

## Algebraic Thinking Profile of Junior High School Students with Reflective Cognitive Style in Solving Mathematics Problems

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### ABSTRACT

The differences in algebraic thinking when solving problems are determined by the characteristics of students. One of the distinguishing characteristics is cognitive style. The study aimed to describe the algebraic thinking profile of students who had reflective cognitive styles in solving mathematics problems. This study used descriptive qualitative research with a case study research design, focusing on 8<sup>th</sup> grade junior high school students with reflective cognitive style. Data were collected using tests and interviews. This study used three types of tests namely Matching Familiar Figure Test (MFFT) to determine students' cognitive style, Mathematics Ability Test (AMT) to measure students' mathematical abilities, and Problem-Solving Test (PST) to obtain data related to students' algebraic thinking profile in solving mathematics problems. Data were analyzed in three stages covering data reduction, data presentation, and conclusion drawing. The results showed that the algebraic thinking of students with reflective cognitive style in solving problems met the three aspects of algebraic thinking indicators namely performing activities to generalize the pattern and determine the next term of the given pattern, representing and comparing data in tabular form, and understanding the meaning of variables and use variables in the form of letters or symbols as a representation of something unknown value in algebraic form. Students with a reflective cognitive style in solving problems could understand the problems given well, be careful and thorough in writing the steps of completion, and straightforward and coherent in answering questions so that the answers given tend to be correct. Thus, the results of this study are expected to be one of the references for teachers or other researchers in developing mathematics learning by considering the cognitive style the students, especially reflective cognitive style.

**Keywords:** *Algebraic Thinking, Problem-Solving, Reflective Cognitive Style.*

## Profil Berpikir Aljabar Siswa SMP dengan Gaya Kognitif Reflektif dalam Menyelesaikan Masalah Matematika

### ABSTRAK

Berpikir aljabar siswa dalam memecahkan masalah berbeda-beda berdasarkan karakteristik yang dimiliki siswa. Salah satu karakteristik yang membedakan yaitu gaya kognitif. Tujuan penelitian ini yaitu mendeskripsikan profil berpikir aljabar siswa dengan

gaya kognitif reflektif dalam memecahkan masalah matematika. Penelitian ini merupakan penelitian deskriptif kualitatif dengan jenis penelitian studi kasus, dengan subjek penelitian adalah siswa kelas 8 SMP dengan gaya kognitif reflektif. Teknik pengumpulan data dengan menggunakan metode tes dan wawancara. Penelitian ini menggunakan tiga jenis tes, yaitu: (1) Tes Matching Familiar Figure Test (MFFT) untuk menentukan gaya kognitif siswa, (2) Tes Kemampuan Matematika untuk mengukur kemampuan matematika siswa, dan (3) Tugas Pemecahan Masalah (TPM) untuk memperoleh data terkait profil berpikir aljabar siswa dalam memecahkan masalah matematika. Hasil penelitian menunjukkan bahwa berpikir aljabar siswa dengan gaya kognitif reflektif dalam memecahkan masalah memenuhi tiga aspek indikator berpikir aljabar yaitu: melakukan aktivitas menggeneralisasikan pola dan menentukan suku selanjutnya dari pola yang diberikan, merepresentasikan dan membandingkan data dalam bentuk tabel, serta memahami makna variabel dan menggunakan variabel yang berupa huruf atau simbol sebagai representasi sesuatu yang belum diketahui nilainya pada bentuk aljabar. Siswa dengan gaya kognitif reflektif dalam memecahkan masalah memiliki kemampuan memahami permasalahan yang diberikan dengan baik, hati-hati dan teliti dalam menulis langkah-langkah penyelesaian, lugas dan runtut dalam menjawab pertanyaan sehingga jawaban yang diberikan cenderung benar. Dengan demikian, hasil penelitian ini diharapkan dapat menjadi salah satu acuan untuk guru atau peneliti lain dalam mengembangkan pembelajaran matematika dengan mempertimbangkan gaya kognitif yang dimiliki siswa, terutama gaya kognitif reflektif.

**Kata Kunci:** Berpikir Aljabar, Pemecahan Masalah, Gaya Kognitif Reflektif

## 1. Introduction

Algebra is one of the important concepts taught in mathematics at school [1]. It is a competency that must be learned by students because it is a provision for the benefit of the development of knowledge and technology [2]. It serves as a mathematical tool to represent and analyze quantitative relationships, model situations, and solve problems [1][3-4]. It is a skill that involves understanding different representations, such as equations, graphs, and solving problems using symbols in the form of letters as a representation of unknown values [5-6].

