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Constructing Analogical Arguments in Solving Mathematical Problem: High School Students' Interactions with ChatGPT

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ABSTRACT

Analogical arguments play an important role in connecting students' prior experiences to solving new problems. However, many students struggle to construct well-structured arguments when solving mathematical problems, they have encountered before. The use of artificial intelligence technology, such as ChatGPT, has emerged as a potential solution to support students in building analogical arguments. This study aims to describe high school students' analogical arguments in solving mathematical problems before and after interacting with ChatGPT. This research employed a descriptive qualitative approach. The participants consisted of two 10th-grade high school students who had studied trigonometry and had prior experience using ChatGPT. The participants were selected based on the completeness of the argument components they demonstrated. Data were collected through analogical argument tasks, semi-structured interviews, and interactions with ChatGPT. Data analysis referred to indicators of analogical arguments that integrate Toulmin's argument components and Bartha's analogical argument structure. The results showed that students' analogical arguments changed before and after interacting with ChatGPT. The structure of the analogical arguments became more organized after the interaction. ChatGPT helped students to reconstruct the arguments they had previously built. This study implies that the use of ChatGPT can be a potential alternative to assist students in developing analogical arguments when solving mathematical problems.

Keywords: *analogical arguments, ChatGPT, high school students, mathematical problem solving*

Konstruksi Argumen Analogis dalam Menyelesaikan Masalah Matematis: Interaksi Siswa SMA dengan ChatGPT

ABSTRAK

Argumen analogis berperan penting dalam menghubungkan pengalaman sebelumnya yang dimiliki siswa untuk menyelesaikan masalah baru. Namun, banyak siswa yang mengalami kesulitan dalam membangun argumen yang terstruktur dengan baik saat menyelesaikan masalah matematis yang pernah mereka temui sebelumnya. Pemanfaatan teknologi kecerdasan buatan, seperti ChatGPT, menjadi salah satu solusi potensial untuk mendukung siswa dalam membangun argumen analogis. Penelitian ini bertujuan untuk mendeskripsikan argumen analogis siswa SMA dalam menyelesaikan masalah matematis sebelum dan sesudah berinteraksi dengan ChatGPT. Penelitian ini menggunakan pendekatan kualitatif deskriptif. Subjek penelitian terdiri dari dua siswa SMA kelas X yang telah mempelajari materi trigonometri dan memiliki pengalaman sebelumnya dalam menggunakan ChatGPT. Subjek dipilih berdasarkan kelengkapan komponen argumen yang mereka miliki. Data dikumpulkan melalui tugas argumen analogis, wawancara semi-terstruktur, dan interaksi dengan ChatGPT. Analisis data mengacu pada indikator argumen analogis yang memadukan komponen argumen Toulmin dan bentuk argumen analogis Bartha. Hasil penelitian menunjukkan bahwa argumen analogis siswa mengalami perubahan sebelum dan setelah berinteraksi dengan ChatGPT. Struktur argumen analogis yang dibangun siswa lebih terstruktur setelah berinteraksi dengan ChatGPT. Interaksi dengan ChatGPT membantu siswa dalam menyusun ulang argumen yang sebelumnya telah mereka bangun. Penelitian ini memberikan implikasi bahwa penggunaan ChatGPT dapat menjadi alternatif solusi dalam membantu siswa membangun argumen analogis ketika menyelesaikan masalah matematis.

Kata Kunci: *argumen analogis, ChatGPT, siswa SMA, pemecahan masalah matematika*

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1. Introduction

An argument can be defined as a series of statements leading to a conclusion (Corneli et al., 2019). In the context of mathematics, an argument plays a crucial role in fostering deeper understanding of mathematics concepts (Wu et al., 2025). In the process of constructing an argument, students automatically put forward reasons that support their statements. Thus, students can understand why their answer aligns with the underlying reasons. Toulmin (2003) states that each statement in an argument has a different function, which can be classified into six types: claim, data, warrants, backings, qualifier, and rebuttals. The process of constructing an argument involves stating a claim, presenting data to support the claim, and providing warrants to bridge the two. Then, three other components, backings, qualifier, and rebuttals, are stated to enrich the argument. This means that the main components of an argument must at least contain a claim, data, and warrant.

One argument that is relevant in solving mathematics problems is an analogical argument. Analogical arguments allow individuals to use their previous experiences to find solutions to new problems (Bartha, 2010). Amir-Mofidi et al., (2012) and Hesse & Klecha (1990) states that people usually use their previous experiences in solving problems to solve new similar problems. This is in line with Angraini et al., (2023) that linking learned concepts to new problems or situations can facilitate the understanding and application of those concepts in different contexts.

An analogical argument involves comparing two similar situations, then drawing a conclusion from the familiar situation to a similar new situation (Shodikin et al., 2023). Juthe (2015) states that analogy in an analogical argument is used as a tool to justify a claim. According to Racharak et al. (2019), an analogical argument is constructed through a comparison between two concepts or problems, highlighting shared characteristics that can support and strengthen a conclusion. Based on these statements, the similarities between two pairs of concepts or problems—referred to as the source and the target—are used to strengthen the argument by showing that what holds true in one case (the source) is also likely to hold true in another similar case (the target). Bartha (2010) illustrates the structure of an analogical argument as follows in Table 1.

