J. Ris. & Ap. Mat. Vol. 06 No. 01 (2022) pp. 63-72
Jurnal Riset dan Aplikasi Matematika
e-ISSN: 2581-0154
URL: journal.unesa.ac.id/index.php/jram

PREDICTING OMICRON DAILY NEW CASES IN INDONESIA AND ITS MEAN RECURRENCE TIME USING MODIFIED WEIGHTED MARKOV CHAIN

RAHMAT AL KAFI[1*], ANGGIA ABYGAIL SIHOMBING[2]

[1,2]Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Indonesia, Depok 16424, Indonesia

*rahmat.alkafi@sci.ui.ac.id

ABSTRACT

This study presents the prediction of daily new cases of the Covid-19 Omicron variant and the average time of appearance using a modified Weighted Markov Chain (WMC) model. The proposed model is the modification of the WMC model and is implemented to Omicron daily new cases in Indonesia, from 12 February 2022 to 25 February 2022, to predict new case on 26 February 2022. The outcome of the modified WMC model is in the form of state represented by the interval where the predicted new case belongs to. The first session of this article is to introduce a new approach in calculating the weight of WMC model. The second and main session provides an analysis of the simulation results of a three-state modified WMC based on the variation of the maximum step in the model. The analysis result shows that the difference in the maximum steps does not affect the final result of the predicted new case. The last stage provides the predicting of the mean recurrence time of Omicron daily new cases based on 3 states in the model. In the long term, the Omicron daily new cases will be in states 1 and 2 every 2.5 days, with the probability of 0.4, and occurs in state 3 every 5 days, with the probability of 0.2.

Keywords: Accessible, Chi-Square, Ergodic, Long-Run Proportion

1 Introduction

Coronavirus Disease 2019 (COVID-19) is an infectious disease caused by the SARS-CoV-2 and has been designated as global pandemic by the World Health Organization (WHO). The policies set by the government are adjusted to the growth of COVID-19 cases in Indonesia, including the emergence of new variant of SARS-CoV-2, which was named the Omicron variant (B.1.1.529). Since the first case report on 24 November 2021 in South Africa, there have been 149 countries that have reported the Omicron variant by far. Based on WHO technical brief on 7 January 2022, it was stated that the transmission rate of the Omicron variant was faster, but based on several preliminary studies in Denmark, South Africa, Canada, the United Kingdom, and the United States, it currently shows that the risk of hospitalization is lower than the delta variant. As of 14 January 2022, Indonesia has reported 644 cases of the Omicron variant, most of which came from overseas travelers (529 cases). While the other cases (115 cases) are local transmissions that have occurred in Indonesia.

Many mathematical models to forecast the COVID-19 have been developed, some of them are listed in the references of this study as [1], [2], [3], [4], [5], and [6]. However, there are no studies that propose the Markov model to predict COVID-19 cases. This study presents

2020 *Mathematics Subject Classification*: 60G07, 60J20
Tanggal Masuk: 03-03-22; direvisi: 13-04-22; diterima: 29-04-22

Omicron daily new cases prediction using Weighted Markov Chain (WMC), and also presents a new weighting scheme in the WMC. The WMC method has been used in some studies as in [7], [8], [9], [10], and [11] since it seems more relevant to accept the stochastic approach than the deterministic approach in predicting the behavior of undetermined event, especially the cases of COVID-19. This model also doesn't require many assumptions and doesn't involve many parameters that have to be estimated. Hence it doesn't take complex calculation in this method. For further analysis of the proposed model, this study carries the calculation of long-run proportions of time, with the intention of knowing the mean recurrence time of every state considered in the model.

2 Literature Review

2.1 Markov Chain and Its Properties

Markov Chain is the stochastic process with the property that the probability of any particular future state, if the present state of the process is known, is (conditionally) independent with the past states. In other words, the future state only depends on the current state. Mathematically, the Markov chain is given by the following definition.

