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Kinds of Mathematical Thinking Addressed in Geometry Research in School: A Systematic Review

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ABSTRACT

Geometry is one of the content of mathematics which is often associated with students' thinking skills such as critical thinking and reasoning abilities. This study aimed to conduct a systematic review of the geometry research in school for identifying the types of mathematical thinking and their interconnections. We searched the Scopus database for articles published from 2003 to 2023 using relevant keywords. We applied the PRISMA method to select and evaluate the studies or articles based on the empirical data. We retrieved and evaluated data from the studies on the various styles of mathematical thinking evolved. Out of 166 titles that were initially obtained, only 10 titles passed the five stages of the systematic review protocol process. We identified 10 types of mathematical thinking that were discussed in the context of learning geometry at school: Creative Mathematical Reasoning (CMR), Computational thinking, Geometric reasoning, Geometric thinking van hiele theory, Geometric thinking (3D geometric thinking with representations), 3D geometry thinking, Visuo spatial reasoning, Geometry Spatial Reasoning, mathematical creative reasoning (MCR), and Inductive reasoning. We also found some connections of literature between these types of mathematical thinking, such as CMR and MCR, Geometric reasoning and Geometric thinking, and Visuo spatial reasoning and Geometry Spatial Reasoning. This systematic review provides an overview of the current state of research on geometry and reasoning in school mathematics and reveals some gaps and directions for future study. It also has implications for teachers who want to enhance their students’ mathematical thinking skills in geometry by exposing them to different types of mathematical thinking and their connections.

Keywords: *Mathematical Thinking; Systematic Review; Geometry*.

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Macam-Macam *Mathematical Thinking* dalam Penelitian Geometri di Sekolah: *A Systematic Review*

ABSTRAK

Geometri merupakan salah satu konten matematika dimana pada banyak studi dikaitkan dengan kemampuan berpikir siswa, seperti berpikir kritis dan kemampuan bernalar atau yang lain. Studi ini bertujuan untuk melakukan tinjauan sistematis terhadap artikel atau literatur yang membahas penalaran dalam geometri sekolah untuk mengidentifikasi jenis-jenis *mathematical thinking* yang dibahas dan bagaimana keterkaitannya. Pencarian artikel dilakukan pada databased scopus yang dipublikasikan dari tahun 2003 hingga 2023 menggunakan kata kunci yang relevan. Kami menerapkan metode PRISMA untuk memilih dan mengevaluasi artikel yang telah didapatkan. Tahap *screening* dan *eligibility* dan menganalisis data tentang jenis-jenis *mathematical thinking* yang muncul dari studi. Dari 166 artikel, hanya 10 judul yang dipilih setelah melalui tahap proses protokol tinjauan sistematis. Dari artikel yang diproses didapatkan 10 jenis *mathematical thinking* yang dibahas dalam konteks pembelajaran geometri di sekolah: *Creative Mathematical Reasoning (CMR)*, *Computational thinking*, *Geometric reasoning*, *Geometric thinking van hiele theory*, *Geometric thinking (3D geometric thinking with representations)*, *3D geometry thinking*, *Visuo spatial reasoning*, *Geometry Spatial Reasoning*, *mathematical creative reasoning (MCR)*, dan *Inductive reasoning*. Kami juga menemukan beberapa keterkaitan antara jenis-jenis *mathematical thinking* ini, seperti CMR dan MCR, *Geometric reasoning* dan *Geometric thinking*, dan *Visuo spatial reasoning* dan *Geometry Spatial Reasoning*. Tinjauan sistematis ini memberikan gambaran umum tentang keadaan penelitian saat ini tentang geometri dan penalaran dalam matematika sekolah dan mengungkapkan beberapa celah dan arah untuk penelitian masa depan. Ini juga memiliki implikasi bagi guru yang ingin meningkatkan keterampilan *mathematical thinking* siswa dalam geometri dengan memperkenalkan mereka pada berbagai jenis *mathematical thinking* dan keterkaitannya.

***Kata Kunci*:** *Mathematical Thinking; Systematic Review; Geometri*

1. Introduction

Geometry is a part of mathematics that is closely related to students' thinking or reasoning skills. Several studies link the topic of geometry with rationale, spatial thinking, critical thinking, and even specifically geometric thinking [1][2] as factors that can influence success in geometry. On the other hand, teachers can make geometry learning tools to improve students' thinking skills, such as critical thinking and reasoning abilities, especially for students at the elementary education level where geometric objects are close to the real world and can be imagined by students.

