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EIGENVALUES OF THE LESLIE MATRIX AND ITS APPLICATION TO FEMALE POPULATION GROWTH RATE IN BANYUMAS REGENCY

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ABSTRACT

The Leslie matrix model is used to estimate the size and growth rate of the female population based on fertility and survival rates in each age group. This study aims to analyze the behavior of the dominant eigenvalues of the Leslie matrix and apply it to estimate the growth rate of the female population in Banyumas Regency. The data used in this study consist of the female population in 2019 and 2024, as well as the number of female births during the period 2019–2024. The research stages include proving that the Leslie matrix has an unique positive eigenvalues and that two consecutive entries in the first row of the Leslie matrix are nonzero. The subsequent stages involve determining age groups, calculating fertility and survival rates, constructing the Leslie matrix and the initial age-distribution vector, and determining the dominant eigenvalues to estimate the growth rate of the female population in Banyumas Regency. The results show that the Leslie matrix has dominant positive eigenvalues. Furthermore, the dominant eigenvalues obtained from the female population growth model in Banyumas Regency is 0.991. It is indicating that the female population growth rate is expected to decline 99,1% in each five-year period. This result suggests a long-term slowdown in the growth of the female population in Banyumas Regency.

Keywords: *Dominant eigenvalues, Female, Growth rate, Leslie matrix, Population.*

1. Introduction

Population growth refers to an increase in the number of individuals in a given region over a certain period, influenced by birth and death rates. This growth is dynamic as it is affected by demographic factors such as fertility rates, survival rates, and age structure, which in turn influence age composition, gender distribution, and geographic distribution. Various methods can be used to estimate population growth, including Arithmetic, Geometric, Exponential, Least Squares, Malthusian, Logistic, Autoregressive Integrated Moving Average (ARIMA), and the Leslie Matrix methods. The study in [1] has discussed about application of the logistic growth model in determining population projections in Banyumas Regency.

The Leslie matrix model is a demographic tool used to estimate population structure and growth based on fertility and survival rates across age groups. This model was developed by P. H. Leslie in 1945 and has been widely applied in population ecology and demographic studies. It is typically applied to female populations since only females contribute to reproduction Population growth rate can be determined using the largest eigenvalues (dominant eigenvalues) of the Leslie matrix. Much research has been done on the use of the Leslie matrix in approach to analyze the growth rate of the female population (See [2,3,4,5,6,7])

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Banyumas Regency is one of the regions with a significant population in Central Java, with a population density of 1,391 people/km² in 2024. The female population plays an essential role in population dynamics because fertility rates in age-structured population models, such as the Leslie matrix, are determined based on female reproductive groups. Therefore, analyzing the growth rate of the female population in Banyumas Regency provides an important case study for understanding long-term population dynamics and stable age distribution patterns. The 2020-2023 Covid-19 outbreak is expected to have a significant impact on the female population in Banyumas.

The previous study [1] has discussed about application of the logistic growth model in determining population projections in Banyumas Regency. Meanwhile, the study in [7] discusses the size and growth rate of the female population in Banyumas Regency for 2027. In this study, the Leslie matrix approach is applied to analyze the growth rate of the female population in Banyumas Regency over a longer period. The data used in this study is data on the number of women in Banyumas at 2017-2022, which is still slightly contaminated by the influence of the Covid-19 outbreak. The study also examines the behavior of the dominant eigenvalues and the convergence of age-distribution proportions toward a stable age distribution.

Based on these considerations, this article will present the behavior of the dominant eigenvalues in Leslie matrix and the convergence of age-distribution proportions in Banyumas Regency using data in [8], i.e the number of women in Banyumas Regency at Covid-19 outbreak in 2019-2024. We will also see whether data from the time of the Covid-19 outbreak will have an impact on the estimated female population in Banyumas Regency in the coming period. The study focuses on the female population using a five-year interval per period. This interval was selected based on simulation results showing that a one-year interval produces larger estimation errors compared to a five-year interval, thus resulting in less accurate estimates.

2. Literature Review

One of the most widely used population growth models in demography is the Leslie matrix model, developed by P. H. Leslie in the 1940s. This model is used to estimate population size and growth rate based on fertility and survival rates. The studies about theory of Leslie matrix can be found in [9,10,11,12,13,14]. In this study, the population considered is human, specifically the female population. The general form of the Leslie matrix is given as follows.

$$\mathbf{L} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & b_{k-1} & 0 \end{bmatrix}$$

The parameter a_i represents the fertility rate of females in the i-th age group, defined as the average number of female births produced by women in that age group. The fertility rate satisfies $a_i \geq 0$ for $i = 1,2,\ldots,k$, with the assumption that at least one age group has $a_i > 0$. The parameter b_i denotes the survival rate of females in the i-th age group, defined as the probability that an individual aged i at time t survives to age $i + 1$ at time $t + 1$. The survival rate satisfies $0 < b_i \leq 1$ for $i = 1,2,\ldots,k-1$ [9].

The Leslie matrix model describes the growth of human or animal populations, particularly female populations. The population is divided into several age groups of equal intervals. If the maximum age is M years and the population is divided into k age groups, then each age group spans an interval of $\frac{M}{k}$ years [10].

