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Fraction Multiplication: Analysis of Student Representation Errors Based on APOS Mental Mechanisms

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ABSTRACT

Most students find fraction material difficult because fractions are rarely encountered in life. Representing fractions in various forms is a problem for students. This research aims to reveal errors in the representation of multiplication of fractions by students at one of the junior high schools in Singosari. The researcher used a qualitative descriptive method with 32 students as potential subjects and 5 students were selected to be interviewed. Triangulation techniques are used by comparing test results, interviews and field notes. Research findings show that the two biggest errors occur in the representation of pictures and language. At the APOS mental mechanism stage, it was found that three out of five subjects made mistakes at the coordination stage. Further research is recommended to use media that can help students understand multiplication of fractions.

Keywords**:** *multiplication of fractions, representation, mental mechanisms*

Perkalian Pecahan: Analisis Kesalahan Representasi Siswa Berdasarkan Mekanisme Mental APOS

ABSTRAK

Sebagian besar siswa menganggap materi pecahan sulit karena pecahan jarang ditemui dalam kehidupan. Representasi pecahan ke berbagai bentuk menjadi masalah tersendiri bagi siswa. Penelitian ini bertujuan untuk mengungkap kesalahan representasi perkalian pecahan siswa pada salah satu SMP di Singosari. Peneliti menggunakan metode deskriptif kualitatif dengan calon subjek sebanyak 32 siswa dan didapatkan 5 siswa terpilih untuk diwawancara. Triangulasi teknik digunakan dengan membandingkan hasil tes, wawancara, dan catatan lapangan. Temuan penelitian menunjukkan dua kesalahan terbesar terjadi pada representasi gambar dan bahasa. Pada tahapan mekanisme mental APOS didapatkan tiga dari lima subjek salah pada tahap koordinasi. Penelitian lanjutan disarankan agar menggunakan media yang dapat memahamkan siswa tentang perkalian pecahan.

Kata Kunci**:** *perkalian pecahan, representasi, mekanisme mental*

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1. Introduction

Understanding fractions is important in everyday life. People who have a good understanding of fractions can do their jobs as carpenters, pharmacists and mechanics well (Murniasih et al., 2020; Fitzsimmons et al., 2020). Carpenters can estimate the amount of wood needed by dividing wood into sections to build houses and buildings. Pharmacists can mix medications according to the dosage required for the patient. Mechanics can cut the metal to make the frame into pieces. Fraction skills are so important in life, research regarding fractions needs to be done.

However, in reality, fractions are a difficult material because in everyday life it is rare to find objects in the form of fractions (Setyawati et al., 2019; Suwanti & Murniasih, 2022; Murniasih, 2021). This is in accordance with the opinion that fractions are a difficult concept so meaningful learning is needed by teachers to explain them (Rahaju & Hartono, 2017; Murniasih et al., 2020). For example, to explain the multiplication of the fraction 3/4 × 1/3 the teacher can give examples from everyday life. Ana has $\frac{3}{4}$ of a chocolate bar, she wants to share $\frac{3}{4}$ of the chocolate with 3 friends. How many chocolates did each friend get?

Researchers from various countries have examined students' representation errors in completing fraction multiplication. Students in Northern Colorado made errors in language representation and gave examples of problems in life when solving multiplication of fractions (Kang, 2022). Many students in Africa experience picture representation errors when multiplying fractions because they do not master the prerequisite material (Makhubele, 2021). Students in Turkiye make many symbol representation errors when completing fraction multiplication (Üzel, 2018). Some students in Saudi Arabia make linguistic representation errors because they assume that fractions are always less than one and thus experience misunderstandings when multiplying fractions by whole numbers (Alghazo & Alghazo, 2017). Students in Kosovo experience misrepresentation of language, pictures, and associations with everyday life when completing fraction multiplication (Kamberi et al., 2022). Based on the explanation above, this research aims to analyze students' errors in representing fraction multiplication based on the APOS (action, process, object, and schema) mental mechanism. This research is important so that teachers can design learning, which can minimize student errors in fraction representation.