Research on geometry and student's reasoning is important because it can provide insights into how students' conceptual understanding of geometric concepts develops [3]. This will certainly be useful for teachers in planning appropriate geometry lessons and being able to support students' understanding of geometry concepts. For instance, a review of literature relating to spatial reasoning and geometry revealed that young children (aged 4 to 8) can demonstrate richer understanding of geometry and spatial reasoning than researchers previously thought possible [3]. This finding highlights the importance of incorporating rich geometry learning experiences into primary school mathematics classrooms. On the other hand, an understanding of the relationship between geometry and student's reasoning can also explore the potential of learning geometry in improving certain cognitive aspects such as students' development of spatial reasoning, geometric visualization, geometric measurement, and geometric reasoning and proving [4].

To explore more about the relationship between geometry and reasoning or thinking based on the published articles, a systematic literature review can be carried out. There is a wealth of systematic reviews research available on the topic of geometry. One such review conducted between 2017-2021 examined the impact of various interventions on geometric thinking [2]. The results showed that these interventions were largely effective, with some producing “very large” effect sizes. These particularly effective interventions included van Hiele’s learning phase and the use of technology-based media and concrete manipulative media. Another systematic review found that the use of Dynamic Geometry Software (DGS) in instruction was effective in improving students’ mathematical achievement. These studies demonstrate the potential for various approaches to enhance achievement and geometric thinking in mathematics [5]. In this case, we conduct a systematic review related to geometry topic to determine which types of mathematical thinking or reasoning are discussed in the context of learning geometry at school and how they are interconnected. This study can benefit researchers, in mathematics education fields, by showing connections between previous information and suggesting new directions for future research. Additionally, teacher can gain insight into students' mathematical thinking concerning the subject of geometry through the explanations and discussions that are presented.

2. Method

This study aimed to conduct a systematic review related to research in geometry and see what types of mathematical thinking were discussed in the articles published over the last 20 years, as well as determining research trends in this field based on the Scopus database. The review followed a structured design comprising several stages. First, studies meeting the criteria were identified and evaluated based on research type and empirical data using the PRISMA method [6]. Data were collected from the Scopus database using relevant keywords, covering the period from 2003 to 2023. The study protocol involved formulating research questions, conducting a search for articles using specific keywords, and selecting relevant titles based on inclusion and exclusion criteria. After acquiring the necessary articles, data quality was assessed and extraction was performed. The final stage involved synthesizing the data to determine the results. To enhance the clarity of the research design, it was recommended to explicitly state the research questions, clearly define the inclusion and exclusion criteria, provide more details about the search strategy, describe the data quality assessment process, explain the data extraction process, justify the chosen synthesis method, ensure figures and diagrams are included, and clarify the timeframe of the review. Incorporating these recommendations would improve the flow and simplicity of the research design, facilitating understanding and replication of the systematic review.

The study used Scopus database to select relevant research using keywords “geometry, thinking and reasoning” using software “publish or perish” from journals and conferences based on the inclusion and exclusion criteria (see Figure 1). The criteria only considered empirical research articles published from 2003 to 2023. Out of 166 titles that were initially obtained, only 10 titles passed the five stages of the systematic review protocol process. The rest of the 156 titles were excluded for not meeting the criteria and limitations we provided. Some of the article findings that we then exclude are studies that do not focus on students or prospective teachers, textbooks, discussion does not focus on thinking or reasoning but on the tools used.