Definition 2.1 A Markov chain $\{\boldsymbol{X_t} | \boldsymbol{t} \in \boldsymbol{T}\}$ is a stochastic process with the property that given the current state $\boldsymbol{X_t}$, then the probability of any future state $\boldsymbol{X_s}$ for $\boldsymbol{s} > \boldsymbol{t}$ is not affected by all the past states $\boldsymbol{X_u}$ for $\boldsymbol{t} > \boldsymbol{u}$, which is mathematically given by the following equation:

$$\Pr\{X_s = j | X_t = i, \dots, X_u = i_u, \dots, X_1 = i_1, X_0 = i_0\} = \Pr\{X_s = j | X_t = i\} = p_{ij}^{(s-t)}$$

for all states $\boldsymbol{i_0}, \boldsymbol{i_1} \dots, \boldsymbol{i_u}, \dots, \boldsymbol{i}, \boldsymbol{j}$ and for all time points $\boldsymbol{t}$. $\boldsymbol{p_{ij}^{(s-t)}}$ is the probability that a process in state $\boldsymbol{i}$ will be in state $\boldsymbol{j}$ after $\boldsymbol{s - t}$ additional transitions (or $\boldsymbol{s - t}$ units of time) [12].

Definition 2.2 State $\boldsymbol{j}$ is said to be accessible from state $\boldsymbol{i}$ if $\boldsymbol{p_{ij}^{(n)}} > \boldsymbol{0}$ for some $\boldsymbol{n} \geq \boldsymbol{0}$ [12].

Definition 2.3 A Markov chain is said to be irreducible if every state is accessible from any other state. [12].

Definition 2.4 State $\boldsymbol{i}$ is said to be positive recurrent if $\boldsymbol{\mu_i} < \infty$, with $\boldsymbol{\mu_i}$ denotes the expected time that it takes the Markov chain to return to state $\boldsymbol{i}$ if the process starting in state $\boldsymbol{i}$ [12].

Definition 2.5 The period of state $\boldsymbol{i}$ is

$$d(i) = \gcd\{n \in \mathbb{N} | p_{ii}^{(n)} > 0\} \tag{1}$$

State $\boldsymbol{i}$ is aperiodic if $\boldsymbol{d(i)} = \boldsymbol{1}$ and periodic if $\boldsymbol{d(i)} > \boldsymbol{1}$ [12].

Definition 2.6 A Markov chain is called periodic if all states are periodic, otherwise the chain is called aperiodic. [12].

2.2 Long-Run Proportion

Theorem 2.7 If an irreducible Markov chain is positive recurrent and aperiodic, then the long-run proportion of time in that the chain is in state $\boldsymbol{j}$, denoted by $\boldsymbol{\pi_j}$, satisfies the following system of equations:

$$\pi_j = \sum_{i=1}^{\infty} \pi_i p_{ij}\,,\ j = 1,2,3,\ldots \tag{2}$$

$$\sum_{j=1}^{\infty} \pi_j = 1 \tag{3}$$

Furthermore, for any initial state, the average time that it takes the Markov chain to return to state $\boldsymbol{i}$ is given by the following equation [12]:

$$\mu_i = \frac{1}{\pi_i} \tag{4}$$

2.3 Weighted Markov Chain

Weighted Markov Chain (WMC) is a generalization of the Markov Chain model. The main difference between these two is the weight of each transition. The following is the procedure in using WMC [7]:

(1) Construct m states. In contrast to research in [7], this study used a quantile method based on a collection of observations. This method classifies data into a certain number of classes with the same (or almost the same) number of observations in each class. The number of observations in each class is given by the following equation [13]:

$$\text{Number of observations per state} = \frac{\text{Total observations}}{\text{Number of states}} \tag{5}$$

(2) Form the frequency matrix, $\boldsymbol{F} = (f_{ij})$, as the following:

$$\boldsymbol{F} = \begin{pmatrix} f_{11} & \cdots & f_{1m} \\ \vdots & \ddots & \vdots \\ f_{m1} & \cdots & f_{mm} \end{pmatrix} \tag{6}$$

where m denotes the number of states, f_{ij} denotes the number of the stochastic process $\{X_i\}$ transitioning one-step from state i to state j.