Table 1. Analogical Argument Representation

Source Domain (S)	Target Domain (T)	
P	P*	[positive analogy]
A	~A*	[negative analogy]
~B	B*	
Q		
	Q*	(plausibly)

Table 1 shows that in an analogical argument, a source domain (S) and a target domain (T) each consist of a set of objects, properties, relations, and functions, along with a set of accepted statements about those elements. To distinguish between the elements of these two domains, different symbols are used. Symbols without an asterisk (A, B, P, Q) refer to elements in the source domain, while symbols with an asterisk (A*, B*, P*, Q*) refer to elements in the target domain. Table 1 illustrates that Q* is likely to apply in the target domain due to several known (or accepted) similarities with the source domain, despite some known (or accepted) differences. When there is a similarity in properties (P and P*) between the source and target domains, it is considered a positive analogy. Conversely, when the properties (A and ~A*; B and ~B*) differ between the source and target domains, it is considered a negative analogy. Bartha (2010) states that it is considered plausible or reasonably supported when Q* holds in the target domain due to certain similarities with the source domain, even if some differences exist. Therefore, it can be concluded that analogical argument involves a source domain (S) and a target domain (T) to draw the conclusion that the target domain likely possesses similar properties to the source domain, even in the presence of some differences. In this study, the source domain and target domain are represented by the source problem and the target problem. Both the source problem and the target problem used are mathematical problems.

Although it has been previously explained that analogical arguments can help students understand and solve problems, many students struggle to construct arguments (Jagadianti & Rosyidi, 2023; Widjaja et al., 2021; Maknun et al., 2018). This is largely due to limited knowledge and a lack of experience in constructing strong arguments. Furthermore, according to Zhou et al., (2021), students' difficulties in arguing may also be caused by teachers not accustoming students to argumentation or teaching them how to construct an argument, as limited teaching time prevents teacher from prioritizing an argumentative classroom culture. Students often provide answers based on the formula taught without understanding the rationale behind the methods (Güner & Erbay, 2021). They tend not to understand why a particular formula should be used for a given problem, so when asked to support their claim, they lack clear reasons to support them.

Students' difficulties in constructing arguments require innovative solutions. Opportunities for advancement in artificial intelligence (AI) technology are emerging to support students' argument development by helping them develop their way of thinking (Szmyd & Mitera, 2024). One of AI technology that has the potential to be used as a tool for constructing arguments in ChatGPT (Ali et al., 2023). This technology can help students develop their way of thinking by providing systematics explanations and feedback, thus encouraging them to refine and strengthen their arguments (Guo & Lee, 2023). This is in line with Essel et al., (2024) that the use of ChatGPT in learning activities has a positive impact on students' critical, creative, and reflective thinking skills. ChatGPT as assistant, can help students reflect on and refine their arguments. However, AI's use in developing students' constructions of analogical arguments is still relatively limited (Allam et al., 2023).

The use of AI in constructing analogical arguments has greater potential for high school students. This is because high school students are at crucial stage in mastering 21st-century skills, such as technological literacy, communication, and adaptability, which are essential amidst the acceleration of digital transformation and the demands of continuous learning (Chaudhry & Kazim, 2022). This urgency is even more evident because some high school students may also directly enter workforce. Furthermore, constructing arguments requires logical thinking, which, according to Jean Piaget, is generally possessed by adolescents, especially high school students in Indonesia (Pakpahan & Saragih, 2022). Students in their teens are already able to understand the form of arguments (Berk, 2015). Therefore, this study will explore the arguments of high school students who are deemed capable of systematically constructing reasons to support a statement and reaching conclusions.

Research on analogical reasoning in mathematics has been extensive, but studies specifically addressing how students' analogical reasoning in solving mathematical problems after using ChatGPT are still relatively rare. Furthermore, the role of AI-based tools like ChatGPT in this context remains underexplored. Therefore, based on this background, this study aims to explore and describe changes in high school students' analogical reasoning in solving mathematical problems after using ChatGPT. Although research on analogical reasoning in mathematics is abundant, studies that specifically examine how students construct analogical arguments after interacting with AI tools like ChatGPT remain scarce. Most existing studies focus either on the cognitive process of analogical reasoning or on the general performance of AI in educational contexts, without connecting the two (Zhai et al., 2021). Therefore, this study fills the gap by highlighting how high school students construct analogical arguments in solving mathematical problems through interaction with ChatGPT. The findings of this study are expected to provide insights into the development of technology-based learning environments that support reasoning and critical argumentation skills in mathematics education.

2. Research Method

This study used a descriptive qualitative approach to explore how students' analogical arguments in solving mathematics problems changed after the use of ChatGPT. Data were collected from 66 students in two grades at a high school in Surabaya in the 2024/2025 academic year. Students were first asked to work on the source problem. Then, based on the students' arguments on the source problem, the correct answers were selected and grouped into two categories: (1) correct source problem arguments with complete argument components and (2) correct source problem arguments with incomplete argument components. One student from each category was purposefully selected to be given the target problem and access to use ChatGPT. This selection was intended for an in-depth qualitative analysis of individual reasoning processes rather than statistical comparison within categories. The selection of participants was done intentionally based on variations in the completeness of the argument components in the source problem.

Here is the framework of the argument model developed by Toulmin (2003), which is used for selecting participants (Figure 1).