2. Method

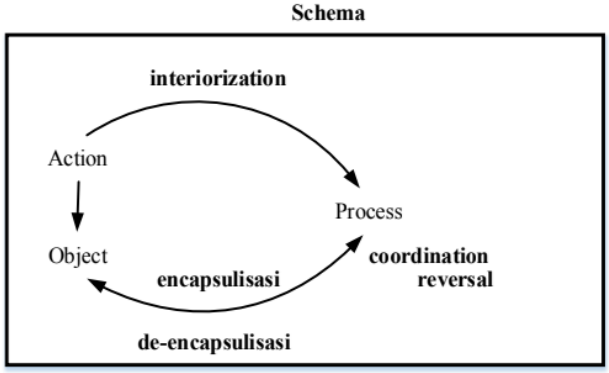
The qualitative descriptive method was chosen because the researcher wanted to describe the problem in detail (Shodikin et al., 2023; Wulandari et al., 2024; Anisa et al., 2024). Errors in representing students' multiplication of fractions were obtained from the results of students' work which was then analyzed. The prospective research subjects were 32 students at one of the junior high schools in Singosari. The reason for choosing the subject was because the subject had taken the material on multiplication of fractions. Subjects who answered correctly were not analyzed, while subjects who answered incorrectly were further grouped based on

representation error indicators. Researchers used several instruments, including: tests, field notes, and interview guides. Tests are used to measure students' ability to solve problems. There were 32 students who took the test. An interview guide was used to explore in depth errors in representing fraction multiplication. Selected subjects for interviews are based on good communication (Murniasih et al., 2020). Field notes are used to record things that have not been recorded through interviews or student work. Researchers conducted direct interviews with selected subjects. The questions for the test were adapted by researchers from Kang's research (Kang, 2022). Indicators of representation errors can be seen in Table 1.

Table 1. Representation Error Indicator (Kang, 2022; Kamberi et al., 2022).

Types of Representation Errors	Representation Error Indicator
R1	Students cannot represent fraction multiplication symbols
R2	Students cannot represent pictures of multiplication of fractions
R3	Students cannot represent multiplication of fractions with language
R4	Students cannot represent multiplication of fractions with everyday problems

APOS analysis is used by researchers because this model is suitable for revealing more depth in students' representations. The APOS stage was chosen by researchers because this stage is suitable for analyzing fraction multiplication (Susiswo et al., 2021).



Picture 1. APOS Mental Mechanism (Susiswo et al., 2021)

3. Result and Discussion

The test questions in this study can be seen as follows.

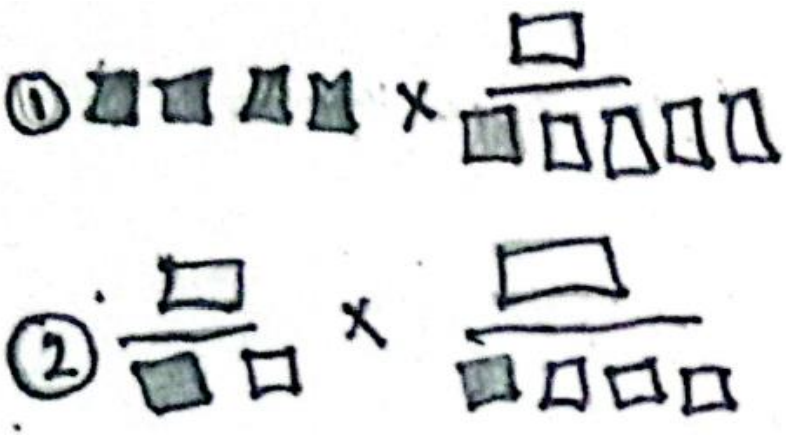
Tuliskan dengan simbol, gambar, bahasa, atau contoh dalam kehidupan sehari-hari untuk menjelaskan:

a. $4 \times \frac{1}{5}$
b. $\frac{1}{2} \times \frac{1}{4}$

Picture 2. Test Questions (adaptation Kang (2022))