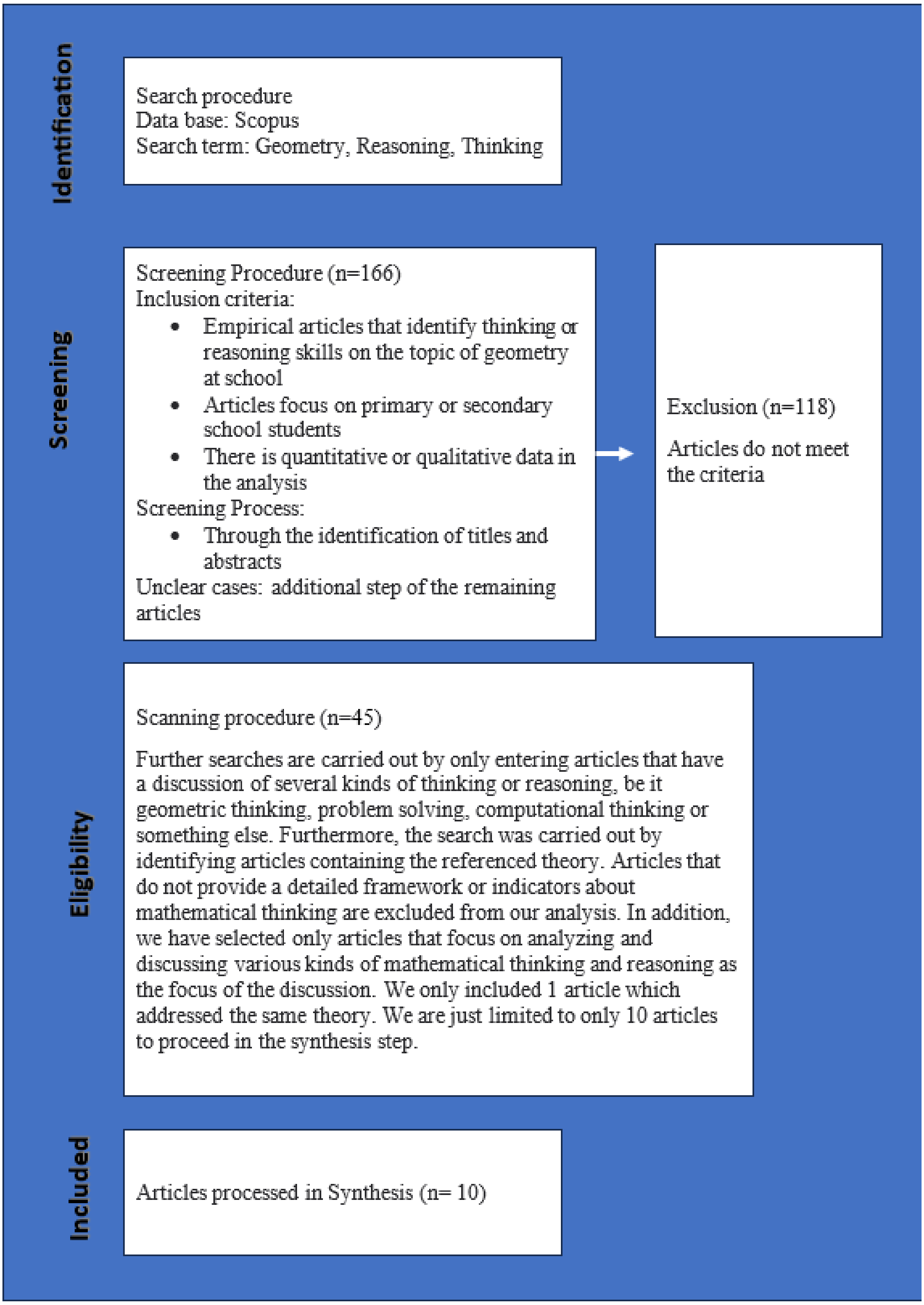

Figure 1 The PRISMA Flow Diagram [6]

3. Results and Discussions

3.1. Publication

Based on the data obtained, there were 45 articles that met the eligibility aspect from 2003 to 2023. These data generally showed how the quantitative condition of the development of articles in the focus of this study. Studies on geometry, reasoning, and thinking showed a significant increase in the number of publications in the Scopus database from year to year, although it had decreased in several years. Data from 2003 to 2005 showed no publications recorded with these keywords. However, from 2006 to 2007, there was a small increase with one publication in 2006 and two publications in 2008. The period from 2008 to 2010 showed fluctuations in the number of publications, with the highest peak occurring in 2010 with three publications. However, in 2011, no publication was recorded. In 2012 and 2013, there was an increase with one publication in 2012 and two publications in 2013. The number of publications increased significantly in 2015 with four publications. From 2016 to 2019, the number of publications continued to increase, reaching peak in 2017. In 2019, there were three publications, which increased to seven publications in 2020. In the last three years, namely 2021 to 2023, the number of publications had remained relatively high, with four publications in 2021, six publications in 2022, and one publication to date in 2023. Overall, the data showed a consistent increasing trend in interest and research on geometry, reasoning, and thinking from 2013.

