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A COMPETITION MODEL FOR THE STUDENTS RECRUITMENT BASED ON THE GENDER

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Abstract

We consider a competition model describing the student competitiveness during recruitment. Here, the student population is divided into two sub-populations, namely male student and female student. We assume that the competition occurs not just on the same sex, but also different sexes. We perform dynamical analysis including the existence and the local stability of equilibria. It is shown that the model has four equilibria. Our analysis shows that the male and female students do not exist, if the intrinsic growth rate is less than one. Furthermore, if the intrinsic growth rate is greater than one, then the coexistence of student equilibria exists. It means that the recruitment process on the student determines the growing effects of the student population at the school. The results show that there has been a student recruitment for SMA Negeri 50 Maluku Tengah in Maluku Province, Indonesia. The dynamics of the recruitment system are confirmed by the numerical simulations.
Keywords*: Competition model, student recruitment, sex, stability.*

1 Introduction

Improving the quality of education in schools requires good planning and implementation. Student management covers the entire process from new student enrollment to graduation [1]. On other's hand, every school is definitely faced with competitiveness. Institutional competitiveness is a topic of discussion in the educational sphere. Considering the importance of this issue, The school is trying to implement various strategies in all fields [2]. The limited supporting capacity available in schools has an impact on competition among students. Through honest competition, students will compete to get student status at a school. This process is very interesting to explore mathematically.

Mathematically, in 1928, Volterra has provided an illustration that when a single resource fought over by two species, the impact is that one species will become extinct. This can be avoided when the ratio of hunting to death is exactly the same [3, 4]. A stable situation for the two species to coexist cannot be achieved. Suppose there is X type of species competing against Y resources. One

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or more types of resources can be consumed by each consumer. Consumers engage in competition for resources but have no direct interaction with each other in any other way. At steady state, the number of consumer species coexisting cannot be greater than the number of available resources, as per the Competition Exclusion Principle, i.e., $X \leq Y$ [5, 4, 6]. For example, competition for prey is limited by carrying capacity [7, 8, 9]. In addition, predators compete to hunt and eat prey [10, 11].

Recently, a mathematical approach has been used to study various real conditions [14, 15]. Due to apply of Lotka–Volterra model for competitive problems, Windarto and Eridani in [12] studied motorcycle sale competition. Next, the problem of competition between iOS and Android is researched by Latif et el. in [13]. Thus, competition models are not only used to investigate the competitive behavior of species, but also industrials sector.

We extend the model given in [12] by carrying capacity to study the dynamics behavior of student recruitment. There are four sections in this paper. The following section studied the Competition Model in Student Recruitment. In Section Analisys and Numerical Simulation, the existence and stability of equilibria are examined through discussions of fundamental mathematical substances. Additionally, we establish the criteria for local stability of the feasible equilibria in the proposed model. Moreover, we offer numerical simulations to reinforce our analytical discoveries. The final section includes the concluding remarks.

2 Application of Competition Model in Student Recruitment

This section utilizes the Lotka-Volterra type competition mathematical model to explain the dynamics of student recruitment. Our focus is on analyzing the competition dynamics between sexes in the school. Here, the student population is divided into two sub-population. Suppose L and P represent the number of annual student recruitment of the male student and the female student at the school. The mathematical model of the student competition is constructed based on the ordinary defferential equation system follow as:

$$\begin{aligned} \frac{dL}{dt} &= \alpha_1 L\left(1 - \frac{L}{C}\right) - \frac{LP}{C}, \\ \frac{dP}{dt} &= \alpha_2 P\left(1 - \frac{P}{C}\right) - \frac{LP}{C}, \end{aligned} \tag{1}$$

where L and P are respectively the densities of male student and female student population at time t. The following assumptions are taken in deriving model (1):

(M-1) The male student growth depends entirely on the competition by the male student and also female student, where the growth is assumed to be logistically with constant intrinsic rate $\alpha_1 > 0$.

(M-2) The female student growth depends entirely on the competition by the male student and also female student, where the growth is assumed to be logistically with constant intrinsic rate $\alpha_2 > 0$.

(M-3) The competition proceses among students in the recruitment are limited by constant carrying capacity $C > 0$.

