problem-solving strategies for the first problem - using two perpendicular auxiliary lines passing through points V and T perpendicular to RS		

Group	Finding	Conclusion (The Characteristic of Metacognitive Planning)	
	- using an auxiliary line forming a trapezoid diagonal to show triangle SVR congruent to triangle RST - proving triangle RVT congruent to triangle STV - using an auxiliary line perpendicular to VT through points R and S • Formulating three problem-solving strategies for the second problem - showing triangle AMN similar to ABC - showing MN never intersects BC - using a parallelogram approach • Selecting problem-solving plans based on awareness of the strengths and weaknesses of each formulated plan.	problem-solving plans.	
Group 3	• Formulating two problem-solving strategies for the first problem - using SAS Postulate - using ASA Postulate • Formulating two problem-solving strategies for the second problem - using the similar triangles approach - using the parallelogram approach • Selecting problem-solving plan by trial and error	The group can formulate two problem-solving plans and choose one solution plan based on trial and error. The group cannot ascertain which strategy is more effective.	A Middle-Level of Metacognitive Planning
Group 4	• Formulating two problem-solving strategies for the first problem - auxiliary lines within the trapezoid - auxiliary lines outside the trapezoid • Formulating two problem-solving strategies for the second problem - similar triangles approach and - parallelogram approach • Selecting problem-solving plan by trial and error		
Group 5	• Formulating one problem-solving strategy for the first problem - showing triangle RMV congruent to SNT • Formulating one problem-solving strategy for the second problem - showing triangles ANM and ACB are congruent	The group only formulates a single problem-solving plan. They immediately implement the problem-solving plan they have determined.	A Low-Level of Metacognitive Planning
Group 6	• Formulating one problem-solving strategy for the first problem - the approach of two congruent triangles • Formulating one problem-solving strategy for the second problem - using the theorem about the midpoint of a triangle		
Group 7	• Formulating one problem-solving strategy for the first problem - using two congruent triangles • Formulating one problem-solving strategy for the second problem - using the approach of two similar triangles		

CONCLUSION

This study explores the planning component in undergraduate students' metacognitive regulation when solving problems collaboratively. To understand the characteristics of planning performed by groups, the researchers presented proof problems in geometry, which could be solved in more than one strategy. We identified three different planning characteristics among the seven groups that solved the proof problems, although the proof results may not necessarily be correct. The first characteristic indicates good planning activity, where groups formulated two or more problem-solving plans and could determine which strategy was more appropriate to implement. The second characteristic shows planning by groups involved formulating two problem-solving plans. Groups selected a strategy to implement in solving the problem through trial and error. The third characteristic indicates groups only formulated a single problem-solving plan. We recommended an encouragement for students to build two or more problem-solving plans to stimulate a better problem-solving process.

Although this study contributes information on the differences in metacognitive planning activities, it is limited to cases where students collaborate on problems. Despite collaboration being an important skill in the 21st century, delving into planning activities when students work individually is also necessary, as it can refer to individual students' abilities. Additionally, this study is limited to seven groups as research subjects. We speculate that further research on larger groups will yield different planning characteristic information from what we found. Because this study only focuses on one component of metacognitive regulation, further research would be interesting to explore how other components are performed by students.

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