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INDUCED (α, β)-DERIVATIONS ON SEMIGROUP RINGS

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ABSTRACT

Derivations and their generalizations play an important role in the study of algebraic structures. Among these, (α, β)-derivations extend the classical notion of derivations by incorporating ring endomorphisms. In this paper, we study the construction of (α, β)-derivations on semigroup rings. Let R be a ring and S a semigroup, and consider the semigroup ring $R[S]$. By using function composition, we construct induced mappings on $R[S]$ arising from endomorphisms and (α, β)-derivations on R. We prove that every ring endomorphism of R naturally induces an endomorphism on the semigroup ring $R[S]$. Moreover, if δ is an (α, β)-derivation on R, then the induced mapping defines an $(\bar{\alpha}, \bar{\beta})$-derivation on $R[S]$. These results provide a natural extension of generalized derivations from rings to semigroup rings and establish a framework for studying derivations on related algebraic extensions, including polynomial rings as a special case of semigroup rings.
Keywords: *Derivation, (α, β)-derivation, Ring endomorphism, Semigroup ring*

1. Introduction

Derivations constitute an important concept in ring theory, serving as algebraic analogues of differential operators. A derivation on a ring R is an additive mapping $d : R \to R$ satisfying the Leibniz rule $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. The concept originated from differential calculus and later became fundamental in the structural study of rings and algebras [1]. In algebra, derivations are frequently used to analyze structural properties of rings, including commutativity, ideal behavior, and the structure of the center, particularly in noncommutative settings [2, 3, 4, 5, 6].

Early investigations revealed that derivations impose strong structural restrictions on rings. Classical results show that certain identities involving derivations may force rings to become commutative or impose constraints on ideals and Lie ideals. In particular, studies on Jordan derivations established that under suitable conditions, Jordan derivations coincide with ordinary derivations in semiprime rings [7, 8]. Related investigations also examined special properties such as nilpotency of derivations and their consequences for ring structure [9, 10]. These foundational results laid the groundwork for the development of a broad theory of generalized derivations.

As ring theory developed, several generalizations of derivations were introduced to capture more complex algebraic interactions. One important extension is the (α, β)-derivation, defined by

$$d(xy) = d(x)\alpha(y) + \beta(x)d(y),$$

where α and β are endomorphisms of the ring. This concept extends the classical Leibniz rule by incorporating ring endomorphisms and has been studied extensively in various classes of rings. In particular, generalized (α, β)-derivations were investigated in semiprime rings and shown to impose significant algebraic constraints on ring elements and ideals [11, 12, 13].

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Further developments have explored several variants of generalized derivations. For instance, generalized (σ, τ)-derivations and their structural consequences in semiprime rings were studied in [14]. Other works analyzed derivation behavior through identities involving commutators, Lie ideals, and homoderivations in prime rings [15]. Recent investigations have also considered (α, β)-Jordan bi-derivations and related structures, highlighting deeper connections between derivations and ring identities [16]. In addition, (α, β)-derivations on matrix rings $M_n(R)$ were investigated in [17], providing further insight into the behavior of generalized derivations on noncommutative ring structures. A comprehensive overview of modern developments in derivation theory, including skew derivations, generalized derivations, and (α, β)-derivations, can be found in the survey by Ali et al. [18].

Beyond classical rings, derivations have also been studied in more general algebraic structures. In operator algebras, derivations play a key role in the analysis of Murray–von Neumann algebras and related functional analytic structures [19]. In the context of group rings, (σ, τ)-derivations have been constructed and investigated through homomorphism pairs, revealing interactions between the group structure and the ring structure [20]. Related studies on group rings and other algebraic constructions further illustrate how derivations interact with underlying algebraic objects [21].

More recently, attention has been directed toward derivations on polynomial rings and related algebraic structures. Several results describe Jordan derivations, nil derivations, and generalized derivations acting on polynomial rings and modules [22, 23, 24, 25, 26, 27]. These works indicate that the behavior of derivations can vary significantly depending on the algebraic structure under consideration.

Another important direction concerns derivations on semigroup rings. A semigroup ring $R[S]$ combines the algebraic structure of a ring R with a semigroup S, producing a rich algebraic object whose properties depend on both components. The general theory of semigroup rings has been extensively developed in the literature [28, 29, 30, 31, 32]. Recent studies have begun to investigate derivations on semigroup algebras, highlighting how the semigroup structure influences derivation behavior [33, 34].