(M-4) Regarding the application to real situations, namely competition in student recruitment, we assume that competition occurs between members of the same sex (L^2 and P^2) and also those of different sexes (LP). In this case, it is realistic to ignore no competition.

3 Analysis and Numerical Simulation

In this section, we discuss existence and stability of equilibria. In addition, we perform numerical simulation to verify analitical results.

3.1 Existence and Stability of Equilibria

It's not difficult to show that system (1) has four non-negative equilibria as follows.

- The equilibria $E_0 = (0, 0)$, which there is no population of student at the school.
- The equilibria $E_1 = (0, C)$, which there is no population of the male student at the school.
- The equilibria $E_2 = (C, 0)$, which there is no population of the female student at the school.
- The equilibria $E^* = (L^*, P^*)$, i.e., the student coexist, where

$$L^* = \frac{\alpha_2 K(\alpha_1 - 1)}{\alpha_1\alpha_2 - 1}, \tag{2}$$
$$P^* = \frac{\alpha_1 K(\alpha_2 - 1)}{\alpha_1\alpha_2 - 1}. \tag{3}$$

If conditions of (4), (5), and (6) are satisfying, a point E^* will be exist.

$$\alpha_1 > 1. \tag{4}$$

$$\alpha_2 > 1. \tag{5}$$

$$\alpha_1\alpha_2 > 1. \tag{6}$$

The local stability of each equilibria of system (1) is shown in the following theorem.

Theorem 3.1. *For the system* (1)*, we have the following stability properties of its equilibria:*
(i) The point E_0 is always unstable.
(ii) The point E_1 is locally asymptotically stable if $\alpha_1 < 1$.
(iii) The point E_2 is locally asymptotically stable if $\alpha_2 < 1$.
(iv) The point E^* is locally asymptotically stable if $\left(\frac{1}{2}(\beta_1 + \beta_2) \pm \frac{1}{2}\sqrt{(\beta_1 - \beta_2)^2 + \frac{4}{K^2}L^*P^*}\right) < 0$.

Proof. The local stability of all equilibria can be studied from the linearization of the system (1). The Jacobian matrix of the system (1) at a point (L, P) is given by.

$$J = \begin{pmatrix} \alpha_1\left(1 - \frac{L}{P}\right) - \frac{\alpha_1 L}{K} - \frac{P}{K} & -\frac{L}{K} \\ -\frac{P}{K} & \alpha_2\left(1 - \frac{P}{K}\right) - \frac{\alpha_2 P}{K} - \frac{L}{K} \end{pmatrix}. \tag{7}$$

By observing the eigenvalues of the Jacobian matrix (7) at each equilibria, we have the following stability properties.

(i) The Jacobian matrix of the system (1) at E_0 has eigenvalues $\lambda_1 = \alpha_1$ and $\lambda_2 = \alpha_2$. Obiviously, $\mathtt{Re}(\lambda_{1,2}) < 0$ and E_0 is always unstable.

(ii) The Jacobian matrix of the system (1) at E_1 has eigenvalues $\lambda_1 = \alpha_1 - 1$ and $\lambda_2 = -\alpha_2$. If

$$\alpha_1 < 1, \tag{8}$$

then $\mathtt{Re}(\lambda_1) < 0$. Thus, point E_1 is locally asymptotically stable.

(iii) The Jacobian matrix of the system (1) at E_2 has eigenvalues $\lambda_1 = -\alpha_1$ and $\lambda_2 = \alpha_2 - 1$. If

$$\alpha_2 < 1, \tag{9}$$

then $\mathtt{Re}(\lambda_2) < 0$. Thus, point E_2 is locally asymptotically stable.

(iv) The Jacobian matrix of the system (1) at E^* has eigenvalues

$$\lambda_1 = \left(\frac{1}{2}(\beta_1 + \beta_2) + \frac{1}{2}\sqrt{(\beta_1 - \beta_2)^2 + \frac{4}{K^2}L^*P^*}\right),$$

and

$$\lambda_2 = \left(\frac{1}{2}(\beta_1 + \beta_2) - \frac{1}{2}\sqrt{(\beta_1 - \beta_2)^2 + \frac{4}{K^2}L^*P^*}\right),$$

where:

$$\beta_1 = \alpha_1\left(1 - 2\frac{L^*}{K}\right) - \frac{P^*}{K},$$

and

$$\beta_2 = \alpha_2\left(1 - 2\frac{P^*}{K}\right) - \frac{L^*}{K}.$$

A point E^* is locally asymptotically stable, when $\mathtt{Re}(\lambda_{1,2}) < 0$.