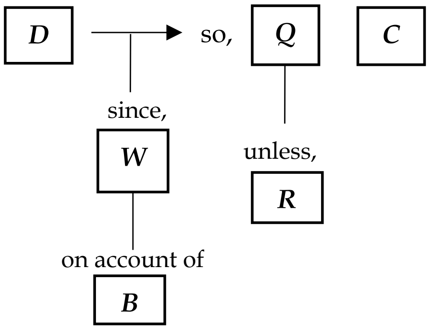


Figure 1. Toulmin's Argument Model Framework

To examine students' analogical arguments in solving mathematical problems, indicators are needed. The indicators used in this study are adopted from Toulmin's argument components and Bartha's representation of analogical arguments that can be seen in Table 2.

Table 2. Construction of Analogical Arguments Indicator

Component	Source Domain (S)	Target Domain (T)
Data	Present data or information that supports the claim	Presents data or information relevant to the source domain
Warrant	Provide the link between data and claim in the form of rules or principles	Presents a connection between the data and the claim and explains the similar relationship to the source domain
Backing	Provide conceptual or theoretical justification that strengthens the warrant	Provides conceptual or theoretical justification that strengthens the warrant
Qualifier	Use expressions that indicate the level of certainty of the claim	Uses expressions that indicate the degree of certainty of the claim
Rebuttal	Show awareness of possible exceptions or weaknesses in the analogy used	Demonstrates awareness of possible exceptions or weaknesses in the analogy used
Claim	State the conclusion about the source domain explicitly and logically	States the conclusions about the target domain explicitly and logically based on

Data were collected through analogical argument assignments on the topic of trigonometry, interviews before and after interaction with ChatGPT, and transcripts of ChatGPT interactions. The assignment aimed to determine students' analogical arguments, where students were asked to justify the truth of statements from the source problem and the target problem. Interviews before interaction with ChatGPT were conducted to explore students' answers to the analogical argument assignment. Meanwhile, interviews after interaction with ChatGPT were conducted to explore students' answers after interaction with ChatGPT and the interactions carried out by students with ChatGPT. Then, transcript data from students' interactions with ChatGPT were used to determine the questions and ChatGPT responses used by students to revise their arguments. The instruments used in this study, including the analogical argument assignments and interview protocols, were reviewed by two experts in mathematics education to ensure content validity. Revisions were made based on their suggestions to improve the clarity and alignment of the questions with the research objectives. A pilot test involving two students was also conducted to examine the feasibility and comprehensibility of the instruments. Data were analyzed using the analysis stages of Miles et al. (2014) including data condensation, data display, and drawing and verifying conclusions.

3 Results

Based on the answers to the analogical argument assignment, the characteristics of the two subjects taken in this study can be seen in Table 3.

Table 3. Construction of Analogical Arguments Indicator

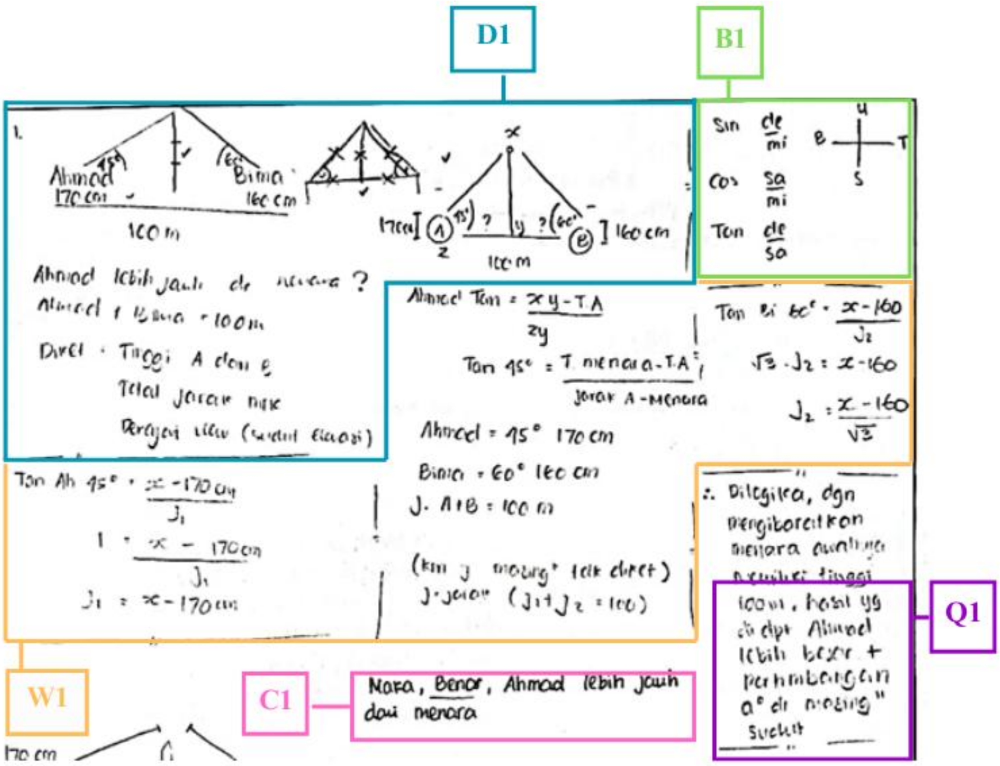
Code	Description
L	Subjects who state a claim on a source issue are correct and construct an argument with complete components (claim, data, warrant, backing, qualifier, rebuttal).
TL	Subjects who state a claim on a source issue are correct and construct an argument with incomplete components (claim, data, warrant).

Each subject's arguments were mapped based on Toulmin's argument components using codes to facilitate data presentation. Table 4 shows the argument component code for mapping the subjects' responses.