However, numerous experts have found that algebraic material is challenging for students to comprehend in learning [7-8]. Causes of students' difficulties in learning algebraic material include difficulties in identifying variables and coefficients, as well as difficulties in determining problem-solving [9]. The results of Setyawati & Ratu's research on 32 seventh-grade junior high school students related to student learning difficulties in algebra material show that student difficulty factors include numeracy difficulties, visual perception difficulties, and lack of understanding of mathematical language [10]. A number of studies show that algebra is often considered one of the most difficult and abstract math materials [11-13]. Furthermore, the results of Malihatuddarajah & Prahmana's research show that some students make errors in solving problems about the operation of algebraic forms, including errors in identifying variables, negative signs, forms of algebraic equations, and solving fractions [14].

Rivera's research conveys that there is a significant difference between the arithmetic learned at the previous level and the algebra students are learning [15-16]. This is because arithmetic is limited to computing numbers whose quantity can be directly imagined. In algebra, students are confronted with symbols (variables) as a general form of something that is unknown. This needs to be considered, especially regarding the algebraic thinking of junior high school students as a tool for solving math problems. Ketterlin-Geller and Chard suggest that developing students' algebraic thinking skills and knowledge is influenced by an understanding of counting [17]. Students must have procedural proficiency, which includes computational fluency and

conceptual understanding, to apply in problem-solving and algebra. By combining skills and knowledge, students are able to use algebraic thinking to solve problems.

Algebraic thinking is an important and fundamental part of mathematical thinking and reasoning skills [18-20]. Lew in his article explains that success in algebra depends on at least six kinds of mathematical thinking skills such as generalization, abstraction, analytic thinking, dynamic thinking, modeling, and organization [21]. Algebraic thinking according to Van de Walle, et al. is a mental activity that involves using patterns that lead to generalizations (especially with operations) from experience with numbers and calculations, formulating mathematical ideas using symbols, and exploring concepts from patterns and functions [22]. Thus, algebraic thinking refers to a mental activity of generalizing numbers and calculations, formulating mathematical ideas using symbols, and exploring the concepts of patterns and functions, as well as performing mathematical modeling to determine the solution of a problem.

Problem-solving is one of the tools used to study the emergence of students' algebraic thinking. Problem-solving plays an important role in the development of algebra as well as being an interesting field to examine thinking and conceptual changes from arithmetic thinking to algebraic thinking [23-24]. Students' algebraic thinking process can be explored of them by using problem-solving [25-27]. According to Suharnan's opinion, most problem-solving or concept-formation activities involve the thinking process [28]. The algebraic thinking process and its characteristics can be studied by involving students in problem-solving situations [29-30].

Problem-solving activities have different characteristics for each student. The difference is possible because there are differences in the cognitive style of each student. Cognitive style is an individual's tendency to comprehend, recall, process, think, interpret information, and solve problems [31]. Cognitive styles that have been found by experts are quite diverse, one of which is a cognitive style based on conceptual differences in tempo or differences in the time used by a person in responding to a stimulus that is classified into two groups, namely reflective cognitive and impulsive cognitive styles [32]. Reflective and impulsive cognitive styles are defined as the characteristics of the cognitive system that combine decision-making time and performance in solving problems [33].

Students with impulsive cognitive styles have the characteristics of being fast in answering a problem but less thorough, so they tend to give answers that are less precise or wrong. Students with reflective cognitive styles have characteristics slow in answering a problem but thorough, so they tend to give the right answer [34]. The results of research by Victor et al. on 90 students who showed that students who have a category of impulsive cognitive style have a tendency of long response latency and higher error scores than students who are categorized as reflective cognitive style [35]. Warli's research reveals that students with a reflective cognitive style have more creativity in problem-solving than those with an impulsive cognitive style [36].

Another study on the profile of mathematical problem-solving of students with reflective-impulsive cognitive styles conducted by Azhil's research shows that reflective cognitive style students have an average value of 75% can solve problems correctly, while impulsive cognitive style students have an average value of 25% can solve problems correctly [37]. The difference in the results of the study is due to the characteristics of reflective students in solving problems that tend to be more careful, thorough, and re-examining the solutions that have been done. This certainly requires a relatively long time compared to students with impulsive cognitive style.

