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OPTIMIZING IMPERFECT INVENTORY PROBLEM USING THE ECONOMIC PRODUCTION QUANTITY MODEL

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ABSTRACT

In production inventory, the goods produced are not always perfect. To ensure that customers get the perfect goods, an inspection process will be carried out before the goods are sent to customers. However, the inspection process is also not perfect. There are two types of inspection errors, i.e., Type-I and Type-II inspection error. In this study, the problem of imperfect inventory will be formulated using the Economic Production Quantity (EPQ) model. This study also considered rework and salvage for inspected goods and returned goods. The purpose of this model is to determine the production size and the length of the inventory cycle that maximizes the expected total profit. In addition, a sensitivity analysis will be carried out to provide the effects of parameter proportion changes optimum expected total profit. The results of the study found that to maximize the expected total profit, the parameter proportions must be decreased. Based on the results of model implementation, the optimum number of production units is 5,755 units, the optimum cycle length is 23 days, and the maximum expected total profit is IDR 1,672,513,135.44.

Keywords: EPQ model; Inspection; Inventory; Production

1 Introduction

Inventory is a collection of goods stored in a warehouse to be used to meet fluctuating customer demand. Inventory in a company can be in the form of raw materials, semi-finished goods, or finished goods. Every company maintains inventory for the continuity of its operational activities, aiming to fulfill customer demand and achieve significant profits [1] Approximately 50% of a company's total capital is in the form of inventory [2].

A manufacturing company is a company that processes raw or semi-finished materials into ready-to-use finished goods through a series of production steps. However, not all items produced in the production process have perfect quality. When the production process is in an "in-control" state, the goods produced can be of high or even perfect quality. Over time, the production process can deteriorate, resulting in defective items or goods with substandard quality [3]. To ensure that customers receive perfect goods, companies inspect their products before shipping them. The inspection process itself is also not perfect or free from errors [4]. In any inspection process, errors can occur in classifying defective and non-defective items. There are two types of inspection errors: Type-I and Type-II inspection errors. A Type-I inspection error is the misclassification of a non-defective item as defective, causing the company to suffer direct losses due to lost profit opportunities. Meanwhile, a Type-II inspection error is the

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misclassification of a defective item as non-defective, leading to customer disappointment and loss of trust [5]. To reduce losses, companies perform rework on defective items so that they can be sold at the price of perfect-quality items. All defective items can be reworked into perfect items, so no items are scrapped [6].

Generally, in inventory problems, all items are assumed to be of perfect quality and free from defects to ensure that customer demand can always be met. In [3] introduced a model demonstrating a significant relationship between production quality and quantity. They discussed an EPQ (Economic Production Quantity) model with an imperfect production process. In [7] described an inventory model for imperfect items considering inspection errors and an imperfect production process. In [8] developed an economic order quantity model where each order lot contains a random number of defective items. In [9] developed an EOQ model that considers a joint lot-sizing and inspection policy where the proportion of defective units is random. Defective units cannot be used and must be replaced with non-defective ones. A large error in quantity optimization can lead to a decrease in the objective function's value [9]. For defective items with a known proportion in a production run, there are fixed and variable inspection costs to identify and remove them [10].

Research on EPQ models for imperfect products continues to evolve by considering more realistic scenarios. For instance, [11] developed an EPQ model that explicitly incorporates factors of human inspection errors and sales returns. In line with this, [12] enhanced the model by integrating the reliability of the production process, where the system can shift from an in-control to an out-of-control state. Model development has also touched on financial aspects, as studied by [13], who considered a trade credit policy in an EPQ model with defective products and a rework process. The model's complexity was further increased by [14], who addressed an EPQ with multiple types of reworkable defective items and the occurrence of shortages. Furthermore, sustainability issues have also begun to be integrated, with [15] proposing an EPQ model that incorporates environmental factors and accounts for variability in defective, repairable, and scrap products under stock-out case.