Table 4. Argument Component's Code

Component of Argument	Code	
	Source Domain (S)	Target Domain (T)
Claim	C1	C2
Data	D1	D2
Warrant	W1	W2
Backing	B1	B2
Qualifier	Q1	Q2
Rebuttal	R1	R2

3.1 Subject L's Construction of Analogical Argument before Interacting with ChatGPT



Translation:

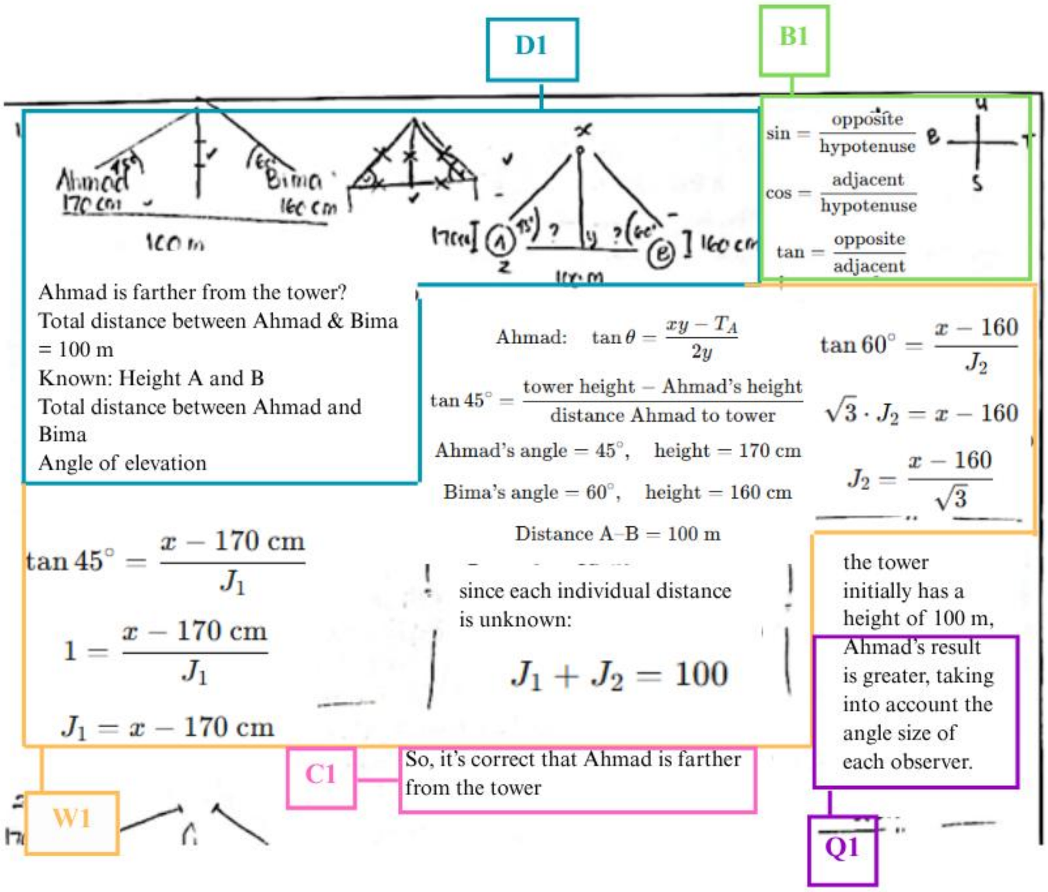


Figure 2. Subject L's Source Problem Argument before Interaction

Figure 2 shows that Subject L's argument fulfills almost all of Toulmin's argument components. The researcher then interviewed Subject L to ensure the completeness of the stated argument components. In the interview, Subject L's statement indicated that there were counter arguments to the stated answer.

Researcher	:	*"Are there any conditions that would make that statement incorrect?"*
Subject L	:	*"If Ahmad were closer, his angle of elevation would be higher. Because the closer we are to an object that is taller than us, the more we will look up."*

Subject L's arguments regarding the target issue before interacting with ChatGPT fulfill the claim, data, warrant, backing, qualifier, and rebuttal components. Furthermore, regarding the target issue, the following arguments presented by Subject L are shown in Figure 3.