3.2 Numerical Simulation

This numerical simulation complements the analysis discussed earlier by providing additional support for the results. The parameter was established based on student recruitment data from SMA Negeri 50 Maluku Tengah, Maluku, Indonesia. By employing the Runge-Kutta Fourth-Order method in Matlab software, we solved the ODEs of model (1) using the parameters provided in Table 1.

Table 1: Parameter Values

Parameter	Value	Reference
α_1	2.0000	Estimation
α_2	2.8181	Estimation
C	250	Assumed

To determine the value of parameters α_1 and α_2 used the following method.

$$\alpha_1 = \alpha_{1LM} - \alpha_{1LK}, \tag{10}$$

$$\alpha_2 = \alpha_{2PM} - \alpha_{2PK}, \tag{11}$$

where α_{1LM} is average number of male students entering per year and α_{1LK} is average number of male students leaving per year. Next, α_{2PM} is average number of female students entering per year and α_{2PK} is average number of female students leaving per year.

By substituting the parameter values in Table 1 into equation system (1), the competitive of student recruitment between male and female, can be illustrated the following form.

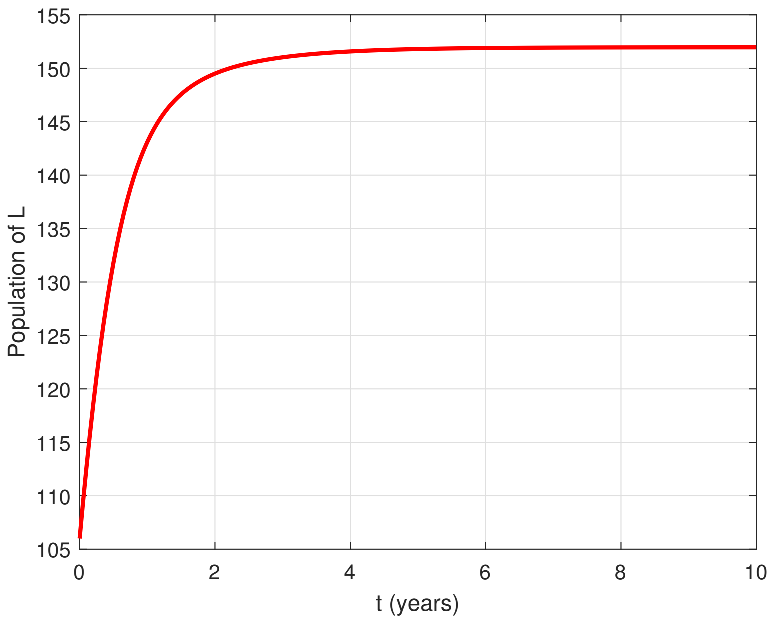

Figure 1: Dynamics of Recruitment for Male Students

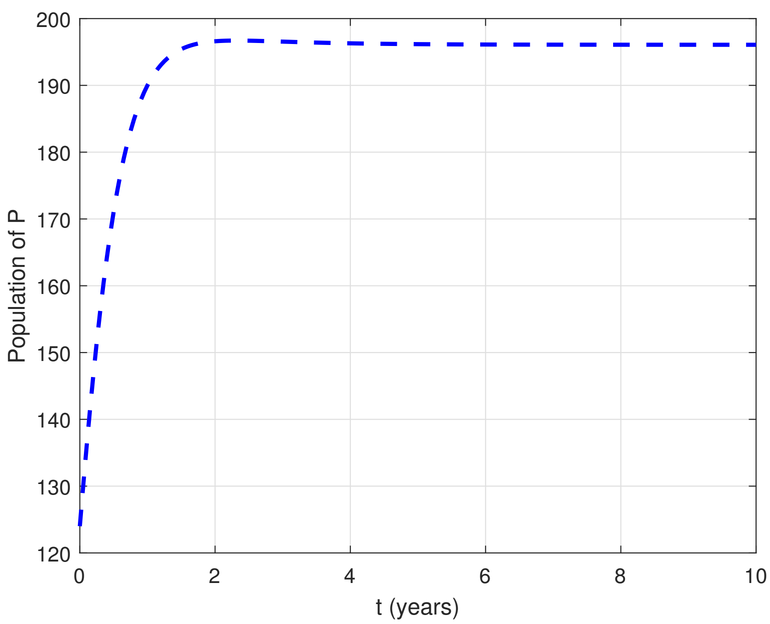

Figure 2: Dynamics of Recruitment for Female Students

$$\frac{dL}{dt} = 2.0000L\left(1 - \frac{L}{250}\right) - \frac{LP}{250}, \tag{12}$$

$$\frac{dP}{dt} = 2.8181P\left(1 - \frac{P}{250}\right) - \frac{LP}{250}. \tag{13}$$

Numerical simulations are carried out from $t = 0$ to $t = 10$ (years). Dynamical recruitments of the male student and female student respectively are plotted in Figure 1 and Figure 2. Next, Figure 3 was illustrated behavior of student recruitment for the both male and female. Further, using the parameter values in Table 1 we get the point $E^* = (151.9617359; 196.0765282)$ is stable. For E^*, we obtained eigenvalues $\lambda_1 = -0.86207 < 0$ and $\lambda_2 = -2.56387 < 0$. The presence