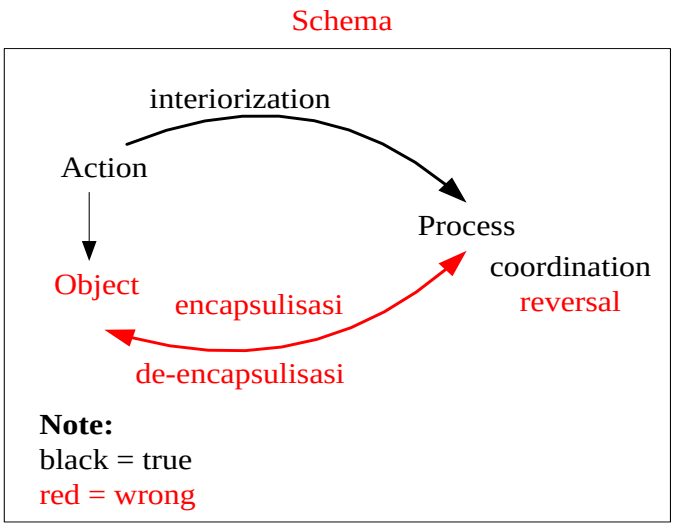
Based on the results of the test, it is analyzed according to the stages of the APOS mental mechanism. A total of 13 out of 32 students answered correctly. The remaining 19 out of 32 students answered incorrectly. The results of students' incorrect work were analyzed and grouped based on indicators of misrepresentation so that 5 subjects were obtained for interviews. Subjects experiencing misrepresentation are described as below.

3.1 GB Subject



Picture 3. GB's Answer

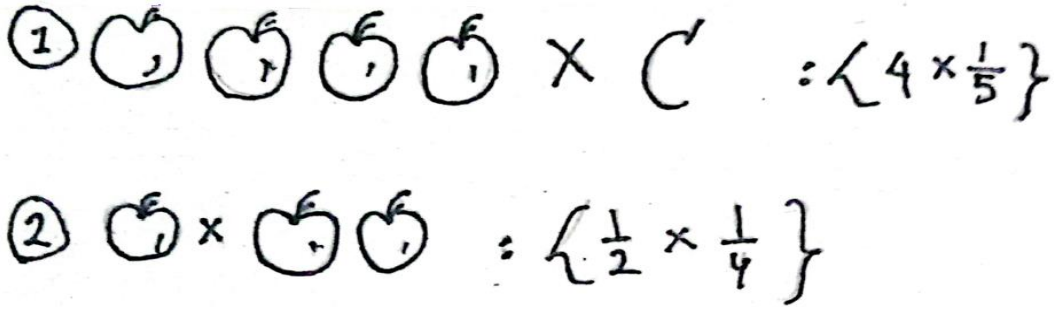
Based on the analysis using the APOS stage, GB students misrepresented the picture (type R2). Subject GB made a mistake at the reversal stage, namely connecting with previous knowledge when representing symbols in pictures (Picture 4). Subject GB was correct when drawing 4 on question number 1 but wrong when drawing $\frac{1}{5}$ on question number 1 and incorrectly drawing $\frac{1}{2}$ and $\frac{1}{4}$ on question 2. At the reversal stage Subject GB could not relate fractions to whole numbers (Susiswo et al., 2021). These results show that GB subjects do not understand that fractions are part of a whole. This is in line with the opinion of Mohamed et al. (2021) who say that well-mastered prerequisite knowledge about integers will make it easier for students to learn fractions with various representations.



Picture 4. GB Error at Reversal Stage

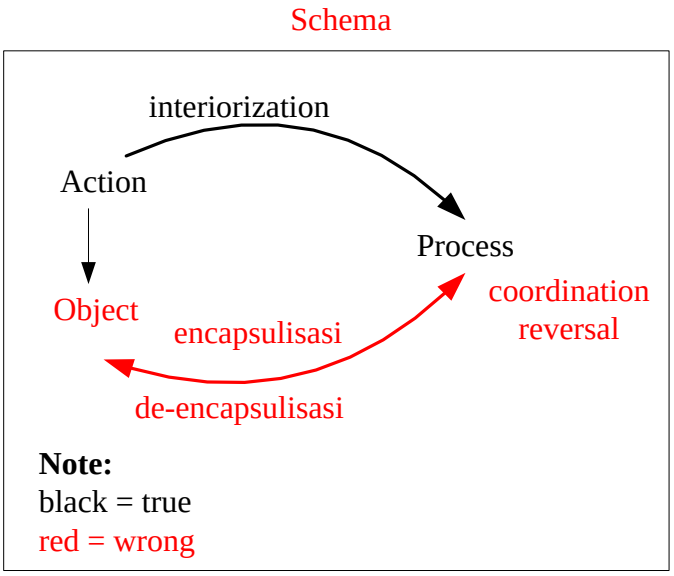
This result is reinforced by an interview that says, that subject GB experienced confusion in drawing $\frac{1}{5}, \frac{1}{2}$ and $\frac{1}{4}$. GB subjects do not understand that fractions are part of a whole (Purnomo et al., 2021).

