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COMPARISON OF FUZZY TIME SERIES MARKOV CHAIN AND AVERAGE BASED FUZZY TIME SERIES MARKOV CHAIN IN FORECASTING COMPOSITE STOCK PRICE INDEX

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ABSTRACT

JCI's movement is a reference for investors to make decisions on whether to sell, hold, or buy shares. In this study, the Fuzzy Time Series Markov Chain (FTS-MC) and Average Based Fuzzy Time Series Markov Chain (Average Based FTS-MC) methods will be compared in forecasting the closing price of the JCI. These two methods have differences in determining the length of the interval where the length of this interval is very influential in the formation of Fuzzy Logical Relationships (FLR). As a result, the length of the interval will affect the forecasting result of the closing price of the JCI. The data used in the study was the closing price of the JCI from January 2016 to April 2022. The accuracy level of these two methods is viewed based on the Mean Absolute Percentage Error (MAPE) value. The test results showed that the Average Based FTS-MC had a smaller MAPE value, which was 1.590415%. In other words, the accuracy rate of forecasting the closing price of the JCI with the Average Based FTS-MC Method has an excellent accuracy rate of 98.41%.

Keywords: Average Based, Fuzzy Logical Relationship, Fuzzy Time Series Markov Chain, MAPE

1 Introduction

The capital market is a means of funding for governments and companies as well as a means of investment for fund owners [1].The capital market experiences an increase (bullish) or decrease (bearish) as can be seen from the rise and fall of the stock price recorded through an index movement. One of the indicators used to see the development of the Indonesian capital market is the Composite Stock Price Index (JCI). JCI is a value used to measure the combined performance of all stocks listed on the Indonesia Stock Exchange (IDX). The movement of the JCI is very important for investors because the stock portfolio of the JCI in general depends on the ups and downs of the index [2]. Thus, the JCI movement can be a reference for investors to make decisions on whether to sell, hold, or buy shares. With the fluctuations in the composite stock price index, a method is needed to predict the value of the JCI that can reduce the risk for investors to invest. One of the methods that can be used for this JCI forecasting is the Fuzzy Time Series Markov Chain.

The Fuzzy Time Series Markov Chain (FTS-MC) was first proposed by Tsaur, who in his research combined the Fuzzy Time Series (FTS) method with the Markov Chain to increase accuracy in predicting the exchange rate of the Taiwan currency with the US dollar [3]. FTS-

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MC widely used in forecasting in various research fields, some of them are listed in the references of this study as [4], [5], and [6]. This FTS-MC has also been used in forecasting stock prices and obtaining an excellent level of accuracy [7]. In the FTS-MC method, the length of the interval is very influential in the formation of Fuzzy Logical Relationships which will later affect the final result of forecasting. One method of determining the interval length that can give better forecasting results is Average Based [8]. The FTS-MC method of determining the interval length using Average Based is known as the Average Based Fuzzy Time Series Markov Chain (Average Based FTS-MC).

Based on this research, FTS-MC and Average Based FTS-MC are two methods that can be used for forecasting where the forecasting results have an excellent level of accuracy. So in this paper, we will discuss the comparison of the Fuzzy Time Series Markov Chain (FTS-MC) and Average Based Fuzzy Time Series Markov Chain (Average Based FTS-MC) methods in forecasting the closing price of the JCI.

2 Literature Review

The Composite Stock Price Index (JCI) is a value used to measure the combined performance of all stocks listed on the Indonesia Stock Exchange (IDX). JCI data is time series data that fluctuated and can be predicted for the next several periods. The JCI movements are important for investors to make decisions about whether to sell, hold, or buy shares. Thus, forecasting JCI can help investors carry out the process of buying and selling shares and reduce the risk for investors to invest. The FTS-MC and Average Based FTS-MC methods were used to forecast the JCI closing price and the accuracy of these two methods will be compared based on the Mean Absolute Percentage Error (MAPE) value to find out which method is better at predicting the JCI closing price.