Every inventory policy affects profitability, so the policy to be implemented is a choice that depends on the company's profitability. This profitability can be achieved, among other ways, by minimizing the total inventory cost. According to [16], the costs that determine a company's profitability are ordering costs, holding costs, shortage costs, and the cost of lost sales.

This research will discuss an inventory model for imperfect items using the EPQ model. The primary source for this study is the article [17] titled "Inventory modeling for imperfect production process with inspection errors, sales return, and imperfect rework process." The objectives of this study include: reformulating the problem of imperfect inventory accompanied by inspection errors, sales returns, and imperfect rework using the EPQ model, and determining the quantity of goods to be produced and the length of the production cycle that maximizes the expected value of total net profit

2 Methods

2.1 Problem Description

This study discusses the imperfect inventory model. This inventory model starts from the production process and inspection process with a production level P of the total production y that takes place during the time period t_1, so it can be seen that $t_1 = {}^{y}/_{P}$. Because the production process is imperfect, there are $x\%$ defective goods produced randomly with a production level d of the total production y. The percentage of defects x is a random variable with a known probability density function $f(x)$. It can also be seen that there are xy defective goods and $(1-x)y$ non-defective goods from the total production y. The imperfect inspection process causes classification errors in the system. There are two types of classification errors in the inspection process, namely Type-I inspection errors and Type-II inspection errors, with their respective proportions being q_1 = proportion of items classified as defective but actually not defective and q_2 = proportion of items classified as not defective but actually defective $(0 < q_1, q_2 < 1)$ followed by their respective probability density functions $f(q_1)$ and $f(q_2)$. Assume, q_1 and q_2 are independent of the defect proportion d. Thus, all items involving inspection errors are interdependent based on q_1, q_2, and y.

Type-I error results in $(1-x)q_1y$ of the total non-defective goods $(1-x)y$ being misclassified as defective, causing losses to the company because of the lost opportunity to increase sales of perfect goods. Type-II error results in penalties and disappointment from customers to the manufacturer because there are xq_2y of the total defective goods xy that are misclassified as non-defective and sold to customers, resulting in sales returns. As a result of customer disappointment, the misclassified defective goods when distributed to customers are continuously re-entered into the system as demand until the production and inspection process ends and are stored in inventory for the cycle period T. The imperfect production and inspection process produces defective and perfect goods, which are $(1-q_2)xy$ and $(1-x)(1-q_1)y$, respectively. Of the total defective goods $((1-q_2)xy + xq_2y + (1-x)q_1y)$which includes actual defective goods, sales returns, and misclassified defective goods will be set aside as scrap and sold at a lower price v before the rework process begins. Suppose the proportion of total defective goods is δ, then $\delta = (1-q_2)x + xq_2 + (1-x)q_1$.