(3) Form the matrix of one-step transition probabilities $\boldsymbol{P} = (p_{ij})$ and the marginal matrix $\boldsymbol{Q} = (q_i)$, whose entries are given by Equation (7) and Equation (8), respectively.

$$p_{ij} = \frac{f_{ij}}{\sum_{j=1}^{m} f_{ij}} \tag{7}$$

$$q_i = \frac{\sum_{j=1}^{m} f_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}} \tag{8}$$

with p_{ij} denotes the probability that a process in state i will be in state j after one-step transition, q_i denotes the probability that a process makes a transition from state i. Furthermore, $\boldsymbol{P^n} = \boldsymbol{P} \times \boldsymbol{P} \times \cdots \times \boldsymbol{P} = \left(p_{ij}^{(n)}\right)$.

(4) Apply the Chi-Square test to check the Markov property of the given stochastic process. Null Hypothesis, H_0, is the stochastic process has no Markov property, against the alternative, H_1, is the stochastic process has a Markov property. The test statistic is

$$\chi^2 = 2\sum_{i=1}^{m}\sum_{j=1}^{m} f_{ij}\left|\ln\frac{p_{ij}}{q_i}\right| \tag{9}$$

with the critical region is rejecting H_0 if $\chi^2 > \chi^2_{\alpha;(m-1)^2}$. α denotes the significance level and $(m-1)^2$ denotes the degree of freedom.

(5) Predict the next observation using WMC that depends on the weight of Markov Chain $w_k \in [0,1]$ which is given by the following equation:

$$w_k = \frac{|r_k|}{\sum_{k=1}^{M}|r_k|} \tag{10}$$

with

$$r_k = \frac{\sum_{t=1}^{n-k}(y_t - \bar{y})(y_{t+k} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2} \tag{11}$$

is the Pearson correlation coefficient from the set of observations, with $k \in \{1,2,\dots,M\}$. M denotes the maximum step used for prediction, which means a process will be in a state in the future time depending on the states at M previous times. y_t denotes the observation at time t, $\bar{y}$ denotes the average of the observed values of y_t, and n denotes the number of observations. Thus, the WMC formula is given by the following equation:

$$\tilde{p}_{ij} = \sum_{k=1}^{M} w_k p_{ij}^{(k)}, \qquad i = 1,2,\dots,m \tag{12}$$

for every $j \in \{1,2,\dots,m\}$. $p_{ij}^{(k)}$ can be obtained from k-step transition probabilities matrix P^k where $p_{ij}^{(k)}$ denotes the probability that a process in state i will be in state j after k-step transition. $\tilde{p}_{ij}$ denotes the probability that the observation y_t will be in state j in the future time given the current observation is in state i. The predicting result of WMC is in the form of state which is denoted by j and obtained as follows:

$$\underset{j\in\{1,2,\dots,m\}}{\arg\max}\, \tilde{p}_{ij} \tag{13}$$

3 The Proposed Weighting Scheme

This study proposes a new weighting scheme in WMC as follows:

$$w_k = \frac{1}{M}, \qquad \forall k = 1,2,\dots,M \tag{14}$$

with M denotes the maximum step used for prediction. It is obvious that w_k given by Equation (14) satisfy the constraint of w_k, that is $w_k \in [0,1]$. This modification aims to reduce the complexity in calculating the weight as in previous research.

4 Results and Discussion

The data used in this study is Omicron daily cases in Indonesia from 12 February 2022 to 25 February 2022 generated by the cumulative cases in [14], as given in the Figure 1.