Each student has a different cognitive style that affects their ability to solve problems. This study aims to describe the algebraic thinking profile of junior high school students with reflective cognitive style in solving math problems. Based on the theory of cognitive development, junior high school students are in the age range of 13-15 years which is included in the formal operational stage. The characteristics of this stage are having the ability to think abstractly, reason logically, and can conclude the information they have obtained [38]. Algebra

is one of the materials related to the ability to think abstractly. So, at this stage, it is necessary to develop a strong understanding and algebraic foundation. Algebraic thinking is one way to improve students' mathematical thinking and reasoning skills [39]. By using relevant research, the author intended to conduct research that focused on the reflective cognitive style of students in solving math problems at the junior high school level.

## 2. Methods

This study used a case study with a qualitative approach. Qualitative research had made humans the main research instrument and described the research data based on facts in the field [40]. Thus, there were two types of instruments in the study; the main instrument was the researcher and the supporting instruments included MFFT (Matching Familiar Figure Test) questions developed by Warli to determine the cognitive style owned by the subject, Mathematics Ability Test (AMT) to measure students' mathematical skills, Problem-Solving Test (PST) in the form of description questions to explore the emergence of algebraic thinking profiles, and interview guidelines. Data were collected using test and interview methods.

Table 1 shows the indicators used in this study to measure algebraic thinking, which was adapted from the description proposed by Wongyai & Kamol [16].

**Table 1 Algebraic Thinking's Indicators.**

<b>Aspect of Algebraic Thinking</b>	<b>Indicators</b>
Pattern	a. Find the terms of the given pattern b. Generalize patterns
Representation	a. Represent data in the form of tables, graphs, or diagrams b. Interpret and compare data in the form of tables, graphs, or diagrams
Variable	Understand and use variables in the form of letters or symbols as a representation of something whose value is unknown in algebraic form

The subjects of this study were students with reflective cognitive style who were in 8<sup>th</sup> grade junior high school. The basis for the selection of subjects in this study was Pitta-Pantazi et al. that the ability of students' algebraic thinking to solve problems in the age range of 13 to 17 years was influenced by a set of cognitive systems [41]. Subject selection began by giving the MFFT test, then the MFFT test data was analyzed by calculating the number of correct answers and the time record of working on all MFFT test items.


Students were said to have a reflective cognitive style if they could answer questions correctly more than or equal to 7 or ( $f \geq 7$ ) and the time used to do the problem more than 7.28 minutes or ( $t > 7.28$ ). Students were said to have an impulsive cognitive style if they could answer questions correctly less than 7 or ( $f < 7$ ) and the time used to do the problem was less than or equal to 7.28 minutes or ( $t \leq 7.28$ ). Table 2 shows the classification of reflective-impulsive cognitive style based on frequency and time.

**Table 2 Classification of Cognitive Styles.**

Cognitive Styles	Time (t)	Correct Answer (f)
Reflective	$t > 7,28$	$f \geq 7$
Impulsive	$t \leq 7,28$	$f < 7$

The problem-solving test data were analyzed based on the algebraic thinking indicators in Table 1 and matched with alternative solutions. Interviews were analyzed by data reduction, data presentation, and conclusion drawing. Data validation using triangulation techniques, namely comparing the data from the problem-solving test results with the interview data. Figure 1 shows the problem-solving test questions used in this study.

Look at the picture of the arrangement of beads of two colors (black and white) below and answer the following questions!



- If there are 14 black beads, then how many white beads are needed for the arrangement as shown? Explain it!
- If there are  $x$  black beads, how many white beads are needed for the arrangement shown? Explain it!
- If 146 white beads are needed to assemble as shown, count the number of black beads!

**Figure 1 Problem-Solving Test Questions**

### 3. Result and Discussion

The subject of this study was a student of class VIII junior high school with a reflective cognitive style hereinafter referred to as SR, who was previously given the Matching Familiar Figures Test (MFFT). SR had completed the MFFT test with a record time of 11 minutes 37 seconds and correct answers of more than 7. This indicated that SR had a reflective cognitive style. Furthermore, the profile of algebraic thinking of junior high school students with reflective cognitive style in solving mathematical problems was analyzed in three indicators, including patterns, representations, and variables, each of which would be discussed as follows.

#### 3.1 Algebraic Thinking of Students with Reflective Cognitive Style in Solving Pattern Problems

The following is a transcript of interview results of students with reflective cognitive style about point A.

P5 : *How do you determine the white beads if you know there are 14 black beads?*

SR5 : *Because each pattern is always different. The black beads from the first, second, and third patterns always increase by one.*

P6 : *Then what about the white beads?*

SR6 : There are four white beads in the first pattern, six in the second, and eight in the third, meaning that from the first pattern to the second and the third, there are two more.