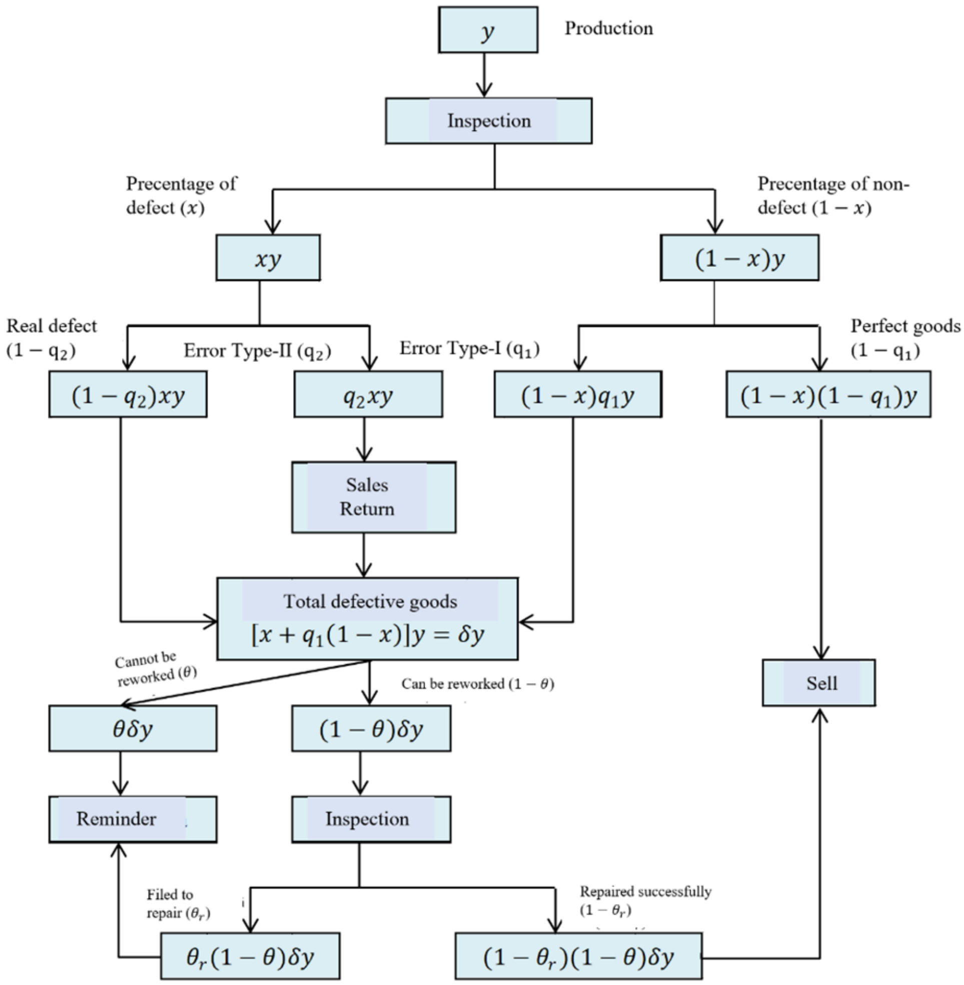

Figure 1 Sequence of inventory events in a cycle

The production level P is limited, so the production level of defective goods can be expressed as the proportion of total defects multiplied by the production level, namely $d = \delta P$. To reduce losses, rework is carried out with θ being the proportion of goods that cannot be reworked from the total defective goods. Therefore, there are $(1-\theta)\delta y$ units to be reworked with rework rate P_r and take place during time period t_2 which is done right after t_1. The rework period t_2 can be calculated as the result of dividing the amount of scrap by the rework rate, $t_2 = {(1-\theta)\delta y}/{P_r}$. After rework, a second inspection will be carried out which is free from errors. The rework process is considered imperfect so that scrap is produced with a proportion of θ_r. The production rate of scrap d_r can be calculated as the result of multiplying the rework rate by the proportion of rework. Thus, the final result of rework and inspection in the form of perfect goods is $(1-\theta_r)(1-\theta)((1-q_2)x + xq_2 + (1-x)q_1)$. The goods that fail in rework are considered as scrap and are sold at a lower price v in the second stage is $(1-\theta)\theta_r((1-q_2)x + xq_2 + (1-x)q_1)y$. The period of one cycle T ends when all the goods are sold out. All the goods will be sold out during the time period t_3, so it can be stated that $T = t_1 + t_2 + t_3$. The sequence of events in one period of inventory cycle is given in Figure 1.

3 Result and Discussion

3.1 Formulation Model

Based on the problem description, a mathematical model of the imperfect inventory problem accompanied by inspection errors, sales returns, and rework can be formulated. The purpose of this model is to determine the optimal length of one cycle (T) and the number of goods produced (y) that maximize the expected value of the total net profit. The model formulation in this problem is carried out based on the sequence of events seen in Figure 1 and the inventory level seen in the illustration in Figure 2.