2.1 Fuzzy Time Series Markov Chain

Fuzzy Time Series Markov Chain (FTS-MC) is a combination of Fuzzy Time Series method with Markov Chain to obtain the largest probability by using a transition matrix in order to obtain a higher level of accuracy than the Fuzzy Time Series method [3]. According to Tsaur [3], the following algorithm is used in the fuzzy time series markov chain method:

(1) Determine the universal set from historical data (Y_t) as follows:

$$U = [Y_{\min} - D_1,\ Y_{\max} + D_2] \tag{1}$$

where $Y_{\min}$ and $Y_{\max}$ represents maximum and minimum history data, whereas D_1 and D_2 was suitable integer such that historical data is contained in the universal set U.

(2) The universal set U can be partitioned into subsets $u_1, u_2, u_3, \ldots, u_n$ with the number of intervals can be determined using the following Sturges formula:

$$n = 1 + 3{,}3 \log N \tag{2}$$

where N represents the number of historical data. The interval length of the subsets $u_1, u_2, u_3, \ldots, u_n$ can be determined by:

$$l = \frac{[(D_{\max} + D_2) - (D_{\min} - D_1)]}{n} \tag{3}$$

Here is the partition of the set U:

$$\begin{aligned} u_1 &= [D_{\min} - D_1,\ D_{min} - D_1 + l] \\ u_2 &= [D_{\min} - D_1 + l,\ D_{min} - D_1 + 2l] \\ &\vdots \\ u_n &= [D_{\min} - D_1 + (n-1)l,\ D_{min} - D_1 + nl] \end{aligned} \tag{4}$$

(3) Determine the fuzzy set. In general, a fuzzy set A_i can be expressed as follows

$$A_i = \left\{\frac{\mu_{ij}}{u_j}\right\}, j = 1,2,\dots,n$$

where μ_{ij} represents membership degree u_j towards A_i which is defined as follows:

$$\mu_{ij} = \begin{cases} 1, & i = j \\ 0.5, & j = i-1 \text{ or } i = j-1 \\ 0, & \text{otherwise} \end{cases}$$

Example:
We will define a fuzzy set A_1 where

$$A_1 = \left\{\frac{\mu_{1j}}{u_j}\right\}, j = 1,2,\dots,n$$

μ_{1j} i.e. membership degree of u_j towards $A_1, j = 1,2,\dots,n$ in the following way:
- for $j = 1 \rightarrow \mu_{11} = 1$ because $i = j$
- for $j = 2 \rightarrow \mu_{12} = 0.5$ because $i = j - 1$
- for toher j, then $\mu_{13} = \mu_{14} = \cdots = \mu_{1n} = 0$

Tuhs, fuzzy set of A_1 can be expressed as follows:

$$A_1 = \left\{\frac{1}{u_1}, \frac{0.5}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6}, \frac{0}{u_7}\right\}$$

(4) Fuzzification of historical data aims to convert numeric variables into linguistics using fuzzy set membership values. If historical data is included in the u_j interval, then the data is fuzzified into A_i

(5) Determining Fuzzy Logical Relationships (FLR). The following is the definition of FLR:
Definition 2.1. Suppose $F(t) = A_i$ is caused by $F(t-1) = A_j$, then FLR is defined as $A_j \rightarrow A_i$.

(6) Determine the Fuzzy Logical Relationship Group (FLRG) based on the grouping of relationships between states into the same state.
Example: If there is an FLR obtained from state A_2, then a transition is made to another state $A_j,\ j = 1,2,\dots,n$, such as $A_2 \rightarrow A_2, A_2 \rightarrow A_2, A_2 \rightarrow A_1$.
Therefore, FLR are grouped into FLRG such as: $A_2 \rightarrow A_1, A_2, A_3$.