Table 1. Age Group Classification

Age Group-i	**Age Interval**
1	$\left[0, \frac{M}{k}\right)$
2	$\left[\frac{M}{k}, \frac{2M}{k}\right)$
3	$\left[\frac{2M}{k}, \frac{3M}{k}\right)$
$\vdots$	$\vdots$
$k-1$	$\left[\frac{(k-2)M}{k}, \frac{(k-1)M}{k}\right)$
k	$\left[\frac{(k-1)M}{k}, M\right]$

Let $n(t)$ denote the female population at time t, where $n_i(t)$ represents the number of females in the i-th age group for $i = 1,2,\dots,k$. The population distribution across age groups at time t, denoted by $n(t)$, forms a column vector called the initial age distribution vector. This vector serves as the basis for estimating the female population in future periods as $n(t+p)$. The vector $\boldsymbol{n}(t)$ can be written as follows.

$$\mathbf{n}(t) = \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ \vdots \\ n_k(t) \end{bmatrix}$$

Let $n_i(t+1)$ denote the number of females in the i-th age group at time $t+1$. The total female population at time $t+1$ is a column vector $\boldsymbol{n}(t+1)$ can be written as follows.

$$\mathbf{n}(t+1) = \begin{bmatrix} n_1(t+1) \\ n_2(t+1) \\ n_3(t+1) \\ \vdots \\ n_k(t+1) \end{bmatrix}$$

The number of females in the first age group at time $t+1$ represents new births and is determined by the fertility rates of all age groups, given by

$$n_1(t+1) = a_1 n_1(t) + a_2 n_2(t) + \cdots + a_k n_k(t) \tag{1}$$

The number of females in the second age group at time $t+1$ is determined by the number of females in the first age group at time t who survive to the next period, given by

$$n_2(t+1) = b_1 n_1(t) \tag{2}$$

Similarly, for higher age groups, the number of females in the k-th age group at time $t+1$ depends on those in the $(k-1)$-th age group at time t who survive to time $t+1$, given by

$$n_k(t+1) = b_{k-1} n_{k-1}(t) \tag{3}$$

Therefore, the Leslie matrix model for the female population over the next p periods is given by

$$\mathbf{n}(t+p) = \begin{bmatrix} n_1(t+p) \\ n_2(t+p) \\ n_3(t+p) \\ \vdots \\ n_k(t+p) \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \cdots & a_{k-1} & a_k \\ b_1 & 0 & \cdots & 0 & 0 \\ 0 & b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & b_{k-1} & 0 \end{bmatrix}^p \begin{bmatrix} n_1(t) \\ n_2(t) \\ n_3(t) \\ \vdots \\ n_k(t) \end{bmatrix} = \mathbf{L}^p \mathbf{n}(t) \tag{4}$$

3. Results and Discussion

3.1. Eigenvalues of The Leslie Matrix

Eigenvalues have an important role in analyzing long-term population dynamics and determining whether a population will increase, decrease, or remain stable. Although the Leslie matrix model can be used to calculate the size of the female population at a given time in the future, it does not directly provide a general description of the population growth rate. Therefore, the behavior of the eigenvalues of the Leslie matrix is investigated to determine the population growth rate in the future. The behavior of the eigenvalues of the Leslie matrix can be formulated through the following theorems.

Theorem 3.1 The Leslie matrix $\mathbf{L}$ has a unique positive eigenvalue, denoted by λ_1 with multiplicity one and a corresponding eigenvector $\boldsymbol{x_1}$ whose entries are all positive.

PROOF.

(i) It will be shown that the Leslie matrix has a unique positive eigenvalue with multiplicity one, namely λ_1. The eigenvalues of the Leslie matrix $\mathbf{L}$ are given by the roots of its characteristic equation. The characteristic equation is given by

$$p(\lambda) = \det(\lambda\mathbf{I} - \mathbf{L}) = 0 \tag{5}$$

Note that

$$\det(\lambda\mathbf{I} - \mathbf{L}) = \det\left(\begin{bmatrix} \lambda - a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \\ -b_1 & \lambda & \cdots & 0 & 0 \\ 0 & -b_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -b_{n-1} & \lambda \end{bmatrix}\right)$$

Use cofactor expansion along the first row, obtained

$$p(\lambda) = \lambda^n - a_1\lambda^{n-1} - a_2b_1\lambda^{n-2} - \cdots - a_nb_1b_2\cdots b_{n-1}$$

so

$$\lambda^n - a_1\lambda^{n-1} - a_2b_1\lambda^{n-2} - \cdots - a_nb_1b_2\cdots b_{n-1} = 0 \tag{6}$$

For $\lambda \neq 0$, dividing both sides by λ^n gives

$$1 = \frac{a_1}{\lambda} + \frac{a_2b_1}{\lambda^2} + \cdots + \frac{a_nb_1b_2\cdots b_{n-1}}{\lambda^n} \tag{7}$$

Let

$$q(\lambda) = \frac{a_1}{\lambda} + \frac{a_2b_1}{\lambda^2} + \cdots + \frac{a_nb_1b_2\cdots b_{n-1}}{\lambda^n}, \tag{8}$$

based on equations (7) and (8), it follows that

$$q(\lambda) = 1 \text{ for } \lambda \neq 0 \tag{9}$$

Suppose that two positive eigenvalues of the Leslie matrix, namely λ_1 and λ_2 satisfy equation (9), then

$$q(\lambda_1) = \frac{a_1}{\lambda_1} + \frac{a_2b_1}{\lambda_1^2} + \frac{a_3b_1b_2}{\lambda_1^3} + \cdots + \frac{a_nb_1b_2\cdots b_{n-1}}{\lambda_1^n} = 1$$

and

$$q(\lambda_2) = \frac{a_1}{\lambda_2} + \frac{a_2b_1}{\lambda_2^2} + \frac{a_3b_1b_2}{\lambda_2^3} + \cdots + \frac{a_nb_1b_2\cdots b_{n-1}}{\lambda_2^n} = 1$$

If $q(\lambda_1) = q(\lambda_2)$, so

$$\frac{a_1}{\lambda_1\lambda_2}(\lambda_2 - \lambda_1) + \frac{a_2b_1}{\lambda_1^2\lambda_2^2}(\lambda_2^2 - \lambda_1^2) + \cdots + \frac{a_nb_1b_2\cdots b_{n-1}}{\lambda_1^n\lambda_2^n}(\lambda_2^n - \lambda_1^n) = 0 \tag{10}$$