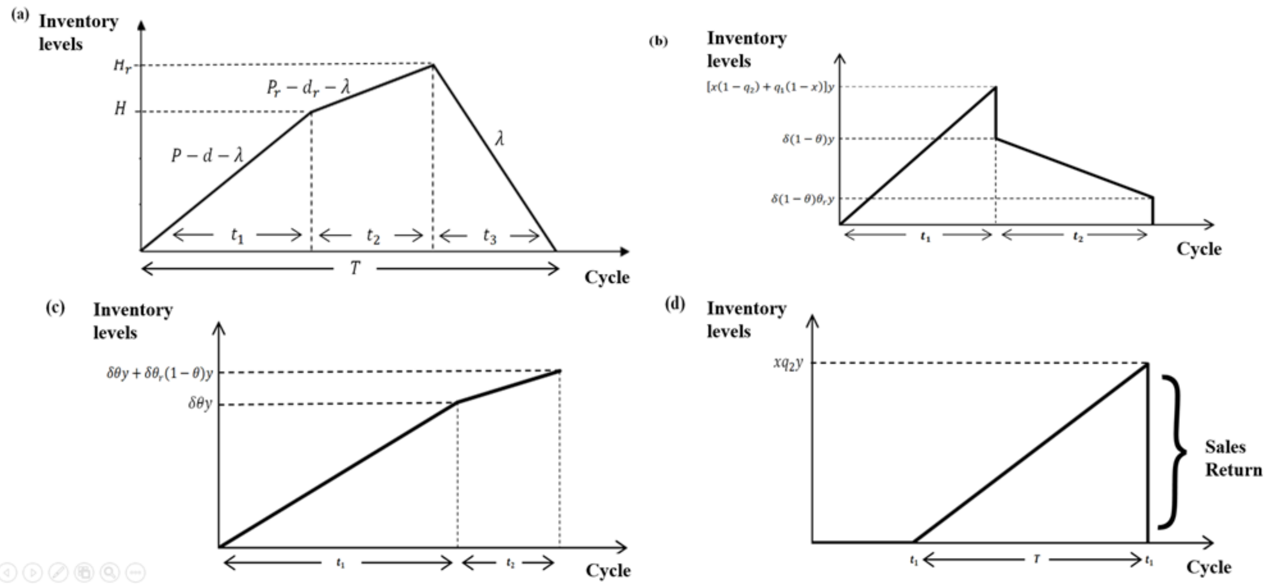

Figure 2 Inventory levels from (a) imperfect production and inspection systems, (b) defective goods sorted through the inspection process, (c) total scrap, (d) sales returns.

Based on Figure 1, it can be seen that the total perfect goods sold are the number of perfect goods when inventory is available $(xq_2 + (1-x)(1-q_1)y)$ and the number of perfect goods after rework $((1-\theta_r)(1-\theta)\delta y)$. The length of one cycle (T) is the total perfect goods sold divided by the demand level, namely:

$$T = \frac{[xq_2 + (1-x)(1-q_1) + [\![\delta(1-\theta]\!]_r)(1-\theta)]y}{\lambda} = \frac{\alpha y}{\lambda} \tag{1}$$

Where,

$$\alpha = \beta + \delta(1-\theta_r)(1-\theta)$$
$$\beta = xq_2 + (1-x)(1-q_1)$$
$$\delta = [x + (1-x)q_1].$$

Since x, q_1, q_2 are random variables, then β, δ are also random variables with the following expected values:

$$E[\beta] = E[x]E[q_2] + (1-E[x])(1-E[q_1]) \tag{2}$$
$$E[\delta] = E[x] + (1-E[x])E[q_1].$$

Since β, δ are random variables, then α is also a random variable with the following expected value:

$$E[\alpha] = E[\beta] + E[\delta](1-E[\theta_r])(1-E[\theta]). \tag{3}$$

Since α is a random variable, then T is also a random variable with the following expected value:

$$E[T] = \frac{E[\alpha]y}{\lambda}. \tag{4}$$

Based on the model analysis, it is known that the inventory available period (t_1) and the rework period (t_2) are as follows:

$t_1 = \frac{y}{P}$ and $t_2 = \frac{\delta y(1-\theta)}{P_r} = \frac{dy(1-\theta)}{PP_r}$.