(7) Forming a transition probability matrix
The transition probability matrix $P_{ij} = (p_{ij})$ where p_{ij} is the one-step transition probability from state i to state j. The transition probability matrix is expressed as follows:

$$P_{ij} = \frac{f_{ij}}{f_i} \tag{5}$$

where, f_{ij}: the number of one-step transition from state A_i to state A_j
f_i: the number of historical data included in state A_i.

(1) Forecasting data at time t, where at time t, data is in state A_i
   a) Calculate the initial forecasted value using the following rules:
   Rule 1. If $A_i \rightarrow \emptyset$, then the forecasted result is $F(t) = m_i$, which is the midpoint of u_i
   Rule 2. If $A_i \rightarrow A_k$, where $p_{ij} = 0$ and $p_{ik} = 1$, $j \neq k$, then the forecasted result is $F(t) = m_k p_{ik} = m_k$ where m_k is the midpoint of u_k:
   Rule 3. If $A_j \rightarrow A_1, A_2, \ldots, A_n$, and data set $Y(t-1)$ at time $t-1$ is in state A_j then the forecasted result is:
   $F(t) = m_1 p_{j1} + \cdots + m_{j-1} p_{j,j-1} + Y(t-1)p_j + m_{j+1} p_{j,j+1} + \cdots + m_n p_{jn}$
   where m_i is midpoint of u_i.
   b) Calculate the adjusted value D_t using the following rules:
   Rule 1. If $A_i \leftrightarrow A_i$, $A_i \rightarrow A_j, i < j$, then the adjusted value D_t is determined by: $D_{t1} = \frac{l}{2}$
   Rule 2. If $A_i \leftrightarrow A_i$, $A_i \rightarrow A_j, i > j$, then the adjusted value D_t is determined by: $D_{t1} = -\frac{l}{2}$
   Rule 3. If $A_i \rightarrow A_{i+s}, 1 \leq s \leq n-i$ then the adjusted value D_t is determined by: $D_{t2} = \left(\frac{l}{2}\right)s$
   Rule 4. If $A_i \rightarrow A_{i-v}, 1 \leq v \leq i$ then the adjusted value D_t is determined by: $D_{t2} = \left(-\frac{l}{2}\right)s$.
   c) The final forecasted value is calculated using the following formula:

$$F^*(t) = F(t) \pm D_{t1} \pm D_{t2} \tag{6}$$

2.2 Average Based Fuzzy Time Series Markov Chain

The algorithm in the Average Based Fuzzy Time Series Markov Chain (Average Based FTS-MC) method is the same as the algorithm in the FTS-MC method. The difference is in determining the length of the interval in step (2). Determination of the length of the interval is very influential on the formation of a Fuzzy Logical Relationship (FLR) which will affect the final forecasting result. According to Xihao and Yimin [8] determination of interval length using the Average Based method can provide better forecasting results. The following are the steps in determining the length of the interval using the Average Based method:

(1) Calculates the average of the absolute value of the difference between Y_t and Y_{t+1}, i.e:

$$M = \frac{\sum_{t=1}^{N-1} |Y_{t+1} - Y_t|}{N-1} \tag{7}$$

(2) Calculates half of the mean value obtained in step (1) which is referred to as interval length (l)
(3) Determine the basis of the interval length based on Table 1 below

Table 1: Interval Basis

Interval Length	Basis
0.1-1.0	0.1
1.1-10	1
11-100	10
101-1000	100
1001-10000	1000

(4) The length of the interval in step (2) is rounded up according to the basis obtained in step (3). Furthermore, the length of this interval is used to create a partition of the universal set U.

2.3 Mean Absolute Percentage Error (MAPE)

After the forecasting results from the two methods are obtained, the next step is to look at the accuracy of the forecasting values of the two methods using the Mean Absolute Percentage Error (MAPE), which can be written mathematically as follows:

$$\text{MAPE} = \frac{1}{N}\sum_{t=1}^{N}\frac{|Y_t - F^*(t)|}{Y_t} \times 100\% \tag{8}$$

where Y_t is the historical data at time t and $F^*(t)$ is the result of forecasting at time t.