3.2 SWP Subject



Picture 5. SWP's Answer

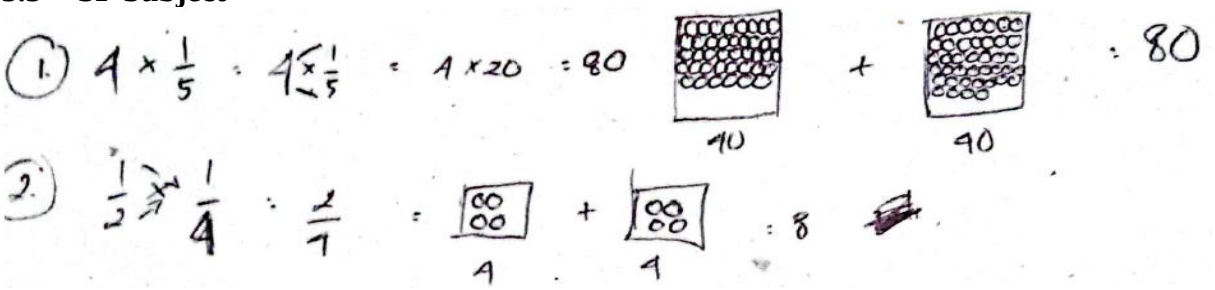
SWP subjects made picture representation errors (R2) at the coordination stage. In number 1, SWP misrepresents 4 $\times \frac{1}{5}$ with the picture of 4 apples $\times \frac{1}{2}$ apples. In question number 2, SWP misrepresents $\frac{1}{2} \times \frac{1}{4}$ with a picture of 1 apple × two apples. SWP cannot coordinate the fraction component in the problem. SWP misrepresent $\frac{1}{5}, \frac{1}{2}$, and $\frac{1}{4}$ with apples.



Picture 6. SWP Error at Coordination Stage

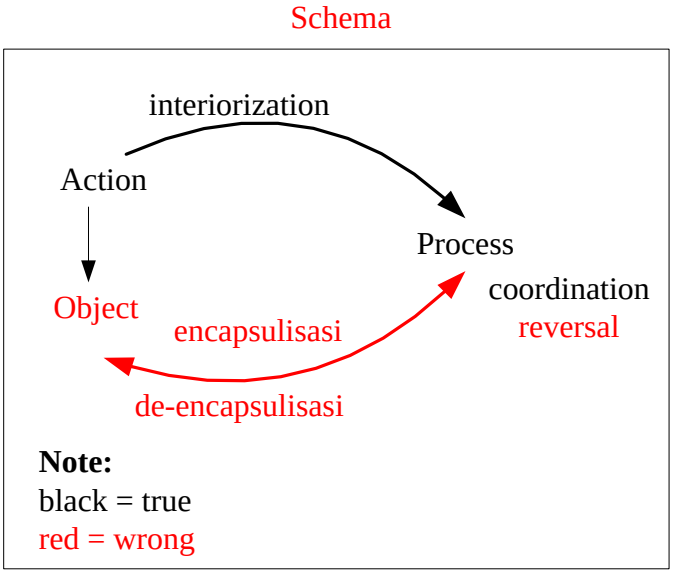
Based on the interview results, the SWP subject incorrectly coordinated the components in the questions, including that the representation of $\frac{1}{5}$ should be depicted with a fifth of an apple in question 1. Meanwhile, in question 2, the SWP subject incorrectly coordinated $\frac{1}{2}$ and $\frac{1}{4}$ by drawing 1 and 2 apples. This result is in accordance with the opinion of Ubah & Bansilal (2018) who said that students who successfully coordinate fraction components at the coordination stage will be able to complete fraction multiplication well.