In Figure 2, it is known that the length of one cycle is the sum of the available inventory period, the rework period, and the out-of-stock period, namely

$$T = t_1 + t_2 + t_3. \tag{5}$$

In time period t_1, the maximum available inventory is H, so that

$$t_1 = \frac{y}{P} = \frac{H}{P-d-\lambda},$$

Where,

$$H = (P-d-\lambda)t_1 = \frac{(P-d-\lambda)y}{P},$$

The rework process causes the maximum inventory to increase to H_r, so that

$$H_r = H + (P_r - d_r - \lambda)t_2 = \frac{(P-d-\lambda)y}{P} + \frac{(P_r - d_r - \lambda)dy(1-\theta)}{PP_r}. \tag{6}$$

After the rework process is finished, the inventory will be exhausted to meet all the demand during the time period t_3, so that $t_3 = \frac{H_r}{\lambda}$.

TC is the total cost of one cycle that occurs when the inventory of goods is imperfect accompanied by inspection errors, sales returns, and imperfect rework, then:

$$\begin{aligned} TC = &(production\ cost\ per\ cycle) + (inspection\ cost\ per\ cycle) \\ &+ (Type-I\ error\ cost\ per\ cycle) + (Type-II\ error\ cost\ per\ cycle) \\ &+ (rework\ cost\ per\ cycle) + (salvage\ cost\ per\ cycle) \\ &+ (storage\ cost\ per\ cycle) \end{aligned}$$

$$TC = PC + IC + EC1 + EC2 + DC + RC + HC$$

$$
\begin{aligned}
TC = {} & K + cy + iy + c_1(1-x)q_1 y + c_2 x q_2 y + c_r\delta(1-\theta) \\
& + u[\theta\delta + \theta_r(1-\theta)\delta]y + \frac{h(P - xP - \lambda)}{2P^2}y^2 \\
& + h(P - xP - \lambda)\frac{x(1-\theta)}{PP_r}y^2 + h(P_r - \theta_r P_r - \lambda)\frac{x^2(1-\theta)^2}{2{P_r}^2}y^2 \\
& + s\frac{h(P - xP - \lambda)^2}{2\lambda P^2}y^2 \\
& + h(P - xP - \lambda)(P_r - \theta_r P_r - \lambda)\frac{x(1-\theta)}{2\lambda PP_r}y^2 + \frac{hx}{2P}y^2 \\
& + \frac{h\alpha x q_2}{2\lambda}y^2 + h_r\frac{x^2(1-\theta)^2}{2P_r}y^2
\end{aligned} \tag{7}
$$

The total revenue per cycle obtained comes from the number of sales of perfect goods, lost income from sales returns due to Type-II errors, sales of perfect goods resulting from rework, and sales of leftover goods.
Total revenue $(TR) = R_1 + R_2 + R_3 + R_4$.

$$
\begin{aligned}
TR = {} & s[xq_2 y + (1-x)(1-q_1)y] - sxq_2 y + s(1-\theta)(1-\theta_r)\delta y \\
& + v[\theta\delta y + (1-\theta)\theta_r\delta y].
\end{aligned} \tag{8}
$$