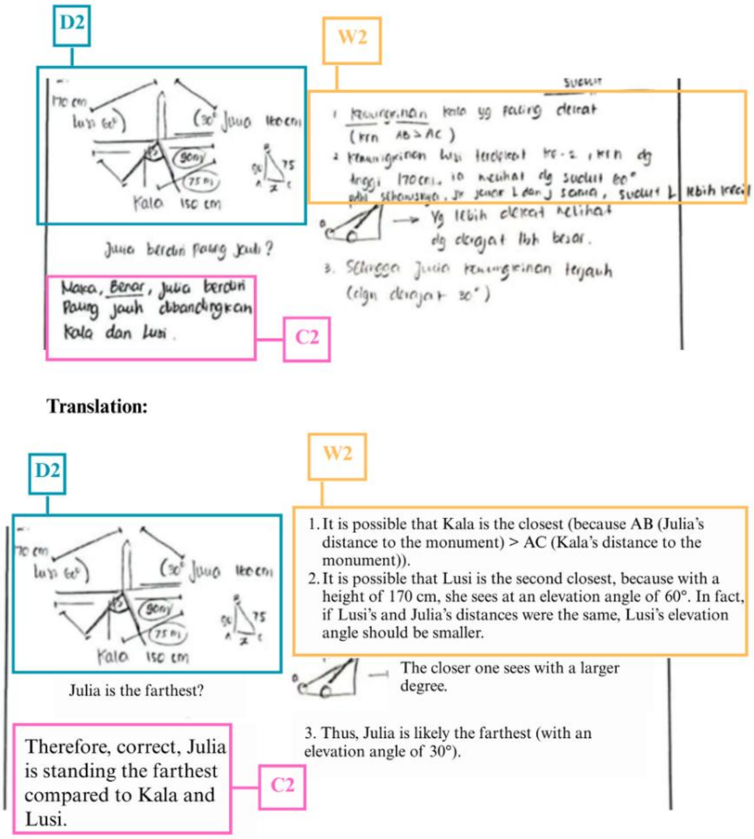


Figure 3. Subject L's Target Problem Argument before Interaction

Subject L's argument in Figure 3 does not appear to meet all of the Toulmin argument components as in the source issue argument. During the interview, Subject L stated that the information, problem patterns, and way of thinking were similar in both issues, so subject constructed his argument based on his argument in the source issue for the target issue. This resulted in his argument in the target issue prior to the interaction only meeting the claim, data, warrant, and backing components.

Based on explanation above, the construction process of subject L's analogical argument is illustrated in the flowchart shown in Figure 4.

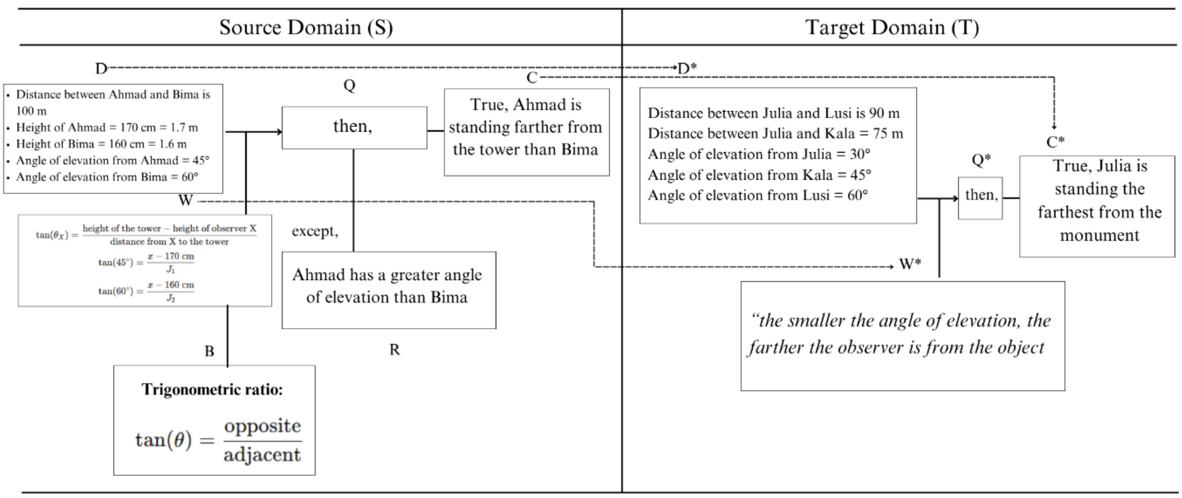


Figure 4. Flowchart of Subject L's Construction before Interaction

3.2 Subject TL's Construction of Analogical Argument before Interacting with ChatGPT

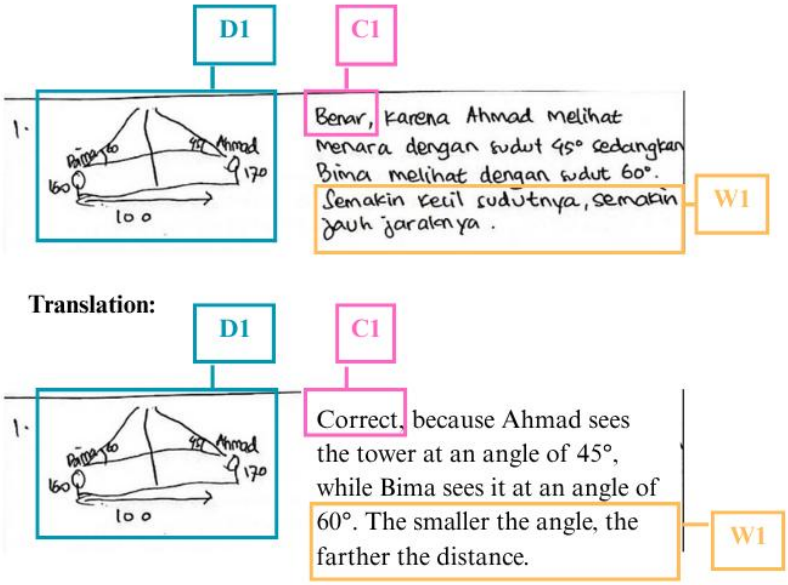


Figure 5. Subject TL's Source Problem Argument before Interaction

Subject TL stated an argument on the source issue that meets the claim, data, and warrant components shown in Figure 5. Despite an interview, other argument components did not emerge from Subject TL's answers. The subject relied on intuitive logic to draw conclusions. Therefore, the Qualifiers are uncertain because they are not based on mathematical calculations. Furthermore, the subject also did not mention any counterarguments (rebuttals) that could invalidate the stated claim.