P7 : What does the number of black beads have to do with the number of white beads?

SR7 : If the black beads increase by one, then the white beads increase by two.

Figure 2 shows written answers of students with reflective cognitive style for question point A.

**Point A**

Diket: 6  
manik<sup>ss</sup> hitam = 14

Dit: 8  
manik<sup>ss</sup> putih?

pola	hitam	putih
1	1	4
2	2	6
3	3	8
4	4	10
5	5	12
6	6	14
7	7	16
8	8	18
9	9	20
10	10	22
11	11	24
12	12	26
13	13	28
14	14	30

Jack, jika banyaknya manik<sup>ss</sup> hitam = 14 maka jumlah manik<sup>ss</sup> putih adalah 30.

↓  
Hitam

*Translated Answer :*

Point A

Known black beads = 14.

Asked the number of white beads?

Patterns	Black Beads	White Beads
1	1	4
2	2	6
3	3	8
4	4	10
5	5	12
6	6	14
7	7	16
8	8	18
9	9	20
10	10	22
11	11	24
12	12	26
13	13	28
14	14	30

So, if the number of black beads is 14 then the number of white beads is 30.

Figure 2 SR Subject's Answer to Question Point A

Based on written answers and interview transcripts, students with reflective cognitive styles could write answers and explain what was meant by the problem correctly and coherently. The mathematical symbols used were also clear and correct. Students with reflective cognitive style performed activities to determine the relationship between patterns to determine the next term of the given pattern. This met the indicators of pattern aspect algebraic thinking, which was generalizing patterns and finding the terms of the given pattern. Then from the answer it was known that students with reflective cognitive style could determine the next term of the given pattern, so they could answer the question point *a* correctly. This was in accordance with the results of research by Maharani et al. that students with a reflective cognitive style wrote the steps of problem solving in detail so that the resulting answers tended to be correct [40].

### 3.2 Algebraic Thinking of Students with Reflective Cognitive Style in Solving Representation Problems

Figure 3 portrays the written answers of students with reflective cognitive style for question point B.

<p><i>Jawab</i></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 33%; text-align: center;">Pola</th> <th style="width: 33%; text-align: center;">hitam</th> <th style="width: 33%; text-align: center;">putih</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">1 } 1</td> <td style="text-align: center;">4 } 2</td> </tr> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">2 } 1</td> <td style="text-align: center;">6 } 2</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">3 } 1</td> <td style="text-align: center;">8 } 2</td> </tr> <tr> <td style="text-align: center;">4</td> <td style="text-align: center;">4 } 1</td> <td style="text-align: center;">10 } 2</td> </tr> <tr> <td style="text-align: center;">...</td> <td style="text-align: center;">...</td> <td style="text-align: center;">...</td> </tr> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">x</td> <td style="text-align: center;"><math>2x + 2</math></td> </tr> </tbody> </table>	Pola	hitam	putih	1	1 } 1	4 } 2	2	2 } 1	6 } 2	3	3 } 1	8 } 2	4	4 } 1	10 } 2	...	...	...	x	x	$2x + 2$	<p><i>Translated Answer:</i></p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 33%;">Patterns</th> <th style="width: 33%;">Black Beads</th> <th style="width: 33%;">White Beads</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">1</td> <td style="text-align: center;">1 </td> <td style="text-align: center;">4 </td> </tr> <tr> <td style="text-align: center;">2</td> <td style="text-align: center;">2  1</td> <td style="text-align: center;">6  2</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">3  1</td> <td style="text-align: center;">8  2</td> </tr> <tr> <td style="text-align: center;">4</td> <td style="text-align: center;">4  1</td> <td style="text-align: center;">10  2</td> </tr> <tr> <td style="text-align: center;">...</td> <td style="text-align: center;">...</td> <td style="text-align: center;">...</td> </tr> <tr> <td style="text-align: center;">x</td> <td style="text-align: center;">x</td> <td style="text-align: center;"><math>2x + 2</math></td> </tr> </tbody> </table>	Patterns	Black Beads	White Beads	1	1	4	2	2  1	6  2	3	3  1	8  2	4	4  1	10  2	...	...	...	x	x	$2x + 2$
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4	4  1	10  2																																									
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x	x	$2x + 2$																																									

Figure 3 SR Subject's Answer to Question Point B

The following is a transcript of interview results of students with reflective cognitive style about point B.