Total profit (TP) is the difference between total income and total costs, then $TP = TR - TC$, so the total profit per unit time (TPU) is $TPU = \frac{TR-TC}{T}$.
Based on the equations (7) and (8), then

$$
\begin{aligned}
TPU = {} & (sxq_2 + s(1-x)(1-q_1) - sxq_2 + s(1-\theta)(1-\theta_r)\delta + v\theta\delta + v(1- \\
& \theta)\theta_r\delta - c - i - c_1(1-x)q_1 - c_2 x q_2 - c_r\delta(1-\theta) - u\theta\delta + u\theta_r(1- \\
& \theta)\delta)\frac{\lambda}{\alpha} - \left(\frac{h(P-xP-\lambda)}{2P^2} + h(P - xP - \lambda)\frac{xP(1-\theta)}{PP_r} + h(P_r - \theta_r P_r - \right. \\
& \lambda)\frac{x^2(1-\theta)^2}{2{P_r}^2} + \frac{h(P-d-\lambda)^2}{2\lambda P^2} + h(P - xP - \lambda)(P_r - \theta_r P_r - \lambda)\frac{x(1-\theta)}{2\lambda PP_r} + \frac{hx}{2P} + \\
& \left.\frac{h\alpha x q_2}{2\lambda} + h_r\frac{x^2(1-\theta)^2}{2P_r}\right)\frac{\lambda y}{\alpha} - \frac{K\lambda}{\alpha y}.
\end{aligned} \tag{9}
$$

Since $x, \theta, \theta\_r, q_1, q_2, \alpha, \beta, \delta$ are random numbers, the expected value of the total profit per unit time $(E[TPU])$ is

$$
\begin{aligned}
E[TPU] = {} & \big(sE[x]E[q_2] + s(1-E[x])\big(1-E[q_{1]}\big) - sE[x]E[q_2] + s(1- \\
& E[\theta])(1-E[\theta_r])E[\delta] + vE[\theta]E[\delta] + v(1-E[\theta])E[\theta_r]E[\delta] - c - \\
& i - c_1(1-E[x])E[q_1] - c_2E[x]E[q_2] - c_rE[\delta](1-E[\theta]) - \\
& uE[\theta]E[\delta] + uE[\theta_r](1-E[\theta])E[\delta]\big)\frac{\lambda}{E[\alpha]} - \left(\frac{h(P-E[x]P-\lambda)}{2P^2} + h(P - \right. \\
& E[x]P - \lambda)\frac{E[x](1-\theta)}{PP_r} + h(P_r - E[\theta_r]P_r - \lambda)\frac{E^2[x](1-E[\theta])^2}{2{P_r}^2} + \\
& \frac{h(P-E[x]P-\lambda)^2}{2\lambda P^2} + h(P - E[x]P - \lambda)(P_r - E[\theta_r]P_r - \lambda)\frac{E[x](1-E[\theta])}{2\lambda PP_r} + \\
& \left.\frac{hE[x]}{2P} + \frac{hE[\alpha]E[x]E[q_2]}{2\lambda} + h_r\frac{E^2[x](1-E[\theta])^2}{2P_r}\right)\frac{\lambda y}{E[\alpha]} - \frac{K\lambda}{E[\alpha]y}.
\end{aligned} \tag{10}
$$

The values of y^* and T^* that maximize $ETPU$ are

$$
y^* = \sqrt{\frac{K}{B}}, \tag{11}
$$

when

$B = \frac{h(P-E[x]P-\lambda)}{2P^2} + h(P - E[x]P - \lambda)\frac{E[x](1-\theta)}{PP_r} + h(P_r - E[\theta_r]P_r - \lambda)\frac{E^2[x](1-E[\theta])^2}{2P_r^{\,2}} + \frac{h(P-E[x]P-\lambda)^2}{2\lambda P^2} + h(P - E[x]P - \lambda)(P_r - E[\theta_r]P_r - \lambda)\frac{E[x](1-E[\theta])}{2\lambda PP_r} + \frac{hE[x]}{2P} + \frac{hE[\alpha]E[x]E[q_2]}{2\lambda} + h_r\frac{E^2[x](1-E[\theta])^2}{2P_r}$