The equation (10) will be valid if $\lambda_1 = \lambda_2$. This implies that $q(\lambda)$ is injective for $\lambda > 0$. Since a_i and b_i are nonnegative, the function $q(\lambda)$ is monotonically decreasing in λ and concave up for $\lambda > 0$. Moreover, $q(\lambda)$ has a vertical asymptote at $\lambda = 0$ and approaches zero as $\lambda \to \infty$. Each eigenvalue λ corresponds to exactly one solution of $q(\lambda)$. Since $q(\lambda)$ is injective for $\lambda > 0$, the equation $q(\lambda) = 1$ has a unique positive solution, say $\lambda = \lambda_1$ satisfies $(\lambda_1) = 1$. Therefore, it is proven that the Leslie matrix has a unique positive eigenvalue. In other words, λ_1 has algebraic multiplicity one, meaning that it is not a repeated root of the characteristic equation of the Leslie matrix.

For $\lambda < 0$, each term in $q(\lambda)$ tends to zero as $\lambda \to -\infty$. Terms involving odd powers of λ, such as $\frac{a_1}{\lambda}$, $\frac{a_3 b_1 b_2}{\lambda^3}$, and so on, are negative, while terms involving even powers, such as $\frac{a_2 b_1}{\lambda^2}$, $\frac{a_4 b_1 b_2 b_3}{\lambda^4}$, and so on, are positive. Since the signs of the terms alternate for $\lambda < 0$, the injectivity of $q(\lambda)$ is not guaranteed in this case. Consequently, for $\lambda < 0$, there may not exist a unique solution to the equation $q(\lambda) = 1$.

(ii) It will be shown that each entry of the eigenvector $\mathbf{x_1}$ is positive.

Let $\mathbf{x_1}$ be an eigenvector of $\mathbf{L}$ corresponding to the eigenvalues λ_1, satisfying $(\lambda_1 \mathbf{I} - \mathbf{L})\mathbf{x_1} = 0$. Let

$$\mathbf{x_1} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}$$

so

$$\begin{bmatrix} (\lambda_1 - a_1)x_1 - a_2 x_2 - \cdots - a_{n-1}x_{n-1} - a_n x_n \\ -b_1 x_1 + \lambda_1 x_2 \\ -b_2 x_2 + \lambda_1 x_3 \\ \vdots \\ -b_{n-2}x_{n-2} + \lambda_1 x_{n-1} \\ -b_{n-1}x_{n-1} + \lambda_1 x_n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

The following system of linear equations is obtained

$$(\lambda_1 - a_1)x_1 - a_2 x_2 - \cdots - a_{n-1}x_{n-1} - a_n x_n = 0 \tag{11}$$

$$-b_1 x_1 + \lambda_1 x_2 = 0 \Leftrightarrow x_2 = \frac{b_1 x_1}{\lambda_1} \tag{12}$$

$$-b_2 x_2 + \lambda_1 x_3 = 0 \Leftrightarrow x_3 = \frac{b_2 x_2}{\lambda_1} \Leftrightarrow x_3 = \frac{b_1 b_2 x_1}{\lambda_1^2} \tag{13}$$

$$\vdots$$

$$-b_{n-2}x_{n-2} + \lambda_1 x_{n-1} = 0 \Leftrightarrow x_{n-1} = \frac{b_{n-2}x_{n-2}}{\lambda_1} \Leftrightarrow x_{n-1} = \frac{b_1 b_2 \ldots b_{n-2} x_1}{\lambda_1^{n-2}} \tag{14}$$

$$-b_{n-1}x_{n-1} + \lambda_1 x_n = 0 \Leftrightarrow x_n = \frac{b_{n-1}x_{n-1}}{\lambda_1} \Leftrightarrow x_n = \frac{b_1 b_2 \ldots b_{n-1} x_1}{\lambda_1^{n-1}} \tag{15}$$

Then, substituting equations (12), (13), (14), and (15) into equation (11), obtained

$$x_1 \left(\lambda_1 - a_1 - a_2 \frac{b_1}{\lambda_1} - a_3 \frac{b_1 b_2}{\lambda_1^2} - \cdots - a_{n-1} \frac{b_1 b_2 \ldots b_{n-2}}{\lambda_1^{n-2}} - a_n \frac{b_1 b_2 \ldots b_{n-1}}{\lambda_1^{n-1}} \right) = 0$$

so

$$x_1 = 0 \tag{16}$$

Since $\mathbf{x_1}$ is a nonzero eigenvector corresponding to λ_1, it follows from equation (16) that if $x_1 = 0$, then the eigenvector corresponding to λ_1 would be the zero vector. Therefore, let $x_1 = u \in \mathbb{R}^+$, then

$$\mathbf{x_1} = u\begin{bmatrix} 1 \\ \dfrac{b_1}{\lambda_1} \\ \dfrac{b_1 b_2}{\lambda_1^2} \\ \vdots \\ \dfrac{b_1 b_2 \dots b_{n-2}}{\lambda_1^{n-2}} \\ \dfrac{b_1 b_2 \dots b_{n-1}}{\lambda_1^{n-1}} \end{bmatrix} \tag{17}$$

with $u \in \mathbb{R}^+$.

Therefore, based on the equation (17), it follows that the eigenspace corresponding to $\mathbf{x_1}$ is one dimension. Thus, its geometric multiplicity is one and all entries of the eigenvector $\mathbf{x_1}$ are positive. ■

Definition 3.1 (Dominant Eigenvalues) Let $\lambda_1, \lambda_2, \dots, \lambda_n$ be the eigenvalues of the Leslie matrix **L** of order $n \times n$. The eigenvalues λ_1 is called the dominant eigenvalues of **L** if $\lambda_1 > |\lambda_k|$ for every $k = 2,3,\cdots,n$.