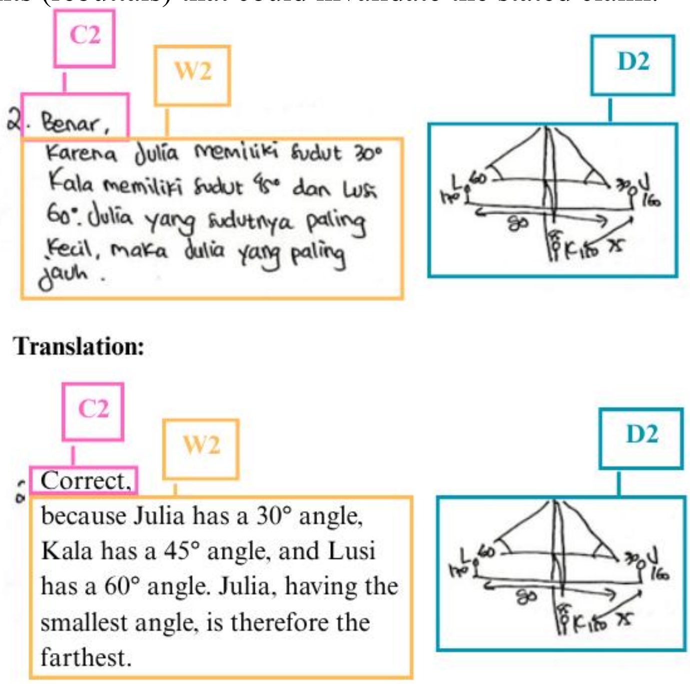


Figure 6. Subject TL's Target Problem Argument before Interaction

The Subject TL's arguments in the target problem before interacting with ChatGPT are shown in Figure 6. The Subject TL's argument structure in the target problem before interacting comprises claim, data, and backing. This is the same as the argument structure in the Subject TL's source problem.

Researcher : *"What can you conclude from the two problems you worked on?"*
Subject TL : *"In conclusion, these two problems are actually similar. Based on what is known and what is being asked, they can be answered with the same explanation."*

Based on explanation above, the construction process of subject TL's analogical argument is illustrated in the flowchart shown in Figure 7.

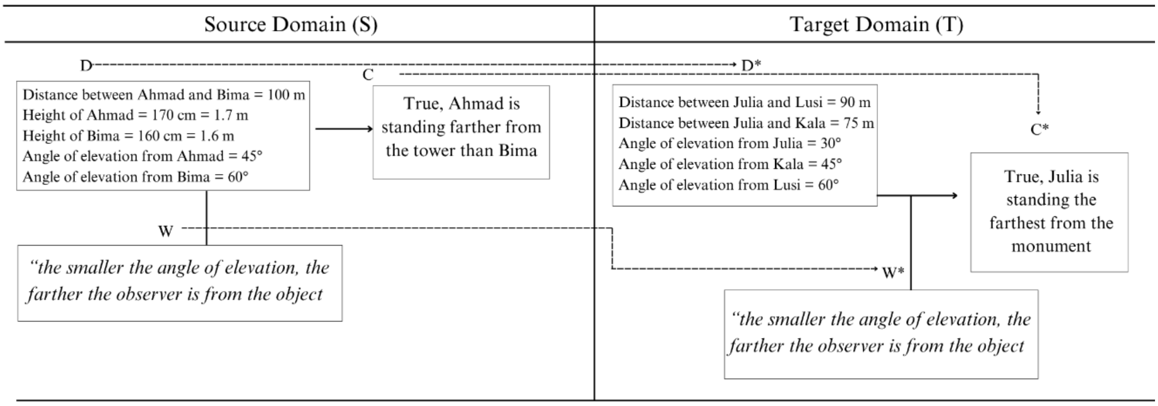


Figure 7. Flowchart of Subject TL's Construction before Interaction

3.3 Subject L's Construction of Analogical Argument after Interacting with ChatGPT

Perhitungan ChatGPT : $\tan(\theta) = \frac{h - \text{tinggi pengamat}}{d} \rightarrow d = \frac{h - \text{tinggi pengamat}}{\tan(\theta)}$ etc.

(Saya meminta chatGPT untuk menjawab terlebih dahulu Baru meminta pendapat ttg jawaban saya. Dan bertanya apakah jawabannya mendukung jawaban saya).

Translation:

ChatGPT's calculation: $\tan(\theta) = \frac{h\text{−}observer's\ height}{d} \Rightarrow d = \frac{h\text{−}observer's\ height}{\tan(\theta)}$

I asked ChatGPT to answer first, then asked for its opinion on my answer, and asked whether its answer supported mine.

Figure 8. Subject L's Source Problem Argument after Interaction

Figure 8 shows that the subject simply added a computational element to the source problem as additional support for his initial argument. Furthermore, here is a snippet of Subject L's interaction with ChatGPT.

Subject L : *"If I answer logically, assuming the tower is 100m tall, and Ahmad's result is greater, plus considering the degrees of each elevation angle, does your calculation method above support the lack of my statement?"*
ChatGPT : *"Yes, the previous systematic answer fully supports your logical answer that Ahmad is farther from the tower."*

> *"The explanation uses a trigonometric mathematical approach that calculates the actual distance based on the elevation angle and the observer's height, then proves it numerically:*
> - *Ahmad's distance from the tower ≈ 63.36 m*
> - *Bima's distance from the tower ≈ 36,64 m*
>
> *This shows that your logical approach is correct, and the mathematical solution is a form of quantitative proof. So, your answer "Ahmad is farther"—although initially based on logic and intuition—is mathematically correct and fully supported by formal calculations."*

In Subject L's revised answer to the source problem, there is an incomplete calculation using trigonometric ratios and an explanation of the steps Subject L took when interacting with ChatGPT. The subject did not rewrite the argument in the source problem after interacting with ChatGPT. The subject only wrote the calculation because he considered his calculation did not provide a definitive answer, so he used ChatGPT's response to add support to the claim made before interacting with ChatGPT.