and

$$T^* = \frac{E[\alpha]y^*}{\lambda}. \tag{12}$$

3.2 Implementation Model

In an inventory problem in a manufacturing company, it is known that the annual demand level from consumers is constant at 90,000 units per year. Because the production and inspection processes are not perfect, rework is carried out, so to meet consumer demand, let's say the company's production level is 200,000 units per year and the rework level is 150,000 units per year. The fixed cost of production is 500 thousand rupiah per cycle, the production cost is thirty thousand rupiah per unit, the inspection cost is ten thousand rupiah per unit and the storage cost is five thousand rupiah per unit. The selling price of perfect goods is sixty thousand rupiah per unit and the selling price of leftover goods is twenty thousand rupiah per unit. Suppose the proportion of imperfect goods, x, has a uniform distribution that is in the range $[a, b]$ with $a = 0$ and $b = 0.1$. The proportion of Type-I and Type-II inspection errors are q_1 and q_2 respectively which are uniformly distributed in the range $[0;\ 0.04]$. The cost due to Type-I and Type-II inspection errors is ten thousand rupiah per unit and twelve thousand rupiah per unit. Suppose that rework is carried out but not perfectly, with the proportion of items that cannot be reworked and failed to be reworked are θ and θ_r which are spread uniformly in the range [0; 0.05]. The salvage cost is twelve thousand rupiah per unit, the rework cost is fifteen thousand rupiah per unit and the storage cost of reworked items is eight thousand rupiah per unit.

In this case, it is known that

P	=	200.000 per year	c_2	=	Rp12.000,00 per unit
P_r	=	150.000 per year	i	=	Rp10.000,00 per unit
λ	=	90.000 per year	s	=	Rp60.000,00 per unit
K	=	Rp500.000,00 per siklus	v	=	Rp20.000,00 per unit
c	=	Rp30.000,00 per unit	u	=	Rp12.000,00 per unit
c_r	=	Rp15.000,00 per unit	h	=	Rp5.000,00 per unit
c_1	=	Rp10.000,00 per unit	h_r	=	Rp8.000,00 per unit

Since x, q_1, q_2, θ, and θ_r have a uniform distribution, the probability density function is as follows.

$$f(x) = \begin{cases} 10, & 0 \le x \le 0{,}1 \\ 0, & x \text{ otherwise} \end{cases}$$

$$f(q_1) = \begin{cases} 25, & 0 \le q_1 \le 0{,}04 \\ 0, & q_1 \text{ otherwise} \end{cases}$$

$$f(q_2) = \begin{cases} 25, & 0 \le q_2 \le 0{,}04 \\ 0, & q_2 \text{ otherwise} \end{cases}$$

$$f(\theta) = \begin{cases} 20, & 0 \le \theta \le 0{,}05 \\ 0, & \theta \text{ otherwise} \end{cases}$$

$$f(\theta_r) = \begin{cases} 20, & 0 \le \theta_r \le 0{,}05 \\ 0, & \theta_r \text{ otherwise} \end{cases}$$

Based on [18], $E(x), E(q_1), E(q_2), E(\theta)$, and $E(\theta_r)$ are as follows

$$E(x) = \frac{0+0{,}1}{2} = 0{,}05,$$
$$E(q_1) = \frac{0+0{,}04}{2} = 0{,}02,$$
$$E(q_2) = \frac{0+0{,}04}{2} = 0{,}02,$$
$$E(\theta) = \frac{0+0{,}05}{2} = 0{,}025,$$
$$E(\theta_r) = \frac{0+0{,}05}{2} = 0{,}025.$$

Based on the equations (2), (3), (10), (11), and (12) in the model formulation, the following values are obtained:

$$E(\beta) = 0{,}9320,$$
$$E(\delta) = 0{,}0690,$$
$$E(\alpha) = 0{,}9976, \text{ and}$$
$$y^* = 5.775 \text{ unit.}$$

The expected cycle length is $T^* = 0.064\ years\ \approx 23\ days.$

The expected total profit is $E[TPU] = Rp1{,}672{,}513{,}135.44.$

The maximum available inventory is obtained, namely $H = 2.88$ units and the maximum available inventory is obtained when the rework process ends, namely $H_r = 2{,}993$ units. Based on the implementation calculation, the total number of defective goods is 399 units with 389 units to be reworked and 10 units cannot be reworked. Of the 389 units that were reworked, there were 10 items that failed to be reworked, so that the total used goods became 20 units. The inventory level in the implementation results can be illustrated in Figure 3 below.