Theorem 3.2 If λ_1 is a unique positive eigenvalue of a Leslie matrix **L** and λ_k is any real or complex eigenvalue of Leslie matrix **L**, then $\lambda_1 \geq |\lambda_k|$ for every $k = 2,3,\cdots,n$.

PROOF. It will be shown that $\lambda_1 \geq |\lambda_k|$ for every $k = 2,3,\cdots,n$. From Theorem 3.1, it has been shown that λ_1 is a unique positive eigenvalue. Furthermore, for $k = 2,3,\cdots,n$, λ_k are eigenvalues that may be real or complex.

Let

$$\lambda_k = re^{i\theta} = r(\cos\theta + i\sin\theta) \tag{18}$$

Then, since $|\lambda_k| = r$, to prove that $\lambda_1 \geq |\lambda_k|$, it suffices to show that $\lambda_1 \geq r$. From equation (9), it is known that

$$q(\lambda) = \frac{a_1}{\lambda} + \frac{a_2 b_1}{\lambda^2} + \frac{a_3 b_1 b_2}{\lambda^3} + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{\lambda^n} = 1$$

with $q(\lambda) = 1$ for $\lambda \neq 0$. Furthermore, since λ_1 and λ_k are eigenvalues, then

$$q(\lambda_1) = \frac{a_1}{\lambda_1} + \frac{a_2 b_1}{\lambda_1^2} + \frac{a_3 b_1 b_2}{\lambda_1^3} + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{\lambda_1^n} = 1 \tag{19}$$

and

$$q(\lambda_k) = \frac{a_1}{\lambda_k} + \frac{a_2 b_1}{\lambda_k^2} + \frac{a_3 b_1 b_2}{\lambda_k^3} + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{\lambda_k^n} = 1 \tag{20}$$

Then, substituting equation (18) into equation (20). Since $q(\lambda_k) = 1$, obtained

$$\frac{a_1}{r}(\cos\theta - i\sin\theta) + \frac{a_2 b_1}{r^2}(\cos(2\theta) - i\sin(2\theta)) + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{r^n}(\cos(n\theta) - i\sin(n\theta)) = 1 \tag{21}$$

If taken only the real part of equation (21), then

$$\frac{a_1}{r}(\cos\theta) + \frac{a_2 b_1}{r^2}(\cos(2\theta)) + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{r^n}(\cos(n\theta)) = 1 \tag{22}$$

Then, since $\cos(i\theta) \leq 1$ for every $i = 1,2,\cdots,n$, and based on equation (22), it follows that

$$1 \leq \frac{a_1}{r} + \cdots + \frac{a_n b_1 b_2 \dots b_{n-1}}{r^n} \tag{23}$$

Then, substituting equation (19) into equation (23), obtained

$$\frac{a_1}{\lambda_1}+\frac{a_2b_1}{\lambda_1^2}+\cdots+\frac{a_nb_1b_2\ldots b_{n-1}}{\lambda_1^n}\leq\frac{a_1}{r}+\frac{a_2b_1}{r^2}+\cdots+\frac{a_nb_1b_2\ldots b_{n-1}}{r^n} \tag{24}$$

Then, since $a_nb_1b_2\cdots b_{n-1}\geq 0$, it follows from equation (24) that $\lambda_1\geq r$, and hence $\lambda_1\geq|\lambda_k|$. Therefore, it is proven that for any λ_k, the inequality $\lambda_1\geq|\lambda_k|$ satisfied for every $k=2,3,\cdots,n$. Furthermore, if $\lambda_1\geq|\lambda_k|$, then λ_1 is called the dominant eigenvalues of the Leslie matrix **L**. ■

Theorem 3.3 If two consecutive entries a_i and a_{i+1} in the first row of a Leslie matrix **L** are both nonzero, then the positive eigenvalues of the Leslie matrix **L** is dominant.

PROOF. The proof will be carried out by contradiction. Suppose λ_1 is the positive eigenvalue that is not dominant, so that there exists λ_k such that $\lambda_1\geq|\lambda_k|$ for some k.

From equation (18), it is known that

$$\lambda_k=re^{i\theta}=r(\cos\theta+i\sin\theta)$$

so

$$\lambda_1=|\lambda_k|=r.$$

Furthermore, since λ_k is eigenvalues, then based on equation (21), obtained

$$\frac{a_1}{r}(\cos\theta-i\sin\theta)+\frac{a_2b_1}{r^2}(\cos(2\theta)-i\sin(2\theta))+\cdots+\frac{a_nb_1b_2\ldots b_{n-1}}{r^n}(\cos(n\theta)-i\sin(n\theta))=1$$

Then, since λ_1 is also an eigenvalue, so from equation (9) and $\lambda_1=|\lambda_k|=r$, it follows that

$$q(r)=\frac{a_1}{r}+\frac{a_2b_1}{r^2}+\frac{a_3b_1b_2}{r^3}+\cdots+\frac{a_nb_1b_2\ldots b_{n-1}}{r^n}=1$$

Then, based on equation (22) and $q(r)=1$, it follows that

$$a_1\left(\frac{\cos\theta-1}{r}\right)+a_2b_1\left(\frac{\cos(2\theta)-1}{r^2}\right)+\cdots+a_nb_1b_2\ldots b_{n-1}\left(\frac{\cos(n\theta)-1}{r^n}\right)=0 \tag{25}$$

Based on equation (25), obtained

$$\theta=2\pi(n-m)=2k'\pi \text{ with } k'=n-m\in\mathbb{Z}$$

If $\theta=2k'\pi$, then from equation (18) it follows that $\lambda_k=r$. Furthermore, since $\lambda_1=r$, we have $\lambda_1=\lambda_k$.