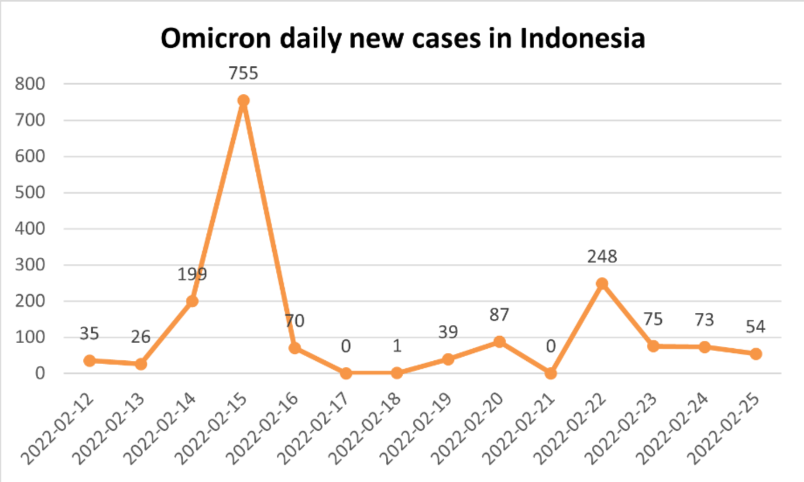

Figure 1: Omicron daily new cases in Indonesia from 12 February 2022 to 25 February 2022

The first stage is to analyze the simulation results of the WMC model in predicting Omicron daily new cases in Indonesia as shown in Figure 1. The following is the calculation result of each step in the WMC algorithm:

(1) This study considered 3 states with each represents low number of cases, mid number of cases, and high number of cases respectively. Hence by Equation (5)

$$\text{Number of observations per state} = \frac{14}{3} = 4{,}6667 \approx 5$$

(2) The result of previous step leads to the following table,

Table 1: State classification

State	Standard for Grading	Interval
1	low number of cases	$0 \le x < 50$
2	mid number of cases	$50 \le x < 100$
3	high number of cases	$100 \le x < 760$

The following Table 2 presents Omicron daily new cases in Indonesia which have been mapped into three states.

Table 2: Omicron daily new cases and state transitions with $\boldsymbol{m} = \mathbf{3}$

Date	Number of Cases	State	State Transition
2022-02-12	35	1	
2022-02-13	26	1	11
2022-02-14	199	3	13
2022-02-15	755	3	33
2022-02-16	70	2	32
2022-02-17	0	1	21
2022-02-18	1	1	11
2022-02-19	39	1	11

2022-02-20	87	2	12
2022-02-21	0	1	21
2022-02-22	248	3	13
2022-02-23	75	2	32
2022-02-24	73	2	22
2022-02-25	54	2	22

(3) According to Table 2, the frequency matrix with $m = 3$ is given by Equation (8).

$$\boldsymbol{F} = \begin{pmatrix} 0.5 & 0.166666667 & 0.333333333 \\ 0.5 & 0.5 & 0 \\ 0 & 0.666666667 & 0.333333333 \end{pmatrix} \tag{15}$$

(4) According to Equation (7) and Equation (8), and Equation (15), the one-step transition probability matrix P and the marginal matrix Q with $m = 3$, respectively, are as follows:

$$\boldsymbol{P} = \begin{pmatrix} 0.5 & 0.166666667 & 0.333333333 \\ 0.5 & 0.5 & 0 \\ 0 & 0.666666667 & 0.333333333 \end{pmatrix} \tag{16}$$

$$\boldsymbol{Q} = \begin{pmatrix} 0.46 \\ 0.307692308 \\ 0.230769231 \end{pmatrix} \tag{17}$$

(5) The Markov property for this process is tested by using Chi-Square test with the significance level of 0.05. Table 3 presents the result of the Chi-Square test.

Table 3: The Chi-Square test result

$\boldsymbol{m}$	$\boldsymbol{\chi^2}$	$\boldsymbol{\chi^2_{0.05;(m-1)^2}}$	**Result**
3	12.68208494	0.711	Reject H_0

Where the test statistic of 12.68208494 is obtained from Equation (9). Since $\chi^2 = 12.68208494 > 0.711 = \chi^2_{\alpha;(m-1)^2}$, then the null hypothesis is rejected which means the stochastic process of Omicron daily new cases in Indonesia follows Markov property.