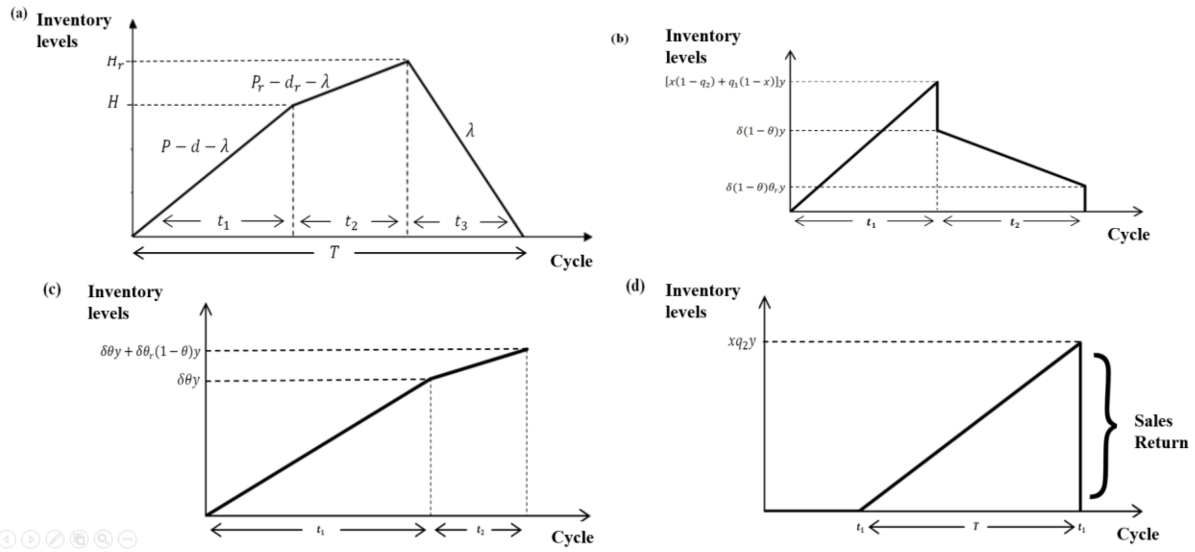

Figure 3 Illustration of inventory level implementation from (a) imperfect production and inspection system, (b) defective goods sorted through inspection process, (c) total remaining goods, (d) sales returns

Figure 3(a) shows the maximum inventory available is 2,888 units and the maximum inventory available when the rework process ends is 2,993 units. Figure 3(b) shows the total number of defective items is 399 units, the number of defective items that can be reworked is 389 units, and the number of items that failed to be reworked is 10 units. Figure 3(c) shows the number of remaining items is 10 units and after the rework process ends the number of remaining items becomes 20 units. Figure 3(d) shows the number of sales returns is 6 units.

4 Conclusion And Suggestions

This scientific paper discusses the imperfect inventory system accompanied by inspection errors, sales returns, imperfect rework using the EPQ method. After conducting research, an inventory problem formulation was obtained to determine the amount of production and the length of the production cycle that maximizes the expected value of total profit. Based on the implementation results, the optimum number of production units is 5,755 units, the optimum cycle length is 23 days, and the maximum expected total profit is Rp 1,672,513,135.44.

This study uses an imperfect inventory model with inspection errors, sales returns, and imperfect rework assuming that inventory shortages are not allowed. This study can be continued by assuming that inventory shortages are allowed.

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