If

$$\lambda_1=\lambda_k$$

then based on equation (18), obtained

$$\lambda_1=re^{i\theta}$$

so

$$\lambda_1=r(\cos\theta+i\sin\theta)\Leftrightarrow\lambda_1+i0=r\cos\theta+ir\sin\theta.$$

Thus, obtained

$$\lambda_1=r\cos\theta \tag{26}$$

$$0=r\sin\theta \tag{27}$$

If equation (26) is multiplied by $\cos\theta$, then

$$\lambda_1\cos\theta=r\cos^2\theta$$

so

$$\lambda_1\cos\theta\neq r \text{ or } \lambda_1\cos\theta-r\neq 0.$$

Furthermore, based on equations (19) and (22), it follows that

$$a_1\left(\frac{\lambda_1\cos\theta-r}{r\lambda_1}\right)+a_2b_1\left(\frac{\lambda_1^2\cos 2\theta-r^2}{r^2\lambda_1^2}\right)+\cdots+a_nb_1b_2\ldots b_{n-1}\left(\frac{\lambda_1^n\cos n\theta-r^n}{r^n\lambda_1^n}\right)=0$$

Meanwhile, since $\lambda_1\cos\theta-r\neq 0$, the solution of the equation would only be obtained if $a_1=0$ and $a_2=0$, which contradicts the initial assumption that $a_1>0$ and $a_2>0$. Therefore, the supposition that $\lambda_1=|\lambda_k|$ is incorrect, and it is proven that $\lambda_1\neq|\lambda_k|$. Consequently, if the female population has two consecutive fertile age classes, a_i and a_{i+1} are both nonzero, then the Leslie matrix **L** has a dominant eigenvalue λ_1. ■

In this section, we assume that the Leslie matrix **L** is diagonalizable to examine the asymptotic behavior of the population growth model. It is not strictly necessary to make this assumption; it simplifies the argument. Under this assumption, the matrix **L** has n eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$, which are not necessarily distinct, and n linearly independent eigenvectors $\mathbf{x_1}, \mathbf{x_2}, \ldots, \mathbf{x_n}$ each corresponding to an eigenvalue. In this ordering, the dominant eigenvalues λ_1 is listed first. Let **P** be the matrix whose columns are the eigenvectors of the Leslie matrix **L**, defined as follows

$$\mathbf{P} = [\mathbf{x_1} \quad \mathbf{x_2} \quad \cdots \quad \mathbf{x_n}] \tag{28}$$

and suppose that the matrix **P** is invertible, with inverse $\mathbf{P^{-1}}$. The diagonalization of the Leslie matrix **L** is given by

$$\mathbf{L} = \mathbf{P}\begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n \end{bmatrix}\mathbf{P}^{-1}$$

Hence, it follows that

$$\mathbf{L}^p = \mathbf{P}\begin{bmatrix} \lambda_1^p & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^p & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n^p \end{bmatrix}\mathbf{P}^{-1}.$$

Let $\mathbf{n}(t)$ denote the initial age distribution vector of a female population with $t = 0$. If both sides of the diagonalization of $\mathbf{L}^p$ are multiplied by $\mathbf{n}(t)$, obtained

$$\mathbf{L}^p\mathbf{n}(t) = \mathbf{P}\begin{bmatrix} \lambda_1^p & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^p & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n^p \end{bmatrix}\mathbf{P}^{-1}\mathbf{n}(t)$$

so

$$\mathbf{n}(t+p) = \mathbf{P}\begin{bmatrix} \lambda_1^p & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^p & 0 & \cdots & 0 \\ 0 & 0 & \lambda_3^p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n^p \end{bmatrix}\mathbf{P}^{-1}\mathbf{n}(t). \tag{29}$$

Let

$$\mathbf{P}^{-1}\mathbf{n}(t) = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ \vdots \\ c_n \end{bmatrix} \tag{30}$$

Then, substituting equations (28) and (30) into equation (29), obtained

$$\mathbf{n}(t+p) = \begin{bmatrix}\mathbf{x_1} & \mathbf{x_2} & \cdots & \mathbf{x_n}\end{bmatrix}\begin{bmatrix}\lambda_1^p c_1 \\ \lambda_2^p c_2 \\ \lambda_3^p c_3 \\ \vdots \\ \lambda_n^p c_n\end{bmatrix}$$

It will be shown that λ_1 is the dominant eigenvalues, as a means of determining the growth rate of the female population. Multiplying both sides by $\frac{1}{\lambda_1^p}$, obtained

$$\frac{1}{\lambda_1^p}\mathbf{n}(t+p) = \mathbf{P}\begin{bmatrix}1 & 0 & 0 & \cdots & 0 \\ 0 & \left(\frac{\lambda_2}{\lambda_1}\right)^p & 0 & \cdots & 0 \\ 0 & 0 & \left(\frac{\lambda_3}{\lambda_1}\right)^p & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \left(\frac{\lambda_n}{\lambda_1}\right)^p\end{bmatrix}\mathbf{P}^{-1}\mathbf{n}(t)$$

Then, since λ_1 is the dominant eigenvalues of the Leslie matrix $\mathbf{L}$, it follows that

$$\left|\frac{\lambda_k}{\lambda_1}\right| < 1 \text{ for } k = 2,3,\cdots,n$$

It is clear that $\left(\frac{\lambda_k}{\lambda_1}\right)^p \to 0$ as $p \to \infty$ for $k = 2,3,\cdots,n$. Therefore, we consider the following limit

$$\lim_{p\to\infty}\left(\frac{1}{\lambda_1^p}\mathbf{n}(t+p)\right) = \mathbf{P}\begin{bmatrix}1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0\end{bmatrix}\mathbf{P}^{-1}\mathbf{n}(t) \tag{31}$$

Then, substituting equations (28) and (30) into equation (31), obtained

$$\lim_{p\to\infty}\left(\frac{1}{\lambda_1^p}\mathbf{n}(t+p)\right) = c_1\mathbf{x_1}$$