(6) This study involved the maximum step of 1, 2, 3, 4, and 5 ($M = 1,2,3,4,5$) in predicting future Omicron daily new cases since it doesn't change the predicting result that has been verified by previous research. Hence by Equation (14), the weight of Markov Chain w_k for each maximum step is presented in Table 4.

Table 4: The weight of the Markov chain of each $\boldsymbol{k}$ for every $\boldsymbol{M = 1, 2, 3, 4, 5}$

$\boldsymbol{k}$	$\boldsymbol{w_k}$				
	$M = 1$	$M = 2$	$M = 3$	$M = 4$	$M = 5$
1	1	0.5	0.333333	0.25	0.2
2		0.5	0.333333	0.25	0.2
3			0.333333	0.25	0.2
4				0.25	0.2
5					0.2

By using Equation (12), Equation (13) and the information in Table 4, the prediction of Omicron daily new case on 26 February 2022 for each maximum step M is presented in the following tables.

Table 5: The predicted Omicron daily new case on 26 February 2022 with $\boldsymbol{M} = \mathbf{1}$

Date	i	k	w_k	$w_k p_{ij}^{(k)}$		
				$j = 1$	$j = 2$	$j = 3$
2022-02-25	2	1	1	0.5	0.5	0
$\tilde{p}_{ij}$				0.5	0.5	0
$\arg\max_{j\in\{1,2,3\}} \tilde{p}_{ij}$				1 and 2		

Table 6: The predicted Omicron daily new case on 26 February 2022 with $\boldsymbol{M} = \mathbf{2}$

Date	i	k	w_k	$w_k p_{ij}^{(k)}$		
				$j = 1$	$j = 2$	$j = 3$
2022-02-25	2	1	0.5	0.25	0.25	0
2022-02-24	2	2	0.5	0.25	0.166667	0.083333
$\tilde{p}_{ij}$				0.5	0.416667	0.083333
$\arg\max_{j\in\{1,2,3\}} \tilde{p}_{ij}$				1		

Table 7: The predicted Omicron daily new case on 26 February 2022 with $\boldsymbol{M} = \mathbf{3}$

Date	i	k	w_k	$w_k p_{ij}^{(k)}$		
				$j = 1$	$j = 2$	$j = 3$
2022-02-25	2	1	0.333333	0.16666667	0.166667	0
2022-02-24	2	2	0.333333	0.16666667	0.111111	0.055556
2022-02-23	2	3	0.333333	0.13888889	0.12037	0.074074
$\tilde{p}_{ij}$				0.47222222	0.398148	0.12963
$\arg\max_{j\in\{1,2,3\}} \tilde{p}_{ij}$				1		

Table 8: The predicted Omicron daily new case on 26 February 2022 with $\boldsymbol{M} = \mathbf{4}$

Date	i	k	w_k	$w_k p_{ij}^{(k)}$		
				$j = 1$	$j = 2$	$j = 3$
2022-02-25	2	1	0.25	0.125	0.125	0
2022-02-24	2	2	0.25	0.125	0.083333	0.041667
2022-02-23	2	3	0.25	0.10416667	0.090278	0.055556
2022-02-22	3	4	0.25	0.10648148	0.094136	0.049383
$\tilde{p}_{ij}$				0.46064815	0.392747	0.146605
$\arg\max_{j\in\{1,2,3\}} \tilde{p}_{ij}$				1		

Table 9: The predicted Omicron daily new case on 26 February 2022 with $\boldsymbol{M} = \mathbf{5}$

Date	i	k	w_k	$w_k p_{ij}^{(k)}$		
				$j = 1$	$j = 2$	$j = 3$

2022-02-25	2	1	0.2	0.1	0.1	0
2022-02-24	2	2	0.2	0.1	0.066667	0.033333
2022-02-23	2	3	0.2	0.08333333	0.072222	0.044444
2022-02-22	3	4	0.2	0.08518519	0.075309	0.039506
2022-02-21	1	5	0.2	0.08117284	0.079733	0.039095
$\tilde{p}_{ij}$				0.44969136	0.39393	0.156379
$\arg\max_{j\in\{1,2,3\}} \tilde{p}_{ij}$				1		

From Table 5 to Table 9, it can be concluded that for each M, Omicron new case on February 26 will be on state 1. The difference in the maximum step M does not change the predicting result.