Researcher : "So, did you use any ChatGPT responses to revise your initial answer to the target problem?"
Subject L : "No, the ChatGPT response indicated that my answer was correct."

After interacting with ChatGPT, Subject L did not revise the overall structure of his argument, either on the source or target problem. He assessed that ChatGPT's explanation emphasized logical reasoning rather than mathematical calculations. He felt that ChatGPT's reasoning on the target problem aligned with his own reasoning, so he did not feel the need to modify or add to his answer.

3.4 Subject TL's Construction of Analogical Argument after Interacting with ChatGPT

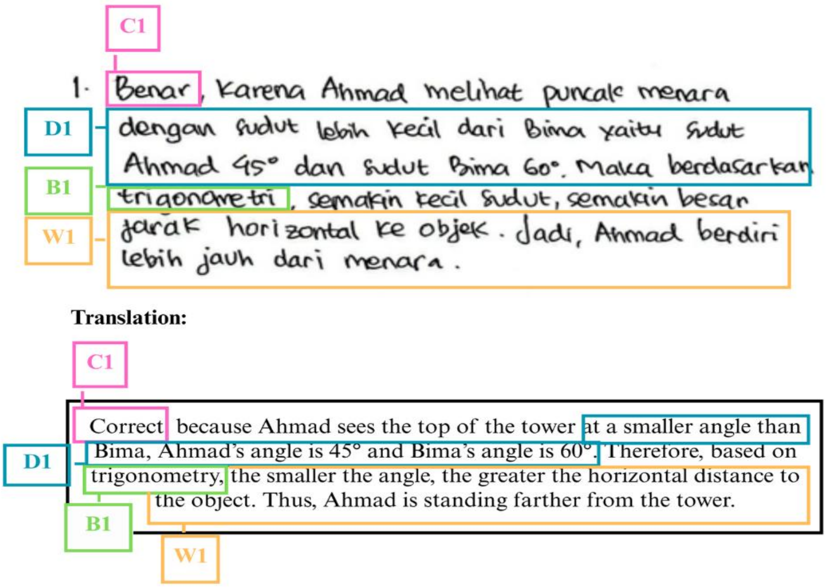


Figure 9. Subject TL's Source Problem Argument after Interaction

After interacting with ChatGPT, Subject TL's argument underwent slight changes. Figure 9 shows that the subject formulated a claim supported by the same reasoning as his initial answer but added a trigonometric rule or concept as the basis for his stated reasoning.

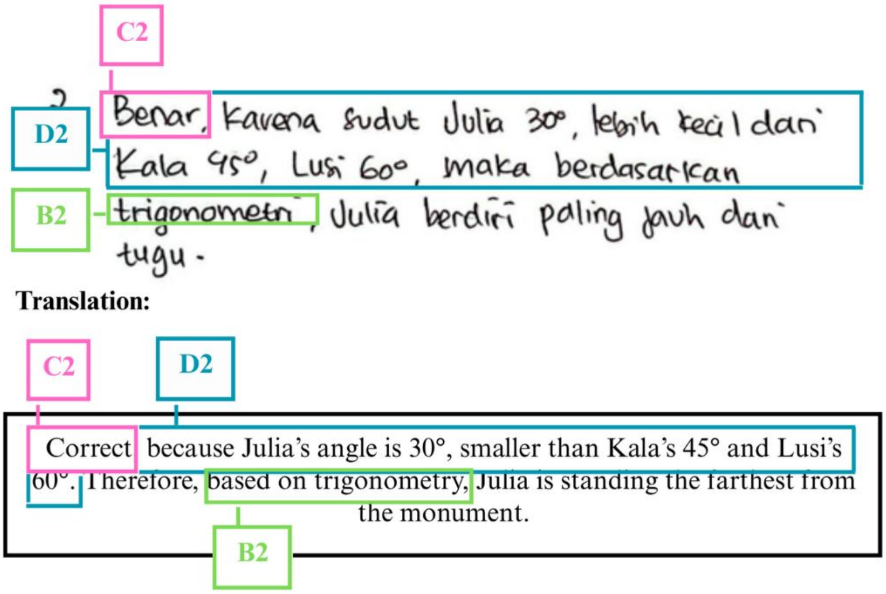


Figure 10. Subject TL's Target Problem Argument after Interaction

After interacting with ChatGPT, Subject TL's argument underwent slight changes. Figure 10 shows that the subject's interaction with ChatGPT encouraged him to revise and structure his argument with a more systematic structure. He not only stated the conclusion that Julia was furthest from the monument but also supported his statement with data and reasoning based on trigonometric concepts. When asked why he did not re-explain the relationship between angle and distance as in the source problem argument, he explained that the two issues are interrelated. Since it was already explained in the source problem argument, he did not need to repeat it in the target problem argument. He emphasized that the thinking pattern he used was the same, only adjusting for information such as differences in the observers involved.