Therefore, the following approximation is obtained

$$\lim_{p\to\infty}\left(\frac{1}{\lambda_1^p}\mathbf{n}(t+p)\right) = c_1\mathbf{x_1}$$

$$\mathbf{n}(t+p) \approx \lambda_1^p c_1\mathbf{x_1}$$

or

$$\mathbf{n}(t+(p-1)) \approx \lambda_1^{p-1} c_1\mathbf{x_1}$$

so that $\mathbf{n}(t+p)$ can be written as

$$\mathbf{n}(t+p) \approx \lambda_1^p c_1\mathbf{x_1} \Leftrightarrow \mathbf{n}(t+p) \approx \lambda_1^1\lambda_1^{p-1} c_1\mathbf{x_1}$$

obtained

$$\mathbf{n}(t+p) \approx \lambda_1\mathbf{n}\big(t+(p-1)\big). \tag{32}$$

Based on equation (32), it follows that for large values of p, each age distribution vector tends to become a scalar multiple of the age distribution vector in the previous period, and the scalar λ_1 is the dominant eigenvalues of the Leslie matrix. Consequently, the proportion of female individuals in each age group approaches a fixed (constant) value, i.e., it converges. Furthermore, if an arbitrary p represents a future period in the female population and it is known that $\lambda_1 = 1$ is the dominant eigenvalues of the Leslie matrix $\mathbf{L}$, then it can be concluded that the age distribution vector in the next period will always be equal to that of the previous period.

Therefore, several possible cases arise when λ_1 is the dominant eigenvalues as follows:

1. If $\lambda_1 < 1$, then the growth rate of the female population in future periods tends to decrease.
2. If $\lambda_1 = 1$, then the growth rate of the female population in future periods remains constant.
3. If $\lambda_1 > 1$, then the growth rate of the female population in future periods tends to increase.

3.2. Application of Eigenvalues of the Leslie Matrix to Female Population Growth Rate in Banyumas Regency

This study used data on the female population in Banyumas Regency. The data consist of the number of female individuals by age group and the number of female births based on the mother's age at childbirth. The female population in Banyumas Regency is divided into 16 age groups, with each group covering a five-year interval.

Table 2. Data on the Female Population and Female Births in Banyumas Regency

Age Group-i	**Interval Age**	**Number of Female in 2019 ($n_i(t)$)**	**Number of Female Births by Mother's Age at Childbirth in 2019-2024 (A_i)**	**Number of Female in 2024 ($n_i(t+1)$)**
1	0-4	61,222	0	67,501
2	5-9	69,071	0	63,987
3	10-14	67,751	0	64,174
4	15-19	68,169	6,447	65,735
5	20-24	65,886	11,879	66,872
6	25-29	64,343	15,591	66,675
7	30-34	62,648	15,372	64,266
8	35-39	71,117	11,691	64,983
9	40-44	69,874	5,655	66,645
10	45-49	65,600	0	67,445
11	50-54	61,370	0	63,913
12	55-59	53,327	0	55,288
13	60-64	42,799	0	47,087
14	65-69	31,243	0	36,901
15	70-74	22,595	0	27,711
16	75+	35,044	0	29,009
Total		912,059	66,635	918,192

Next, the fertility rates (a_i) and survival rates (b_i) of the female population will be determined. The fertility rates are obtained using the following equation

$$a_i = \frac{A_i}{n_i(t)} \tag{33}$$

Meanwhile, the survival rates are obtained using the following equation

$$b_i = \frac{n_{i+1}(t+1)}{n_i(t)} \tag{34}$$

Table 3. Fertility and Survival Rates of the Female Population

Age Group-i	**Interval Age**	**Fertility Rates (a_i)**	**Survival Rates (b_i)**
1	0-4	0	1.045
2	5-9	0	0.929

3	10-14	0	0.970
4	15-19	0.095	0.981
5	20-24	0.180	1.012
6	25-29	0.242	0.999
7	30-34	0.245	1.037
8	35-39	0.164	0.937
9	40-44	0.081	0.965
10	45-49	0	0.974
11	50-54	0	0.901
12	55-59	0	0.883
13	60-64	0	0.862
14	65-69	0	0.887
15	70-74	0	1.284
16	75+	0	-

Next, the Leslie matrix can be constructed after obtaining the fertility rates (a_i) and survival rates (b_i) of the female population. Since there are 16 age groups, a Leslie matrix of order 16×16 can be formed.

$$\mathbf{L} = \begin{bmatrix}
0 & 0 & 0 & 0.095 & 0.180 & 0.242 & 0.245 & 0.164 & 0.081 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.045 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.929 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.970 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.981 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1.012 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.999 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1.037 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.937 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.965 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.974 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.901 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.883 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.862 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8087 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.284 & 0
\end{bmatrix}$$

The Leslie matrix growth model used to estimate the female population in the next period is given by

$$\mathbf{n}(t+p) = \mathbf{L}^p\mathbf{n}(t)$$

where $\mathbf{L}$ is the Leslie matrix, p is the growth period, and $\mathbf{n}(t)$ is the initial age distribution vector. The initial population $\mathbf{n}(t)$ used is the female population in 2019, and the duration of each growth period p is five years. Thus, the Leslie matrix growth model can be written as follows

$$\mathbf{n}(2019+p) = \mathbf{L}^p\mathbf{n}(2019) \tag{35}$$

Next, the estimation is carried out for several future periods to examine the long-term trend of population growth, for instance, up to $p = 50$.