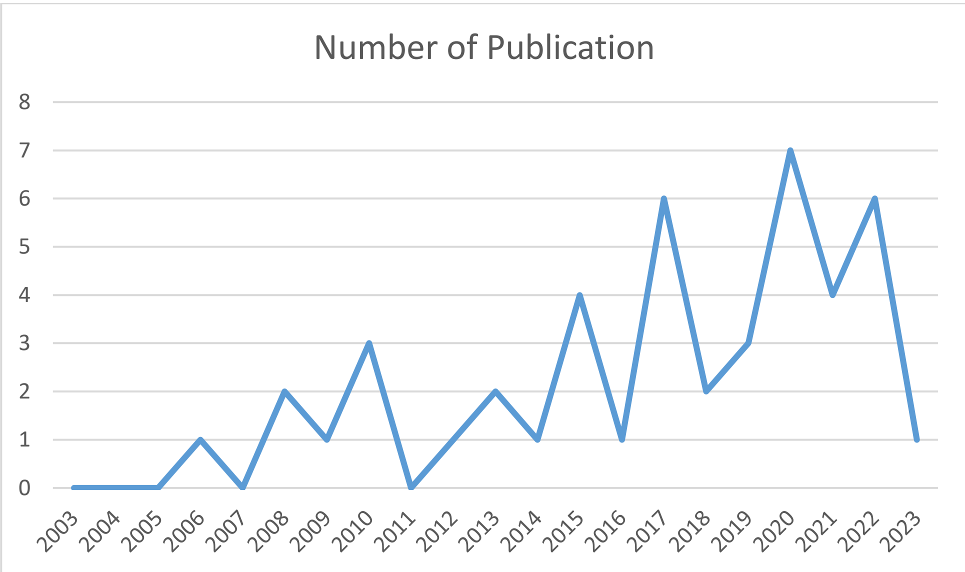

Figure 2. Number of Studies Relate to Geometry, Reasoning, and Thinking in Scopus Databased (based on data that meets the eligibility aspect)

3.2. The Characteristic of The Publications

From the published data we obtained, the following were some of the characteristics of the studies we found. The studies covered different grade levels, including elementary, middle, high school, vocational high school, and higher education settings. By examining mathematical thinking and learning across these grade levels, researchers aimed to gain insights into the progression of students' mathematical understanding and problem-solving skills throughout their educational journey.

Technological tools and environments played a significant role in several studies, with researchers exploring the impact of virtual manipulatives, whiteboard systems, smart pen technology, computer algebra systems, dynamic geometry environments, touchscreendragging, and digital environments on mathematical thinking and learning. These studies investigated how the integration of technology enhanced students' conceptual understanding, spatial skills, and reasoning skills in geometry. On the other hand, the studies highlighted the importance of cognitive development and reasoning in mathematics education. Researchers examined students' cognitive growth, levels of geometric thinking, creative reasoning, adaptive reasoning skills, visual-spatial thinking, problem-posing skills, and the use of reasoning processes in geometry. The goal was to promote students' mathematical understanding, critical thinking, and higher-order cognitive skills.

Instructional approaches had been a focal point of many studies, as researchers explored effective strategies to enhance students' conceptual understanding of mathematical concepts. These approaches included problem-based learning, concept-based inquiry models, ethnomathematics learning, and think-talk-write learning. The studies investigated the effectiveness of these instructional models in improving students' spatial skills, geometrical reasoning, mathematical performance, and transfer of learning.

The studies listed in the provided list of references spanned various educational contexts and countries, providing insights into mathematics education practices and challenges across different cultural and educational settings. By conducting research in diverse contexts, researchers contributed to the understanding of effective instructional strategies, technological tools, and cognitive processes related to geometric thinking and mathematical reasoning.

In general, the studies listed in the references covered a wide range of topics, grade levels, research methods, technological environments, cognitive development, instructional approaches, and educational contexts. They collectively contributed to the field of mathematics education by shedding light on the complexities of geometric thinking, spatial reasoning, and mathematical cognition, and informing effective instructional practices for promoting mathematical understanding and critical thinking skills among students.