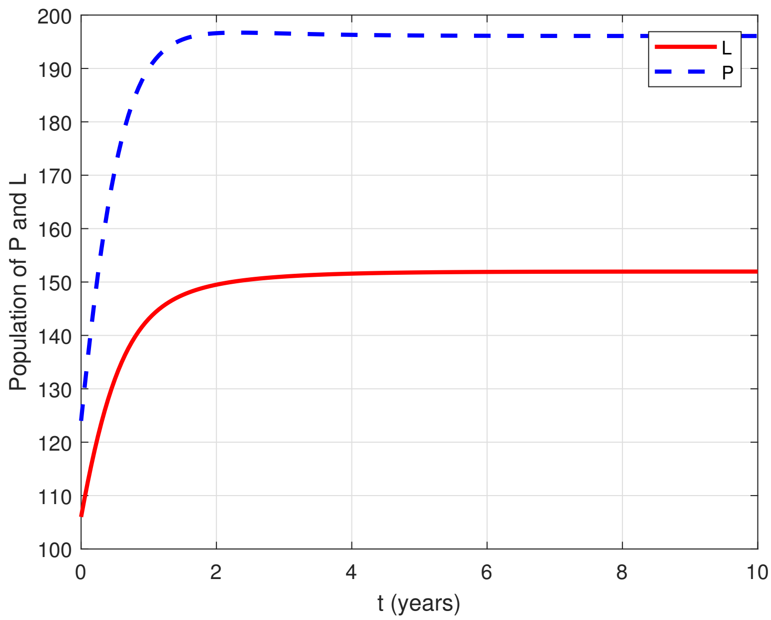

Figure 3: Dynamics of Recruitment for Male and Female Students

of students guarantees the schools survival. The number of male students at any given time will convergent to 151.9617359. Next, different things happen to the number of female students which convergent to 196.0765282.

The parameters in the system (1) offer a straightforward explanation of the recruitment conditions. To enhance realism, numerical simulations are used to estimate real events occurring in schools. Our research reveals that recruitment plays a crucial role in the movement of student population growth. The system (1) also indicate the presence of the student population through stability analysis. Mathematically, the stability analysis of the model demonstrated through trajectories is highly realistic. Thus, the recruitment process causes an increase in the student population, as seen.

4 Conclusion

A modification of the Lotka–Volterra model for the student competitive problem in recruitment process is developed. The model system (1) has been used by us to describe student competition based on the sex, where the data is cited from SMA Negeri 50 Maluku Tengah, Maluku, Indonesia. Due to the recruitment process, the student population is devided into two sub-population, i.e., male and female students. Every equilibria that generates is a solution of the system. Mathematically, from our findings there is absolutely equilibria to consider, which is a point E^* has a positive value and gives two negative eigenvalues. The simulation model shows the recruitment conditions for the male and female students to continue to coexist.

5 Acnknowlegments

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