3 Results and Discussion

The data used is monthly JCI closing price data taken from https://finance.yahoo.com from January 2016 to April 2022 (76 months) obtained from [9]. JCI closing price movement from January 2016 to April 2022 can be seen in Figure 1 below:

Figure 1: JCI closing price data from January 2016 to April 2022

Based on Figure 1, it can be seen that the closing price of the Composite Stock Price Index (JCI) from January 2016 to April 2022 always fluctuated. The lowest JCI closing price occurred in January 2016 while the highest JCI closing price occurred in March 2020. Based on this data, the JCI closing price was forecasted using the FTS-MC and Average Based FTS-MC methods. Then, the level of accuracy of each method is calculated using MAPE to find out which method is better in forecasting the closing price of the JCI.

The following is the calculation result of each step in the FTS-MC algorithm:

(1) Based on historical data obtained $Y_{\max} = 7228.91$ and $Y_{\min} = 4538.93$.
The values of D_1 and D_2 used are $D_1 = 8.93$ and $D_2 = 1.09$, so based on Equation (1), we get $U = [4530{,}7230]$.

(2) The number of intervals obtained based on Equation (2) is: $n = 7.2066849 \approx 7$, and by using Equation (3) the length of the interval is: $l = 385.71484$. So that the fuzzy set is obtained according to equation (4) as follows:

$$u_1 = [4530, 4915.72]$$

$$u_2 = [4915.72, 5301.43]$$

$$\vdots$$

$$u_7 = [6844.29, 7230]$$

(3) Determine the fuzzy set A_i for the entire universal set U where $i = 1,2, \dots, n$.
Based on the rules of membership degree u_i in step (3) before, the fuzzy set A_i are defined as:

$$A_1 = \left\{\frac{1}{u_1}, \frac{0.5}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6}, \frac{0}{u_7}\right\}, A_2 = \left\{\frac{0.5}{u_1}, \frac{1}{u_2}, \frac{0.5}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0}{u_6}, \frac{0}{u_7}\right\}, \dots,$$

$$A_7 = \left\{\frac{0}{u_1}, \frac{0}{u_2}, \frac{0}{u_3}, \frac{0}{u_4}, \frac{0}{u_5}, \frac{0.5}{u_6}, \frac{1}{u_7}\right\}$$

(4) Based on the fuzzy set that has been formed, each historical data will be fuzzified and then the FLR will be determined from the closing price of the JCI.

Table 2. Fuzzification and FLR result of JCI Closing Price with the FTS-MC

Date	JCI Closing Price	Fuzzification	FLR
Jan-16	4615.16	A_1	
Feb-16	4770.96	A_1	$A_1 \longrightarrow A_1$
Mar-16	4845.37	A_1	$A_1 \longrightarrow A_1$
Jun-16	4838.58	A_1	$A_1 \longrightarrow A_1$
⋮	⋮	⋮	⋮
Jan-22	6631.15	A_6	$A_6 \longrightarrow A_6$
Feb-22	6888.17	A_7	$A_6 \longrightarrow A_7$
Mar-22	7071.44	A_7	$A_7 \longrightarrow A_7$
Apr-22	7228.91	A_7	$A_7 \longrightarrow A_7$

Suppose the closing price of JCI in June 2016 was 4838.58 which is included in the interval u_1, then Y_4 is fuzzified into A_1.

Based on the formed FLR, it can be seen the relationship between current state and the next state. For example, $A_6 \longrightarrow A_7$ means state A_6 transitions to state A_7.