3.3 SF Subject



Picture 7. SF's Answer

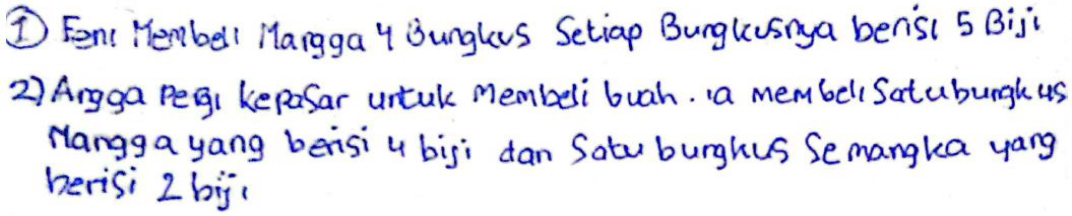
SF subjects made errors in representing symbols (R1) and pictures (R2) at the reversal stage. These results indicate the existence of overlapping representation errors in SF work (Aliustaoğlu et al., 2018).



Picture 8. SF Error at Reversal Stage

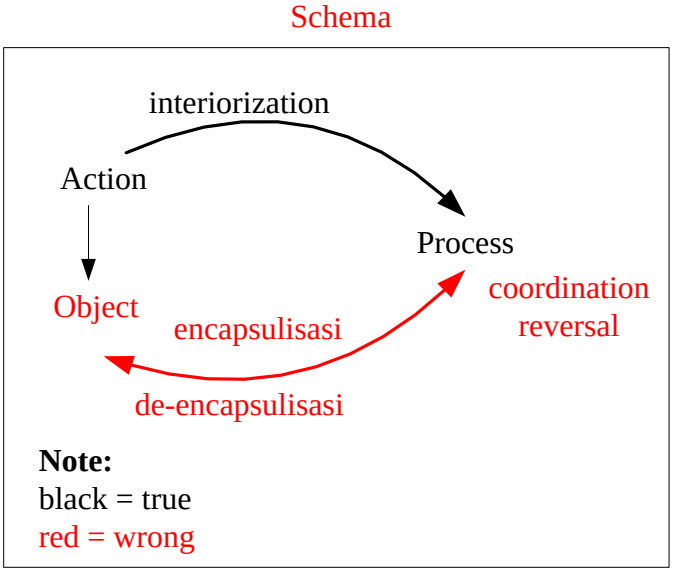
The interview results showed that at the reversal stage, SF Subjects misrepresented the symbols (R1) and pictures (R2) in both questions 1 and 2. SF Subjects multiplied fractions crosswise, namely the numerator and denominator and vice versa. The interview results also showed that Subject SF said that multiplication would produce something bigger. This applies to positive integers but SF gets confused when multiplying fractions. This is in line with research which states that most students think that multiplication will produce something bigger (Gibim et al., 2023).

3.4 MMA Subject



Picture 9. MMA's Answer

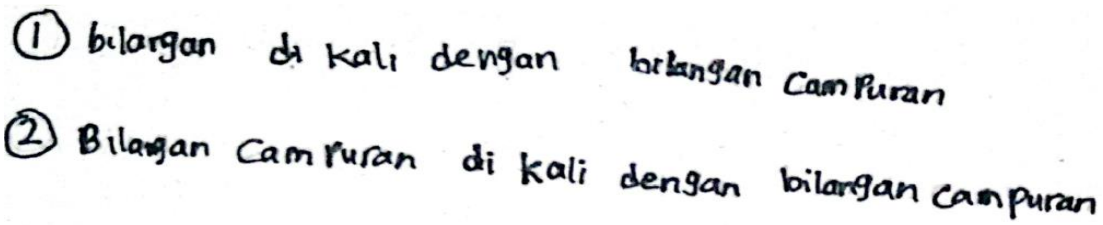
MMA subjects made errors in representing language (R3) and everyday problems (R4) at the coordination stage. In question number 1, MMA incorrectly represented $4 \times \frac{1}{5}$ with the representation of 4 packs of mangoes, each containing 5 mangoes. In question number 2, MMA misrepresented $\frac{1}{2} \times \frac{1}{4}$ with the representation of one pack of mangoes containing 4 fruits plus one pack of watermelon containing 2 fruits. These results also show that there is overlapping representation in MMA (Aliustaoğlu et al., 2018).