Despite these developments, the study of (α, β)-derivations on semigroup rings remains relatively limited. Most existing results focus on classical rings, polynomial rings, or group rings, while the interaction between endomorphism-based derivations and semigroup structures has not been thoroughly explored. In particular, the algebraic behavior of (α, β)-derivations on semigroup rings $R[S]$ and the influence of the semigroup operation on derivation identities have not been systematically investigated.

Motivated by this gap, this paper studies (α, β)-derivations on semigroup rings $R[S]$. The main objective is to analyze how the semigroup structure affects derivation behavior and to establish several structural properties arising from the interaction between ring endomorphisms and semigroup elements. The results presented here extend existing derivation theory from classical rings and group rings to the broader setting of semigroup rings, thereby contributing to the ongoing development of generalized derivations in algebra.

2. Preliminaries

This section recalls several fundamental concepts from semigroup theory and ring theory that will be used throughout the paper. Standard references for these concepts include [6, 29, 31, 28].

We begin with the notion of a binary operation, which serves as the basic building block for many algebraic structures.

Definition 2.1. *[29] Let S be a nonempty set. A* binary operation *on S is a mapping*

$$* : S \times S \to S$$

that assigns to each pair $(a, b) \in S \times S$ an element $a * b \in S$.

Binary operations naturally give rise to algebraic structures such as semigroups.

Definition 2.2. *[29, 30] A nonempty set S equipped with a binary operation $*$ is called a* semigroup *if the operation is associative, that is,*

$$(a * b) * c = a * (b * c)$$

for all $a, b, c \in S$.

Semigroups also appear naturally in ring theory, since the multiplicative structure of a ring forms a semigroup.

Definition 2.3. *[6, 31] A nonempty set R together with two binary operations + and ·, denoted* $\langle R, +, \cdot \rangle$, *is called a* ring *if the following conditions hold:*

1. $(R, +)$ *is an Abelian group,*
2. $(R, \cdot)$ *is a semigroup,*
3. *the distributive laws hold for all* $a, b, c \in R$,

$$a(b + c) = ab + ac, \qquad (a + b)c = ac + bc.$$

Mappings that preserve the algebraic structure of rings play an important role in the study of generalized derivations.

Definition 2.4. *[4] Let R be a ring. A mapping* $\phi : R \to R$ *is called a* ring endomorphism *if*

$$\phi(a + b) = \phi(a) + \phi(b), \qquad \phi(ab) = \phi(a)\phi(b)$$

for all $a, b \in R$. *In other words, a ring endomorphism is a homomorphism from R into itself.*

The main algebraic object considered in this paper is the semigroup ring, which combines the structure of a ring with that of a semigroup.

Definition 2.5. *[28] Let R be a ring and* $(S, *)$ *a semigroup. The* semigroup ring $R[S]$ *is the set of all functions*

$$f : S \to R$$

with finite support, that is,

$$\text{supp}(f) = \{s \in S \mid f(s) \neq 0\}$$

is finite. Addition is defined pointwise by

$$(f + g)(s) = f(s) + g(s),$$

and multiplication is defined by convolution

$$(fg)(s) = \sum_{t*u=s} f(t)g(u).$$

With these operations, $R[S]$ *forms a ring called the* semigroup ring of S over R. *A classical example occurs when* $S = \mathbb{Z}_0^+$, *in which case* $R[S]$ *is isomorphic to the polynomial ring* $R[x]$.

After introducing these algebraic structures, we recall the concept of derivations, which plays a central role in this study.

Definition 2.6. *[18] Let R be a ring. An additive mapping* $d : R \to R$ *is called a* derivation *if it satisfies the Leibniz rule*

$$d(ab) = d(a)b + ad(b)$$

for all $a, b \in R$.

Several generalizations of derivations have been introduced in order to incorporate ring endomorphisms into the Leibniz rule.

Definition 2.7. *[11] Let R be a ring and let α, β be endomorphisms of R. An additive mapping $d : R \to R$ is called an (α, β)-derivation if*

$$d(xy) = d(x)\alpha(y) + \beta(x)d(y)$$

for all $x, y \in R$.