Table 4. Estimated Number of the Female Population

Period- $\boldsymbol{p}$	**Estimated Number of the Female Population** $(\mathbf{n}(\boldsymbol{t}+\boldsymbol{p}))$	**Period-** $\boldsymbol{p}$	**Estimated Number of the Female Population** $(\mathbf{n}(\boldsymbol{t}+\boldsymbol{p}))$
0	912,059	26	760,025
1	917,195	27	752,934
2	929,024	28	746,104
3	934,492	29	739,507
4	933,142	30	733,010
5	926,579	31	726,476

6	917,957	32	719,874
7	907,553	33	713,261
8	895,958	34	706,741
9	885,698	35	700,359
10	879,951	36	694,094
11	873,304	37	687,892
12	864,719	38	681,704
13	854,611	39	675,526
14	845,240	40	669,390
15	837,250	41	663,321
16	833,332	42	657,337
17	825,699	43	651,426
18	818,049	44	645,567
19	810,048	45	639,743
20	802,083	46	633,959
21	794,562	47	628,221
22	787,662	48	622,543
23	781,091	49	616,926
24	774,362	50	611,366
25	767,303		

Next, the growth rate of the female population will be determined by computing the eigenvalues of the Leslie matrix. The eigenvalues of the Leslie matrix indicate whether the growth rate tends to increase, remain constant, or decrease. The eigenvalues of the Leslie matrix for the female population will be determined using equation (5).

$$\det\left(\begin{vmatrix}
\lambda & 0 & 0 & -0.095 & -0.180 & -0.242 & -0.245 & -0.164 & -0.081 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1.045 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -0.929 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -0.970 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -0.981 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1.012 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -0.999 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1.037 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.937 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.965 & \lambda & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.974 & \lambda & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.901 & \lambda & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.883 & \lambda & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.862 & \lambda & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.8087 & \lambda & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1.284 & \lambda
\end{vmatrix}\right) = 0$$

The dominant eigenvalues of the Leslie matrix **L** is computed using Visual Studio Code, given by $\lambda_1 = 0.991$. The distribution of the eigenvalues of the Leslie matrix is shown in the following figure

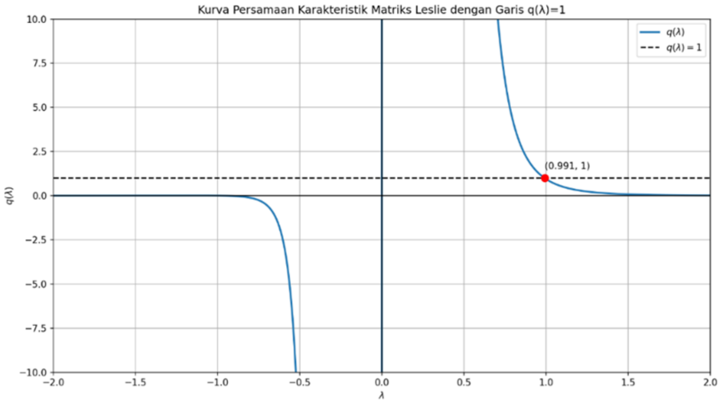

Figure 1. Graph of $\boldsymbol{q(\lambda)}$ for the Leslie Matrix

Based on Figure 1, the distribution of the nonzero eigenvalues of a Leslie matrix can be observed from the graph of $q(\lambda)$ for $\lambda \neq 0$. The nonzero eigenvalues of the Leslie matrix correspond to the intersection points between the graph of $q(\lambda)$ and the line $q(\lambda) = 1$. The positive eigenvalues obtained is $\lambda_1 = 0.991$. Thus, from the eigenvalues computation, the dominant eigenvalues of the Leslie matrix $\mathbf{L}$ is $\lambda_1 = 0.991$. Since $\lambda_1 = 0.991 < 1$, this indicates that the growth rate of the female population in Banyumas Regency tends to decrease over each five-year period.

Next, an analysis is conducted on the proportion of the age distribution of the female population in Banyumas Regency over a sufficiently long period, up to $p = 50$ in order to examine the asymptotic behavior of the Leslie matrix model in this study. The results of the computation of the age distribution proportions of the female population are presented in Table 5 obtained by calculating the ratio between the number of female individuals in the i-th age group at period p and the total female population at period p in order to illustrate the tendency toward convergence to a stable age distribution.