3.3. Results and Discussions

In the synthesis stage, we only selected 10 articles for in-depth analysis which generally had different types or theories about mathematical thinking discussed on the topic of geometry. In general, these 10 articles adequately represented the 45 articles included in the eligibility stage based on the type of mathematical thinking discussed. Based on the selected included articles, we identified various types of mathematical thinking that emerged from the data analysis. These types of mathematical thinking were: Creative Mathematical Reasoning (CMR), which involved generating and exploring new ideas; Computational Thinking, which encompassed using algorithmic and logical thinking to solve problems; Geometric Reasoning, which comprised using spatial visualization, transformation, and deduction to reason about shapes and properties; Geometric Thinking van Hiele Theory, which described the levels of geometric understanding from visualization to formal deduction; Geometric Thinking (3D Geometric Thinking with Representations), which covered using different representations such as drawings, models, and software to explore and communicate 3D geometric concepts; 3D Geometry Thinking, which included understanding and applying 3D geometric properties and relationships; Visual Spatial Reasoning, which involves using mental imagery and manipulation to solve spatial problems; Geometry Spatial Reasoning, which involved using spatial intuition and reasoning to solve geometry problems; Mathematical Creative Reasoning (MCR), which covered using divergent and convergent thinking to generate and evaluate mathematical ideas; and Inductive Reasoning, which encompassed using specific examples and patterns to make generalizations and conjectures. Table 1 summarizes the list of mathematical thinking types, their definitions, and the articles that exemplified them.

TABLE 1 Kind of Mathematical Thinking Addressed in Geometry Research in School.

Reference	Kinds of Mathematical Thinking	Explanation of Definition, framework or indicators addressed in research
Marsitin, R. [7]	Creative Mathematical Reasoning (CMR)	Creative Mathematical Reasoning Criteria and Indicators were divided into three categories: Mathematics Foundation, Plausible, and Novelty Criteria and Indicators. These categories involved identifying and explaining information, arithmetic operations, formulas, and concepts in solving problems; developing and implementing strategies or steps to prove the truth in mathematical concepts; and obtaining connectedness of information to solve problems.
Guimaraes, V. [8]	Computational thinking	Computational thinking was supported by four pillars: decomposition, pattern recognition, abstraction, and algorithm. These pillars involved breaking down a problem into smaller parts, recognizing similarities between situations, filtering out irrelevant information, and developing a set of steps to solve the problem.
Afifah, A.H. [9]	Geometric reasoning	Five levels of reasoning about shapes: Level 1 (Visual reasoning), Level 2 (Descriptive reasoning), Level 3 (Analytic reasoning), Level 4 (Relational-inferential property-based reasoning), and Level 5 (Formal deductive proof). These levels included recognizing and identifying shapes by their appearances and properties, using informal and formal language to describe them, understanding the relationships between their properties, and constructing arguments based on their properties.
Stols, G. [10]	Geometric thinking van hiele theory	There were five levels of reasoning about shapes: Level 1 (visual recognition), Level 2 (analysis of properties), Level 3 (informal deduction), Level 4 (formal deduction), and Level 5 (rigor). These levels encompassed recognizing shapes as a whole, analysing their properties, understanding the relationships between their properties, constructing multistep proofs, and using logical reasoning to construct valid arguments.
Fujita, T. [11]	Geometric thinking (3D geometric thinking with representations)	The assessment framework for 3D geometric thinking with representations. The framework included six categories: Category 0 (no ability to manipulate or reason with 3D representations), Category 1 (limited ability to perform simple mental manipulations), Category 2-A (able to perform simple manipulations but flawed deductions for complex problems), Category 2-B (able to perform simple manipulations and add lines or draw nets for complex problems), Category 2-C (able to perform simple manipulations and activate knowledge of useful properties but influenced by visualization of geometric shapes), and Category 3 (able to manipulate the representation and demonstrate valid reasoning for complex problems).
Pittalis, M. [12]	3D geometry thinking	According to the cognitive model of Duval of geometrical thinking, geometrical thinking involves visualization, construction, and reasoning processes.