In this paper, derivations defined on the base ring R are extended to the semigroup ring $R[S]$. Given ring endomorphisms α and β on R, we construct induced mappings $\bar{\alpha}$ and $\bar{\beta}$ on $R[S]$, together with a mapping $\bar{\delta}$ induced by a derivation δ on R. The main objective is to examine whether these induced mappings preserve the algebraic structure of $R[S]$. In particular, we show that $\bar{\alpha}$ and $\bar{\beta}$ define ring endomorphisms on $R[S]$, and we investigate conditions under which $\bar{\delta}$ becomes an $(\bar{\alpha}, \bar{\beta})$-derivation on $R[S]$. Examples are provided to illustrate the construction and to highlight the influence of the semigroup structure on these derivations.

3. Main Results

In this section we establish the main results concerning the construction of (α, β)-derivations on the semigroup ring $R[S]$.

Lemma 3.1. *Let R be a ring, S a semigroup, and let $\alpha : R \to R$ be a ring endomorphism. For any $f, g \in R[S]$ the following hold:*

1. *$\alpha \circ f \in R[S]$;*
2. *$\alpha \circ (f + g) = (\alpha \circ f) + (\alpha \circ g)$;*
3. *$\alpha \circ (fg) = (\alpha \circ f)(\alpha \circ g)$.*

PROOF. Let $f, g \in R[S]$ and $s \in S$.

1. If $(\alpha \circ f)(s) \neq 0$, then $\alpha(f(s)) \neq 0$, hence $f(s) \neq 0$. Thus $\text{supp}(\alpha \circ f) \subseteq \text{supp}(f)$, which is finite, and therefore $\alpha \circ f \in R[S]$.

2. For $s \in S$,

$$(\alpha \circ (f + g))(s) = \alpha(f(s) + g(s)) = \alpha(f(s)) + \alpha(g(s)) = ((\alpha \circ f) + (\alpha \circ g))(s).$$

3. Using the convolution product in $R[S]$,

$$\begin{aligned}(\alpha \circ (fg))(s) &= \alpha((fg)(s)) \\ &= \alpha\left(\sum_{t*u=s} f(t)g(u)\right) \\ &= \sum_{t*u=s} \alpha(f(t)g(u)) \\ &= \sum_{t*u=s} \alpha(f(t))\alpha(g(u)) \\ &= ((\alpha \circ f)(\alpha \circ g))(s).\end{aligned}$$

■

The following example illustrates how the composition of a function $f \in R[S]$ with a ring endomorphism α again produces an element of the semigroup ring $R[S]$, as stated in Lemma 3.1.

Example 3.1. *Let*

$$R = \left\{ \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} \,\middle|\, a, c, d \in \mathbb{Z} \right\}$$

and let $S = \mathbb{Z}$ under multiplication. Define a map $\alpha : R \to R$ by

$$\alpha\left(\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}.$$

It is straightforward to verify that α preserves addition and multiplication in R, hence α is a ring endomorphism. Define $f : S \to R$ by

$$f(s) = \begin{cases} \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}, & s = 1, \\ 0, & s \neq 1. \end{cases}$$

Then $\operatorname{supp}(f) = \{1\}$*, so $f \in R[S]$. Moreover,*

$$(\alpha \circ f)(1) = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix},$$

and $(\alpha \circ f)(s) = 0$ for $s \neq 1$, which shows that $\alpha \circ f \in R[S]$.

The previous lemma allows us to construct an induced map on the semigroup ring.

Lemma 3.2. *Let R be a ring and S a semigroup. If $\alpha : R \to R$ is a ring endomorphism, then the map*

$$\bar{\alpha} : R[S] \to R[S], \qquad \bar{\alpha}(f) = \alpha \circ f$$

defines a ring endomorphism on $R[S]$.

PROOF. For $f \in R[S]$, Lemma 3.1(1) shows that $\bar{\alpha}(f) \in R[S]$, so the map is well defined. Moreover, Lemma 3.1(2) and (3) imply

$$\bar{\alpha}(f + g) = \bar{\alpha}(f) + \bar{\alpha}(g), \qquad \bar{\alpha}(fg) = \bar{\alpha}(f)\bar{\alpha}(g).$$

Hence $\bar{\alpha}$ is a ring endomorphism on $R[S]$. ■

Analogously, if $\beta : R \to R$ is a ring endomorphism, then the map

$$\bar{\beta}(f) = \beta \circ f$$

defines a ring endomorphism on $R[S]$. We now present an example demonstrating the induced endomorphism $\bar{\alpha}$ on the semigroup ring $R[S]$ described in Lemma 3.2.