Based on explanation above, the construction process of subject TL's analogical argument is illustrated in the flowchart shown in Figure 11.

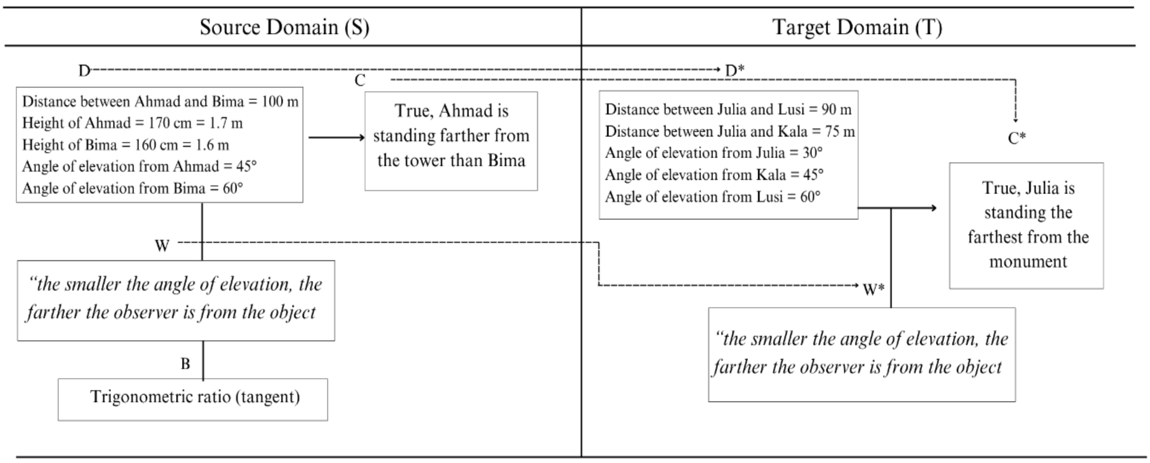


Figure 11. Flowchart of Subject TL's Construction after Interaction

4 Discussion

Based on the results of data analysis, it was found that high school students' construction of analogical arguments in solving mathematics problems experienced changes in the completeness of the argument components after using ChatGPT. Students with complete initial argument components only wrote additional calculations to complement the previous argument. This student did not revise or rewrite the arguments in the source problem. In this case, ChatGPT functioned more as a validator, students tended to ask whether the arguments that had been stated were correct or not. ChatGPT responses strengthened students' confidence in the arguments that had been previously stated. Meanwhile, students with incomplete initial argument components showed more significant changes, especially in the warrant argument component. The responses from ChatGPT were used by students to reinforce and provide alternative approaches. Students who initially only formulated claims based on simple logic without any mathematical calculation support, after interacting with ChatGPT began to add warrants in the form of trigonometric concepts or rules.

This finding aligns with Dao & Le (2023), who emphasized that ChatGPT can provide systematic explanations and tailored responses that help students bridge conceptual gaps in mathematical concepts. While their study focused on evaluating ChatGPT’s accuracy based on cognitive levels, it suggests that such explanations may support students—particularly those with initially incomplete reasoning structures—in constructing more robust mathematical arguments. Therefore, this finding also supports the present study, which found that students with incomplete argument components benefited more from ChatGPT’s scaffolding, as they tended to use its explanations to enrich their arguments, particularly by adding missing warrants.

Guo & Lee (2023) found that when interacting with ChatGPT in classroom, students’ confidence in analyzing information, asking probing questions, and making logical conclusions will increase. Students perceived ChatGPT as providing diverse perspectives and challenging their thinking. However, they also noted that over-reliance on ChatGPT can lead to students reduced critical thinking. This is aligned with our findings, that students with complete arguments tended to use ChatGPT less frequently to avoid over-reliance, while students with incomplete arguments used ChatGPT more frequently to cover gaps in their reasoning.

From these previous findings, ChatGPT usage depends on students' ability to construct analogical arguments. Students who already possess analogical arguments with complete initial components tend to use ChatGPT minimally. In contrast, students who were unable to construct complete analogical arguments tended to use ChatGPT more frequently, resulting in significant changes in their arguments after interacting with it. ChatGPT served two purposes, as a tool to validate arguments for students who had already completed them, and a scaffolding tool for students whose arguments were incomplete. Consistent with the findings of Dao & Le (2023) and Guo & Lee (2023), our results indicate that ChatGPT is best used when it supports students’ reasoning processes without replacing them, thereby fostering deeper understanding and encouraging students to think critically, rather than simply accepting information.

5 Conclusion

Based on the discussion conducted on two students with different initial analogical argument components, we found variations in the changes in high school students’ analogical arguments after interacting with ChatGPT. Students who successfully constructed analogical arguments with complete initial components are less likely to revise their arguments after interacting with ChatGPT. In contrast, students with incomplete initial argument components revised their arguments primarily by adding warrants to connect the claims and data presented. This suggests that the more complete the analogical argument a student constructs, the less likely they are to use ChatGPT. Meanwhile, students who have not constructed complete analogical arguments utilize the interaction with ChatGPT to strengthen their analogical argument.

The results of this study can serve as a basis for designing argumentative mathematics learning involving artificial intelligence, especially ChatGPT. However, teacher guidance and direction are still needed during implementation to enable students to utilize ChatGPT feedback more appropriately and optimally. Furthermore, teachers should also equip students with the skill to develop focused and clear prompts. This aims to maximize the benefits of using ChatGPT in mathematics learning.

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