Picture 10. MMA Error at Coordination Stage

The interview results showed that MMA subjects represented whole numbers with the number of plastic bags and fractions with the contents of the plastic bags. These results indicate that MMA subjects do not understand that fractions are repeated additions in problem 1. Apart from that, MMA also does not understand that fractions are part of a whole (Kang, 2022).

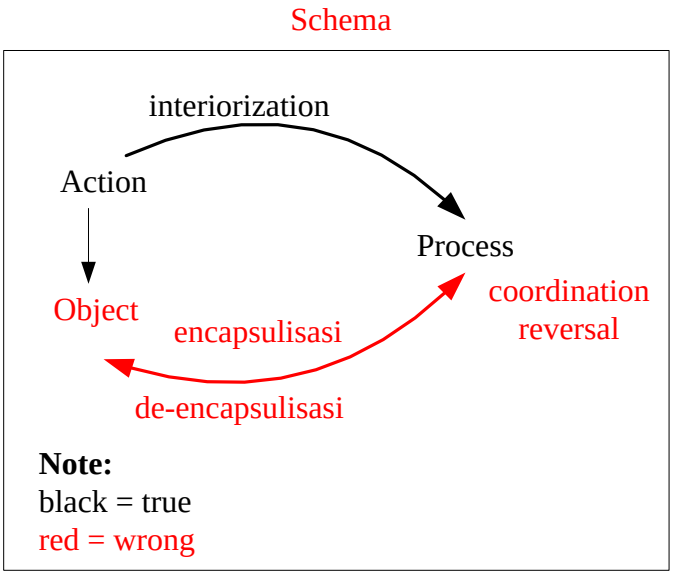
3.5 OA Subject



Picture 11. OA's Answer

OA subjects made language representation errors (R3). A good understanding of the language is necessary when representing fractions in various ways (Viseu et al., 2021).

The results of the interview showed that subject OA made a mistake at the coordination stage, namely representing the number 4 by saying "number" and representing fractions by saying "mixed fraction". Errors at the coordination stage cause OA students to make mistakes at the reversal stage, resulting in the wrong schema for representing fractions (Bintoro et al., 2021). Based on the explanation above, representation errors can be seen in Table 2.



Picture 12. OA Error at Coordination

Table 2. Subject Representation Errors Based on Mental Mechanism Stages

Subject	Type of Representation Error	Interiorization	Coordination	Reversal	Encapsulisasi	De-encapsulisasi
GB	R2	√	√	x	x	x
SWP	R2	√	x	x	x	x
SF	R1, R2	√	√	x	x	x
MMA	R3, R4	√	x	x	x	x
OA	R3	√	x	x	x	x

Note:
√ = right stage
X = wrong stage

Based on Table 2, the biggest error occurred in the picture representation made by 3 subjects, namely GB, SWP, and SF. The next biggest mistake was the language representation made by 2 subjects, namely MMA and OA. Meanwhile, according to the mental mechanism stages, 3 subjects made mistakes starting from the coordination stage, namely SWP, MMA, and OA. Based on the research results, educators should teach fraction representation using learning media so as to minimize student errors (Yurniwati & Yarmi, 2020).

4. Conclusion

The results of students' incorrect work were analyzed and grouped based on indicators of misrepresentation so that 5 were selected. Based on the data presentation and discussion, it can be concluded that the two biggest errors occur in the representation of pictures and language. Meanwhile, based on the stages of the APOS mental mechanism, it was found that three subjects made mistakes starting at the coordination stage and two other subjects made mistakes starting at the reversal stage. These results indicate that a good understanding of the prerequisite material is needed before students learn fractions. Further research is recommended to teach fraction multiplication using media that can minimize fraction representation errors.

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