Example 3.2. *Using the endomorphism α defined in the previous example, define*

$$\bar{\alpha} : R[S] \to R[S], \qquad \bar{\alpha}(f) = \alpha \circ f.$$

Let $f, g \in R[S]$ be functions supported at $1 \in S$ with

$$f(1) = \begin{pmatrix} a & 0 \\ c & d \end{pmatrix}, \qquad g(1) = \begin{pmatrix} a' & 0 \\ c' & d' \end{pmatrix}.$$

Then

$$\bar{\alpha}(f)(1) = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}, \qquad \bar{\alpha}(g)(1) = \begin{pmatrix} a' & 0 \\ 0 & d' \end{pmatrix}.$$

A direct computation shows that

$$\bar{\alpha}(f + g) = \bar{\alpha}(f) + \bar{\alpha}(g), \qquad \bar{\alpha}(fg) = \bar{\alpha}(f)\bar{\alpha}(g).$$

Hence $\bar{\alpha}$ defines a ring endomorphism on $R[S]$.

We now consider the behavior of (α,β)-derivations under composition with functions in $R[S]$.

Lemma 3.3. *Let R be a ring, S a semigroup, and let $\alpha,\beta : R \to R$ be ring endomorphisms. Suppose that $\delta : R \to R$ is an (α,β)-derivation. For $f \in R[S]$ define $(\delta \circ f)(s) = \delta(f(s))$. Then for all $f, g \in R[S]$ the following hold:*

1. *$\delta \circ f \in R[S]$;*
2. *$\delta \circ (f + g) = (\delta \circ f) + (\delta \circ g)$;*
3. *$\delta \circ (fg) = (\delta \circ f)(\alpha \circ g) + (\beta \circ f)(\delta \circ g)$.*

PROOF. Let $f, g \in R[S]$ and $s \in S$.

1. If $(\delta \circ f)(s) \neq 0$, then $\delta(f(s)) \neq 0$. Since $\delta(0) = 0$, this implies $f(s) \neq 0$, hence $\operatorname{supp}(\delta \circ f) \subseteq \operatorname{supp}(f)$ and $\delta \circ f \in R[S]$.
2. For $s \in S$,
$$(\delta \circ (f + g))(s) = \delta(f(s) + g(s)) = \delta(f(s)) + \delta(g(s)) = ((\delta \circ f) + (\delta \circ g))(s).$$
3. Using the convolution product,
$$\begin{aligned}(\delta \circ (fg))(s) &= \delta((fg)(s)) \\ &= \delta\left(\sum_{t*u=s} f(t)g(u)\right) \\ &= \sum_{t*u=s} \delta(f(t)g(u)) \\ &= \sum_{t*u=s} \big(\delta(f(t))\alpha(g(u)) + \beta(f(t))\delta(g(u))\big),\end{aligned}$$
which equals
$$((\delta \circ f)(\alpha \circ g))(s) + ((\beta \circ f)(\delta \circ g))(s).$$

■

The previous lemma describes the behavior of the composition $\delta \circ f$ with respect to the convolution product in $R[S]$. This property plays a crucial role in establishing the extension of (α,β)-derivations from R to the semigroup ring $R[S]$, which is presented in the following theorem.

Theorem 3.1. *Let R be a ring and S a semigroup. Let $\alpha,\beta : R \to R$ be ring endomorphisms and let $\delta : R \to R$ be an (α,β)-derivation. Define*
$$\bar{\delta} : R[S] \to R[S], \qquad \bar{\delta}(f) = \delta \circ f.$$
Then $\bar{\delta}$ is an $(\bar{\alpha},\bar{\beta})$-derivation on $R[S]$.

PROOF. By Lemma 3.3(1), $\bar{\delta}$ is well defined. Lemma 3.3(2) shows that $\bar{\delta}$ is additive. Finally, by Lemma 3.3(3),
$$\bar{\delta}(fg) = \bar{\delta}(f)\bar{\alpha}(g) + \bar{\beta}(f)\bar{\delta}(g),$$
which proves that $\bar{\delta}$ is an $(\bar{\alpha},\bar{\beta})$-derivation on $R[S]$. ■

The following example illustrates the extension of an (α,β)-derivation from a ring R to the semigroup ring $R[S]$, as described in Theorem 3.1.