		The role of visual representation of a geometrical statement was discussed in five aspects: the ability to manipulate different representational modes of 3D objects, the ability to recognize and construct nets, the ability to structure 3D arrays of cubes, the ability to recognize 3D shapes' properties and compare 3D shapes, and the ability to calculate the volume and area of solids.
Zahari, C.L. [13]	Visuo spatial reasoning	Representational competence referred to the ability to determine when and why to choose one representation over others.
Dhlamini, Z. [14]	Geometry Spatial Reasoning	To clarify learners' conceptual thinking in connection to spatial geometry, or geometry spatial reasoning, the four AR (adaptive reasoning) constructs were used. The AR consisted of four sections: (1) explaining particular problem elements; (2) using logical reasoning to explore mathematical concepts; (3) justifying statements; and (4) reflecting on acceptable mathematical notions. Therefore, the AR was used to record students' memory and manipulation of spatial items within particular situations. To throw light on important facets of the reasoning process, the idea of AR was used.
Dwirahayu, G. [15]	mathematical creative reasoning (MCR)	Lithner mentioned the following MCR (Mathematical Creative Reasoning) competencies: 1. Creativity: This referred to a student's capacity to approach mathematical issues in novel ways rather than merely memorizing formulas or copying past problem-solving techniques. It entailed coming up with fresh methods for tackling problems. 2. Plausibility: The ability of a student to offer logical, accurate, and reasonable reasons to support answers to mathematical problems was referred to as plausibility. It underlined how crucial it was to provide well-supported arguments rather than depending on educated assumptions because educated guesses were difficult to support. 3. Anchoring: The capacity of students to successfully resolve mathematical problems was known as anchoring. It entailed offering answers that were strongly based in the circumstances and specifications of the specific problem at hand.
Gagani, R.F.M. [16]	Inductive reasoning	In the context of research in geometry, researchers divided inductive reasoning into two levels. At level 1, students could illustrate and establish obvious connections within a task but did not use the significance of the connection to complete the task. In contrast, at level 2, students further investigated the task and demonstrate an improved ability to reason out inductively by meeting only a few task requirements.

The ten aspects provided all relate to mathematical reasoning and problem-solving within the context of geometry. Several of the aspects focused specifically on evaluating and developing skills and abilities related to geometric reasoning [17-23], such as Levels of Reasoning in Shape Recognition and Understanding, Levels of Shape Recognition and Reasoning, Assessment Framework for 3D Geometric Thinking with Representations, and

The Cognitive Model of Duval of Geometrical Thinking. These aspects provided frameworks for understanding how individuals progress in their ability to recognize and reason about shapes and manipulate representations of geometric objects.

Other aspects, such as Creative Mathematical Reasoning Criteria and Indicators, Computational Thinking, Representational Competence, Adaptive Reasoning (AR), Mathematical Creative Reasoning (MCR), and Inductive Reasoning in the Context of Research in Geometry, emphasized the importance of developing general skills such as creative thinking, logical reasoning, problem-solving, and the ability to manipulate and reason with representations [24-32]. These skills were essential for success in geometry as well as other areas of mathematics.

Overall, these ten aspects provided a comprehensive view of the skills involved in mathematical reasoning and problem-solving within the context of geometry. They highlighted the importance of developing a range of skills, from basic skills such as recognizing shapes and manipulating representations to more advanced skills such as creative thinking and logical reasoning. By focusing on these aspects, student could develop a strong foundation in geometric reasoning and problem-solving.

Afterward, we tried to look at the relationships and links between nine out of ten articles (Figure 3). We excluded 1 article because it was not included in the database used by the litmap application. There were three articles that had a direct relationship namely Pittalis & Christou[12] and Fujita [11] discussing 3D geometric dan Dhlamini [14] discussing geometry spatial reasoning. While there was no direct relationship through reference with the other articles. The three publications discussed in this study had the greatest citations and had been topics of conversation for a long time. Pittalis [12] cited Duval geometrical thinking as one of the earliest references pertaining to the creation of frameworks for geometrical thought. All three frameworks highlighted the importance of visual representation and the ability to manipulate and reason with geometric shapes in the development of geometrical thinking and spatial reasoning, especially for 3D object.