P18 : *How did you determine the number of white beads?*

SR18 : *The black one always adds one, the white one always adds two, and the white one is always even. If the number of black beads is x, I remember I was taught that the even number pattern is  $2x + 2$ , Then I tried it to the answer to point A and it turned out to be correct.*

P19 : *How do you know that your steps are correct?*

SR19 : *I recalculated; I checked it.*

Based on written answers and interview transcripts, the student with reflective cognitive styles used generalization patterns that had been found to determine problem-solving. Students model the situation using tables to determine problem-solving. This fulfilled the indicator of representation algebraic thinking (e.g., representing data in the form of tables, graphs, or diagrams). Students determined the next term by analyzing the relationship between numbers using patterns that had been found through the relationship between the number line and the picture to solve the problem.

Students modeled or represented problems using mathematical expressions. So, based on this description, students fulfilled the indicator of representation algebraic thinking (e.g., interpreting and comparing data to tables, graphs, or diagrams) in solving problems. In addition, in the SR19 transcript, the student did a double-check to ensure that the answers were correct. This was in accordance with the results of research undertaken by Satriawan et al. that students with a reflective cognitive style were very careful so that if there was an error, they would realize and correct it [41].

### 3.3 Algebraic Thinking of Students with Reflective Cognitive Style in Solving Representation Problems

Figure 4 conveys written answers of students with reflective cognitive style for question point C.

<p>Point C:</p> <p>Diket: manik<sup>xx</sup> putih = 146  Dita: manik<sup>xx</sup> hitam?</p> <p>Jawab: <math>2x + 2 = 146</math>  <math>2x = 146 - 2</math>  <math>2x = 144</math>  <math>x = 144 : 2</math>  <math>x = 72</math></p> <p>Jadi, banyak manik<sup>xx</sup> hitam adalah <u>72</u></p>	<p>Translated Answer :</p> <p>Point C  Known that amount of white beads is 146  Asked how many black beads?  Answer :</p> $2x + 2 = 146$ $2x = 146 - 2$ $2x = 144$ $x = 144 : 2$ $x = 72$ <p>So, amount of black beads is 72</p>
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Figure 4 SR Subject's Answer to Question Point C

The following is a transcript of interview results of students with reflective cognitive style about point C.

P23 : *How do you solve the problem from the information you already know?*

SR23 : *We know that there are 146 white beads, so we can find the black beads using  $2x+2$  from point B earlier.*

P24 : *What is  $2x+2$  and what is  $x$ ?*

SR24 : *That's the general pattern of many white beads,  $x$  is many black beads.*

P25 : *What did you do next?*

SR25 : *Because the number of white beads is 146, I made it like this  $2x+2=146$ . Then I calculated it until I found  $x$ , now that's the number of black beads 72.*

Based on the written answers and interview transcripts, the student with reflective cognitive style used patterns that had been found through the relationship between the number line from the previous problem (Point B). This results, compared with relevant studies, suggested that students with reflective cognitive style tended to use the same way that had been found or planned in solving problems [41]–[44]. Students understood the meaning of the value of  $x$  in the problem as a symbol or representation of something whose value was unknown, then students could also operate algebraic forms to determine the final result of problem-solving. Based on this description, the student with a reflective cognitive style fulfilled the variable indicator which was understanding and using variables in the form of letters or symbols as a representation of something whose value was unknown in algebraic form.

#### 4. Conclusion

Based on the results of data analysis and discussion, it could be concluded that the algebraic thinking of students with reflective cognitive styles in solving problems related to patterns meets two indicators namely (1) generalizing patterns and (2) finding terms from the given pattern. From the answers given, the student can write answers and explain what the question means correctly and coherently. Moreover, the mathematical symbols used are also clear and correct. The algebraic thinking of the student with reflective cognitive style in solving problems related to representation meets two indicators namely (1) representing data in the form of tables, graphs, or diagrams, in this case, students model the situation using tables to determine problem-solving and (2) interpreting and comparing data to tables, graphs, or diagrams, in this



case, students analyze the relationship between numbers using patterns that have been found and model or represent problems using mathematical expressions. Furthermore, the algebraic thinking of student with a reflective cognitive style in solving problems related to variables fulfills the indicators of variables in accordance with understanding and using variables in the form of letters or symbols as a representation of something whose value is unknown in algebraic form. Student with a reflective cognitive style in solving problems appear to understand the problems given, careful and thorough in writing the steps of completion, straightforward and coherent in answering questions so that the answers given tend to be correct. Thus, the results of this study are expected to be one of the references for teachers or other researchers in developing mathematics learning by considering the cognitive style the students, especially reflective cognitive style.

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