(5) After obtaining the FLR, then the FLRG can be determined which is the grouping of each JCI closing price transition:

Table 3. FLRG result of JCI Closing Price with the FTS-MC

Current State		Next State
A_1	→	$7(A_1), 3(A_2)$
A_2	→	$1(A_1), 4(A_2), 3(A_3)$
A_3	→	$1(A_1), 1(A_2), 4(A_3), 2(A_4)$
A_4	→	$1(A_3), 18(A_4), 5(A_5)$
A_5	→	$4(A_4), 7(A_5), 4(A_6)$
A_6	→	$3(A_5), 4(A_6), 1(A_7)$
A_7	→	$2(A_7)$

(6) Based on Equation (5) it can be formed a one-step probability matrix P_{ij} is obtained as follows:

$$P_{ij} = \begin{bmatrix} 0.7 & 0.3 & \cdots & 0 \\ 0.125 & 0.5 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It represents probability that stock price is in state A_j at time $n+1$ given that the stock price was in state A_i at the previous time n. Suppose $p_{12} = 0.125$, this shows that the probability of transition from state A_1 to state A_2 is 0.125.

(7) JCI closing price forecast is calculated using the rules of initial forecasted value, adjusted value, and final forecasted value based on Steps 8a), 8b), and 8c). Here are the results of the calculations obtained:

Table 4. The Results of Forecasting JCI Closing Price with the FTS-MC

Date	Initial Forecasted Value	Adjusted Value	Final Forecasted Value
Jan-16	4763.186	0	4763.186
Feb-16	4872.241	0	4872.241
Mar-16	4924.331	0	4924.331
Juni-16	4919.58	0	4919.58
Juli-22	4890.38	192.8574	5083.238
⋮	⋮	⋮	⋮
Feb-22	6544.863	192.8574	6737.72
Mar-22	6888.171	0	6888.171
Apr-22	7071.442	0	7071.442

The accuracy level of this method is calculated using MAPE (Mean Absolute Percentage Error) in Equation (8). The MAPE value of the forecasting the JCI closing price using the Fuzzy Time Series Markov Chain method is 1.672888%.

Next, JCI closing price forecasting is carried out using the Average Based Fuzzy Time Series Markov Chain method. The following is the calculation result of each step in the Average Based FTS-MC algorithm:

(1) Specifies the set of universes U i.e. $U = [4530,7230]$.

(2) Determine the length of the interval using average based as follows:
   a. Based on Equation (7) the average of the absolute value of the difference between Y_t and Y_{t+1} is 162.52, then half of the average value is 81.26.
   b. From Step (1), the length of the interval is 81.26. Based on Table 1, then the base length of such intervals is 10.
   c. The length of the interval is rounded according to its base and obtained the length of the interval: $l = 90$.

(3) The number of the intervals obtained based on Equation (2) are: $n = 30$. So that the Fuzzy Set is obtained according to Equation (4) as follows

$$u_1 = [4530, 4620]$$
$$u_2 = [4620,4710]$$
$$\vdots$$
$$u_{29} = [7050, 7140]$$
$$u_{30} = [7140, 7230]$$

(4) Specifies the fuzzy set A_i for the entire set of universes U where $i = 1,2, \dots, n$. Based on the rules of the degree of membership u_i in step (3) before, the fuzzy set of A_i are defined as:

$$A_1 = \left\{\frac{1}{u_1}, \frac{0.5}{u_2}, \frac{0}{u_3}, \dots, \frac{0}{u_{30}}\right\}, A_2 = \left\{\frac{0.5}{u_1}, \frac{1}{u_2}, \frac{0.5}{u_3}, \dots, \frac{0}{u_{30}}\right\}, \dots,$$

$$A_{30} = \left\{\frac{0}{u_1}, \frac{0}{u_2}, \frac{0}{u_3}, \dots, \frac{1}{u_{30}}\right\}$$

(5) Based on the fuzzy set that has been formed, each historical data will be fuzzified and then determined by the FLR from the closing price of the JCI.