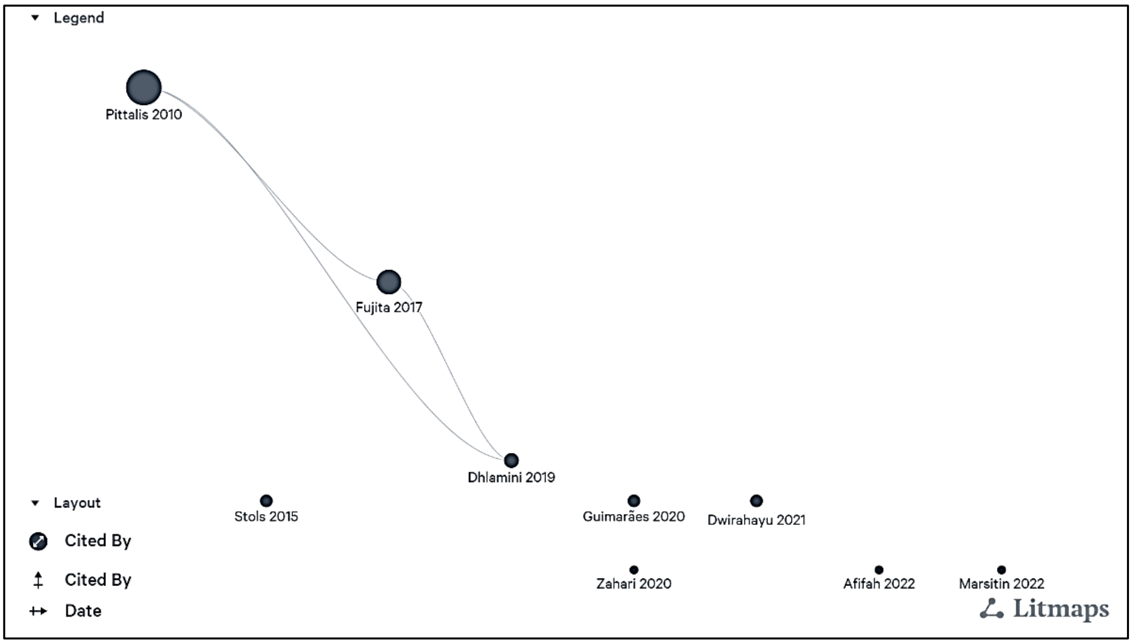

Figure 3 The Relationships between articles included in the study

An interesting finding from this study was that there were many other aspects of thinking discussed apart from geometric thinking. Dhlamini [14] identified spatial reasoning using adaptive reasoning theory. This of course also indicated that researchers could also use other reasoning theories to identify an ability by modifying or developing a new framework that could be used to explain an aspect of a particular ability. There were still many research opportunities about geometry in schools involving other aspects of mathematical thinking, for example analogical reasoning or others.

This study had some limitations required to be acknowledged. The first limitation was the small number of articles that were included in the systematic review, which might not reflect the full range of research on geometry and mathematical thinking in school. Second, the use of Scopus database as the only source of literature, which might have excluded some relevant studies that were published in other databases or journals. The lack of a clear definition and classification of the types of mathematical thinking that emerged from the studies, which might have caused some ambiguity and inconsistency in the analysis and comparison. In addition, the focus on the connections between the types of mathematical thinking, rather than on their impact on students' learning outcomes and achievement in geometry. These limitations suggested some directions for future research, such as conducting a more comprehensive and inclusive literature search, developing a more rigorous and coherent framework for identifying and categorizing the types of mathematical thinking, and examining the effects of different types of mathematical thinking on students' performance and attitudes in the geometry topic.

3 Conclusion

Based on analysis using PRISMA systematic review of the literature on geometry research in schools, we identify 10 different types of mathematical thinking addressed in the studies namely Creative Mathematical Reasoning (CMR), Computational thinking, Geometric reasoning, Geometric thinking van hiele theory, Geometric thinking (3D geometric thinking with representations), 3D geometry thinking, Visuo spatial reasoning, Geometry Spatial Reasoning, mathematical creative reasoning (MCR), and Inductive reasoning. Geometry research in schools covers a wide range of mathematical thinking skills and processes, besides the traditional notion of geometric thinking. We also discuss the similarities and differences among these types of thinking and its implications for teaching and learning geometry. Our work contributes to the understanding of the diversity and complexity of mathematical thinking in geometry education and provides a useful finding and information for future research and practice. We hope that our work will be of interest to both researcher and teachers who wish to study or support mathematical reasoning and who must negotiate the diversity of concepts related to mathematical reasoning as well as the diversity of research around it.

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