Table 5. Age Distribution Proportions of the Female Population

Period-*p*	**Age Group-*i* (%)**															
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
0	6,7	7,6	7,4	7,5	7,2	7,1	6,9	7,8	7,7	7,2	6,7	5,8	4,7	3,4	2,5	3,8
1	7,3	7,0	7,0	7,2	7,3	7,3	7,0	7,1	7,3	7,4	7,0	6,0	5,1	4,0	3,0	3,2
2	7,1	7,5	6,4	6,7	6,9	7,3	7,2	7,2	6,6	6,9	7,1	6,2	5,3	4,4	3,5	3,8
3	7,1	7,4	6,9	6,2	6,5	7,0	7,2	7,4	6,7	6,3	6,7	6,3	5,4	4,5	3,9	4,5
4	7,0	7,4	6,9	6,7	6,1	6,6	7,0	7,5	6,9	6,5	6,1	6,0	5,6	4,7	4,0	5,0
5	6,9	7,4	6,9	6,7	6,6	6,2	6,7	7,3	7,1	6,7	6,3	5,6	5,4	4,9	4,2	5,2
6	6,8	7,3	6,9	6,8	6,7	6,8	6,2	7,0	6,9	6,9	6,6	5,8	5,0	4,7	4,4	5,4
7	6,8	7,2	6,8	6,8	6,7	6,8	6,9	6,5	6,6	6,7	6,8	6,0	5,1	4,3	4,2	5,7
8	6,9	7,2	6,8	6,7	6,7	6,9	6,9	7,2	6,2	6,5	6,6	6,2	5,4	4,5	3,9	5,5
9	7,0	7,3	6,8	6,6	6,7	6,9	7,0	7,2	6,8	6,1	6,4	6,1	5,5	4,7	4,0	5,0
10	7,0	7,3	6,8	6,6	6,5	6,8	6,9	7,3	6,8	6,6	5,9	5,8	5,4	4,8	4,2	5,2
11	6,9	7,4	6,9	6,6	6,5	6,7	6,8	7,2	6,9	6,6	6,5	5,4	5,1	4,7	4,3	5,4
12	6,9	7,3	6,9	6,7	6,6	6,7	6,7	7,1	6,8	6,7	6,5	5,9	4,8	4,5	4,2	5,6
13	6,9	7,3	6,9	6,8	6,7	6,7	6,7	7,1	6,8	6,7	6,6	6,0	5,3	4,2	4,0	5,4
14	6,9	7,3	6,9	6,8	6,7	6,8	6,8	7,1	6,7	6,6	6,6	6,0	5,3	4,6	3,8	5,2
15	6,9	7,3	6,8	6,7	6,7	6,9	6,9	7,1	6,7	6,5	6,5	6,0	5,4	4,6	4,1	4,9
16	6,9	7,3	6,8	6,7	6,6	6,8	6,9	7,2	6,7	6,5	6,4	5,9	5,3	4,6	4,1	5,3
17	6,9	7,3	6,8	6,7	6,6	6,8	6,9	7,2	6,8	6,5	6,4	5,8	5,2	4,6	4,2	5,3
18	6,9	7,3	6,9	6,7	6,6	6,7	6,8	7,2	6,8	6,6	6,4	5,8	5,2	4,6	4,1	5,4
19	6,9	7,3	6,9	6,7	6,6	6,7	6,8	7,1	6,8	6,6	6,5	5,8	5,2	4,5	4,1	5,4
20	6,9	7,3	6,9	6,7	6,7	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,5	4,0	5,3
21	6,9	7,3	6,8	6,7	6,7	6,8	6,8	7,1	6,7	6,6	6,5	5,9	5,3	4,5	4,0	5,2
22	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,7	6,5	6,5	5,9	5,3	4,6	4,1	5,2
23	6,9	7,3	6,8	6,7	6,6	6,8	6,9	7,2	6,7	6,5	6,4	5,9	5,3	4,6	4,1	5,3
24	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,2	6,8	6,6	6,4	5,8	5,2	4,6	4,1	5,3
25	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,2	6,8	6,6	6,5	5,8	5,2	4,5	4,1	5,3
26	6,9	7,3	6,9	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,5	4,1	5,3
27	6,9	7,3	6,9	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,5	4,1	5,3
28	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,7	6,6	6,5	5,9	5,3	4,6	4,1	5,3
29	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,7	6,6	6,5	5,9	5,3	4,6	4,1	5,3
30	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,2	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
31	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,2	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
32	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
33	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,5	4,1	5,3
34	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
35	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
36	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
37	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
38	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
39	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
40	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
41	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
42	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
43	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
44	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
45	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
46	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
47	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
48	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
49	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3
50	6,9	7,3	6,8	6,7	6,6	6,8	6,8	7,1	6,8	6,6	6,5	5,9	5,2	4,6	4,1	5,3

Based on Table 5, the age distribution proportions of the female population in Banyumas Regency up to $p = 50$ show convergence toward stability, beginning around $p \geq 30$, where changes between successive periods become very small and relatively constant. This indicates that the population reaches a stable age distribution. The proportion pattern shows that age groups 1-10 have relatively larger and more stable proportions compared to age groups 11-16, which are smaller but still stable. This is consistent with Leslie matrix theory, where for large p, the age distribution vector becomes a scalar multiple of the previous one, leading to convergence of proportions. However, since the dominant eigenvalues is $\lambda_1 = 0.991 < 1$, the total female population tends to gradually decrease. Thus, in the long term, the female population in Banyumas Regency exhibits a stable age structure with a decreasing population size.

Compared with the study in [7] which focused on estimating population size for a particular year, this study emphasizes the analysis of the behavior of the dominant eigenvalues and its interpretation in describing long-term population growth rates. This approach provides a clearer description of the future trend of female population growth. The dominant eigenvalues in [7] is $\lambda_1 = 1.23 > 1$, the total female population tends to gradually increase. Thus, in the long term, the female population in Banyumas Regency exhibits a stable age structure with a increasing population size.

We used real data from 2019-2024, and the figure of 0.991 is historically accurate for describing the crisis period. However, if this figure is used at face value to project the Banyumas population for the next 30-50 years, the projections are unrealistic (biased) because they assume the severity of the pandemic will persist annually. The value of $\lambda_1 = 0.991 < 1$ is possible driven by a temporary decline in fertility and an increase in mortality due to Covid-19 as pandemic anomaly.

Meanwhile, if we compare with the study in [1], the advantage of the Leslie matrix method lies in its ability to systematically describe population dynamics based on age structure, fertility rates, and survival rates. Through the analysis of the dominant eigenvalues, this model can also be used to predict the stability of age distribution and long-term population growth trends.

4 Conclusion

Based on the results and discussion, it can be concluded that the Leslie matrix has an unique dominant positive eigenvalues with a corresponding eigenvector whose entries are all positive. Furthermore, the application of the Leslie matrix eigenvalues to the growth rate of the female population shows the existence of dominant positive eigenvalues $\lambda_1 = 0.991$. This indicates that the growth rate of the female population tends to decrease over each five-year period in the future. It is indicating that the female population growth rate is expected to decline 99,1% in each five-year period. The projections are unrealistic (biased) because they assume the severity of the Covid-19 pandemic will persist annually.

This decreasing trend may reflect changes in fertility patterns, demographic structure, or socio-economic conditions in Banyumas Regency. Therefore, the results of this study may serve as a reference for future demographic planning and population policy evaluation, particularly those related to age structure and long-term population sustainability. Further studies may consider additional demographic factors, such as migration and mortality variations, to obtain a more comprehensive population model.

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