Table 5. Fuzzification and FLR result of JCI Closing Price with the Average Based FTS-MC

Date	JCI Closing Price	Fuzzification	FLR
Jan-16	4615.16	A_1	
Feb-16	4770.96	A_3	$A_1 \longrightarrow A_3$
Mar-16	4845.37	A_4	$A_3 \longrightarrow A_4$
Jun-16	4838.58	A_4	$A_4 \longrightarrow A_4$
⋮	⋮	⋮	⋮
Jan-22	6631.15	A_{24}	$A_{23} \longrightarrow A_{24}$
Feb-22	6888.17	A_{27}	$A_{24} \longrightarrow A_{27}$
Mar-22	7071.44	A_{29}	$A_{27} \longrightarrow A_{29}$
Apr-22	7228.91	A_{30}	$A_{29} \longrightarrow A_{30}$

Suppose the closing price of the JCI in June 2016 is 4838.58 which is included in the interval u_4, then Y_4 is fuzzified into A_4.

Based on the FLR formed, it can be known the relationship between current state and the next state. Suppose, $A_{23} \longrightarrow A_{27}$ means that the state A_{23} transitions to the state A_{27}.

(6) After obtaining the FLR, it can then be determined the FLRG which is a grouping of each JCI closing price transition:

Table 6. The Results of Forecasting JCI Closing Price with the Average Based FTS-MC

Current State		Next State
A_1	→	$2(A_3)$
A_2	→	$\emptyset$
A_3	→	$1(A_3), 1(A_4), 4(A_5), 2(A_6)$
⋮		⋮
A_{28}	→	$\emptyset$
A_{29}	→	$1(A_{30})$
A_{30}	→	$\emptyset$

(8) Based on Equation (5) it can be formed a one-step probability matrix P_{ij} is obtained as follows:

$$P_{ij} = \begin{bmatrix} 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0.25 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(7) JCI closing price forecast is calculated using the rules of the initial forecasted value, adjusted value, and final forecasted value result based on Steps 8a), 8b), and 8c). Here are the calculation results:

Table 7. The Results of Forecasting JCI Closing Price with the Average Based FTS-MC

Date	Initial Forecasted Value	Adjusted Value	Final Forecasted Value
Jan-16	4755	90	4845
Feb-16	4893.99	45	4939
Mar-16	4905.12	0	4905.1
Juni-16	4902.86	-45	4857.9
Juli-22	4900.47	135	5035.5
⋮	⋮	⋮	⋮
Feb-22	6735	135	6870
Mar-22	7095	90	7185
Apr-22	7185	45	7230

The accuracy level of this method is calculated using MAPE (Mean Absolute Percentage Error) in Equation (8). The MAPE value of the forecasting JCI closing price using Average Based Fuzzy Time Series Markov Chain method is 1.590415%.

Based on the two methods discussed earlier, there are differences in determining the length of the interval. In the Average Based Fuzzy Time Series Markov Chain, the length of the interval is determined using the Average Based method. The number of intervals obtained in the Average Based FTS-MC method is more than in the FTS-MC method and greatly affects the formation of Fuzzy Logical Relationships (FLR). As a result, there is no significant transition (state shift), so the forecasting results obtained in the Average Based FTS-MC method are closer to the actual closing price of the JCI. This can be seen from the error value for forecasting the closing price of the JCI using MAPE. FTS-MC and Average Based FTS-MC, respectively, have MAPE of 1.672888% and 1.590415%. It is found that the Average Based FTS-MC method has a smaller MAPE value.

4 Conclusion

In this study, it has been explained about the Fuzzy Time Series Markov Chain and Average Based Fuzzy Time Series Markov Chain methods to predict the closing price of the JCI. By looking at the MAPE values for both methods, FTS-MC and Average Based FTS-MC, respectively, have MAPE of 1.672888% and 1.590415%. According to those MAPE values, the Average Based FTS-MC is more reliable in forecasting the JCI closing price. For further research, several things can be developed, one of which is by performing different schemes in determining the set of universes and the length of the interval to obtain better forecasting results.

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