Example 3.3. *Let*

$$R = \left\{ \begin{pmatrix} a & 0 \\ c & d \end{pmatrix} \middle| \, a, c, d \in \mathbb{Z} \right\}, \qquad S = \mathbb{Z}$$

under multiplication. Define endomorphisms $\alpha, \beta : R \to R$ *by*

$$\alpha\left(\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & 0 \\ 0 & d \end{pmatrix}, \qquad \beta\left(\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ 0 & d \end{pmatrix}.$$

Define $\delta : R \to R$ *by*

$$\delta\left(\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\right) = \begin{pmatrix} 0 & 0 \\ c & 0 \end{pmatrix}.$$

One can verify that δ *satisfies*

$$\delta(xy) = \delta(x)\alpha(y) + \beta(x)\delta(y),$$

so δ *is an* (α, β)*-derivation on* R*. Define* $\bar{\delta} : R[S] \to R[S]$ *by*

$$\bar{\delta}(f) = \delta \circ f.$$

Then

$$\bar{\delta}(fg) = \bar{\delta}(f)\bar{\alpha}(g) + \bar{\beta}(f)\bar{\delta}(g),$$

which shows that $\bar{\delta}$ *is an* $(\bar{\alpha}, \bar{\beta})$*-derivation on* $R[S]$.

4. Conclusion

In this paper, we investigated the extension of endomorphisms and (α, β)-derivations from a ring R to the semigroup ring $R[S]$. It was shown that every ring endomorphism $\alpha : R \to R$ induces a natural endomorphism $\bar{\alpha} : R[S] \to R[S]$ defined by $\bar{\alpha}(f) = \alpha \circ f$ for all $f \in R[S]$. Furthermore, if $\delta : R \to R$ is an (α, β)-derivation on R, then the induced map $\bar{\delta} : R[S] \to R[S]$ defined by $\bar{\delta}(f) = \delta \circ f$ forms an $(\bar{\alpha}, \bar{\beta})$-derivation on the semigroup ring $R[S]$. These results show that the structure of (α, β)-derivations on a ring can be naturally extended to its associated semigroup ring through function composition.

This construction provides a systematic approach for studying generalized derivations on larger algebraic structures obtained from rings, and may serve as a basis for further investigations of derivations on related algebraic extensions. Future work may explore similar extensions for other generalized derivations or for special classes of semigroup rings such as group rings and polynomial rings.

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References

[1] T. Archibald, C. Fraser, and I. Grattan-Guinness, "The history of differential equations, 1670–1950," *Oberwolfach Reports*, vol. 1, pp. 2729–2794, 2005.

[2] I. N. Herstein, "Jordan derivations of prime rings," *Proceedings of the American Mathematical Society*, vol. 8, pp. 1104–1110, 1957.

[3] I. N. Herstein, "Sui commutatori degli anelli semplici," *Seminario Matematico e Fisico di Milano*, vol. 33, pp. 80–86, 1963.

[4] I. N. Herstein, *Noncommutative Rings*. Mathematical Association of America, 1968.

[5] T. W. Hungerford, *Algebra*. New York: Springer, 1974.

[6] D. S. Dummit and R. M. Foote, *Abstract Algebra*. Hoboken: Wiley, 3rd ed., 2004.

[7] M. Bresar, “Jordan derivations on semiprime rings,” *Proceedings of the American Mathematical Society*, vol. 104, no. 4, pp. 1003–1006, 1988.

[8] M. Bresar and J. Vukman, “Jordan derivations on prime rings,” *Bulletin of the Australian Mathematical Society*, vol. 37, pp. 321–322, 1988.

[9] L. O. Chung and J. Luh, “Nilpotency of derivations,” *Canadian Mathematical Bulletin*, vol. 26, pp. 344–346, 1983.

[10] L. O. Chung and J. Luh, “Nilpotency of derivations ii,” *Proceedings of the American Mathematical Society*, vol. 91, pp. 357–358, 1984.

[11] F. Ali and M. A. Chaudhry, “On generalized (α,β)-derivations of semiprime rings,” *Turkish Journal of Mathematics*, vol. 35, no. 3, pp. 399–404, 2011.

[12] N. Garg and R. K. Sharma, “Generalized (α,β)-derivations in prime rings,” *International Journal of Algebra*, vol. 4, no. 15, pp. 735–742, 2010.

[13] N. Garg and R. K. Sharma, “On generalized (α,β)-derivations in semiprime rings,” *International Mathematical Forum*, vol. 7, no. 33, pp. 1621–1628, 2012.

[14] B. Dhara and S. Pattanayak, “Generalized (σ,τ)-derivations of semiprime rings,” *International Journal of Algebra*, vol. 6, pp. 1–14, 2012.

[15] N. Bera and B. Dhara, “Jordan homoderivation behavior of generalized derivations in prime rings,” *Ukrains’kyi Matematychnyi Zhurnal*, vol. 75, no. 9, pp. 1178–1194, 2023.

[16] W. Ahmed, A. S. Alali, and M. R. Mozumder, “Characterization of (α,β)-jordan bi-derivations in prime rings,” *AIMS Mathematics*, vol. 9, no. 6, pp. 14549–14557, 2024.

[17] D. Azzahra, Fitriani, and A. Faisol, “(α,β)-derivation on matrix ring $m_n(r)$,” *Euler: Jurnal Ilmiah Matematika, Sains dan Teknologi*, vol. 14, no. 1, pp. 260–269, 2026.

[18] S. Ali, N. N. Rafiquee, and V. Varshney, “Certain types of derivations in rings: A survey,” *Journal of the Indonesian Mathematical Society*, vol. 30, no. 2, pp. 256–306, 2024.

[19] A. Ber, K. Kudaybergenov, and F. Sukochev, “Notes on derivations of murray–von neumann algebras,” *Journal of Functional Analysis*, vol. 279, no. 1, pp. 1–26, 2020.

[20] D. Chauduri, “(σ,τ)-derivations of group rings.” arXiv:1803.09418, 2018.

[21] R. Waluyo, A. Faisol, and Fitriani, “(σ,τ)-derivasi pada ring grup,” *Euler: Jurnal Ilmiah Matematika, Sains dan Teknologi*, vol. 13, no. 2, pp. 142–146, 2025.

[22] D. E. Sitompul, Fitriani, S. L. Chasanah, and A. Faisol, “Jordan derivation on the polynomial ring $r[x]$,” *Integra: Journal of Integrated Mathematics and Computer Science*, vol. 2, no. 2, pp. 41–47, 2025.

[23] D. L. Mursyidah, Fitriani, B. H. S. Utami, and A. Faisol, “Nil derivations and d-ideals on polynomial rings,” *BAREKENG: Journal of Mathematics and Its Applications*, vol. 20, p. 0325–0334, 2025.

[24] Fitriani, I. E. Wijayanti, A. Faisol, and S. Ali, “On f-derivations on polynomial modules,” *Journal of Algebra and Its Applications*, vol. 24, no. 6, 2025.

[25] A. B. Thomas, N. P. Puspita, and Fitriani., “Derivation on several rings.,” *Barekeng: Jurnal Ilmu Matematika dan Terapan*, vol. 18, no. 3, pp. 1729–1738, 2024.

[26] N. A. Syaharani, Fitriani, S. L. Chasanah, and A. Faisol, “(α',β')-derivation on the polynomial ring $\mathcal{K}[x]$,” *Journal of the Indonesian Algebra Society*, vol. 1, no. 1, pp. 17–29, 2026.

[27] A. Faisol and Fitriani, "A study of derivations and linear mappings on skew generalized power series modules," *Barekeng: Jurnal Ilmu Matematika dan Terapan*, vol. 19, no. 4, pp. 3047–3058, 2025.

[28] R. Gilmer, *Commutative Semigroup Rings*. Chicago: University of Chicago Press, 1984.

[29] P. A. Grillet, *Abstract Algebra*. Springer, 2007.

[30] B. Surodjo and Y. Susanti, *Teori Semigrup*. Yogyakarta: Gadjah Mada University Press, 2023.

[31] S. Wahyuni, I. E. Wijayanti, D. A. Yuwaningsih, and A. D. Hartanto, *Teori Ring dan Modul*. Yogyakarta: UGM Press, 2021.

[32] Fitriani and A. Faisol, *Grup*. Yogyakarta: Matematika, 2022.

[33] A. Alekseev and A. Arutyunov, "Derivations in semigroup algebras," *Eurasian Mathematical Journal*, 2024.

[34] I. Ernanto, "Sifat-sifat ring faktor yang dilengkapi derivasi," *Journal of Fundamental Mathematics and Applications*, vol. 1, no. 1, pp. 12–21, 2018.