The last stage provides the average time of occurrence of Omicron daily new cases based on 3 states in the model. This result is obtained by using Theorem 2.7. However, the implications of Theorem 2.7 prevail if the chain met the sufficient conditions. It will be guaranteed beforehand that this Markov chain is an irreducible, a positive recurrent, and an aperiodic chain.

Based on the one-step transition probability matrix in Equation (16), it can be obtained that $p_{11}^{(1)} = 0.5,\ p_{12}^{(1)} = 0.16667,\ p_{13}^{(1)} = 0.33333,\ p_{21}^{(1)} = 0.5,\ p_{22}^{(1)} = 0.5,$

$$p_{23}^{(2)} = \sum_{k=1}^{3} p_{2k}^{(1)} p_{k3}^{(1)} = p_{21}^{(1)} p_{13}^{(1)} + p_{22}^{(1)} p_{23}^{(1)} + p_{23}^{(1)} p_{33}^{(1)} = (0.5)(0.33333) = 0.166665$$

$$p_{31}^{(2)} = \sum_{k=1}^{3} p_{3k}^{(1)} p_{k1}^{(1)} = p_{31}^{(1)} p_{11}^{(1)} + p_{32}^{(1)} p_{21}^{(1)} + p_{33}^{(1)} p_{31}^{(1)} = (0.66667)(0.5) = 0.333335$$

$p_{32}^{(1)} = 0.66667$ and $p_{33}^{(1)} = 0.33333$. It means all states are accessible to each other. So based on Definition 2.3, this Markov chain is irreducible. Since this finite Markov chain is irreducible, then $\mu_1, \mu_2, \mu_3 < \infty$ which means this chain is a positive recurrent.

Afterwards, by using Definition 2.5, Equation (16), as well as the n-step transition probability matrix $P^n = (p_{ij}^{(n)})$, we get

$$d(1) = \gcd\left\{n \in \mathbb{N} \middle| p_{11}^{(n)} > 0\right\} = \gcd\{1,2,3,\dots\} = 1$$

$$d(2) = \gcd\left\{n \in \mathbb{N} \middle| p_{22}^{(n)} > 0\right\} = \gcd\{1,2,3,\dots\} = 1$$

$$d(3) = \gcd\left\{n \in \mathbb{N} \middle| p_{33}^{(n)} > 0\right\} = \gcd\{1,2,3,\dots\} = 1$$

This means that all states in this Markov chain are aperiodic, hence the chain is aperiodic by Definition 2.6. Therefore, this Markov chain satisfies all the sufficient conditions of Theorem 2.7.

Based on Theorem 2.7, the solutions of Equation (2), Equation (3), and Equation (4) are the long-run proportions and the recurrence period for each state. In this Markov chain, the three equations can be written as the following system of equations

$$\begin{cases} \pi_1 = \frac{1}{2}\pi_1 + \frac{1}{2}\pi_2 \\ \pi_2 = \frac{1}{6}\pi_1 + \frac{1}{2}\pi_2 + \frac{2}{3}\pi_3 \\ \pi_3 = \frac{1}{3}\pi_1 + \frac{1}{3}\pi_3 \\ \mu_i = \frac{1}{\pi_i}, \qquad i = 1,2,3 \end{cases}$$

The solution of the above system of equations is presented in Table 10.

Table 10: Long-run proportion and recurrence period for each state

State (i)	1	2	3
π_i	0.4	0.4	0.2
μ_i	2.5	2.5	5

From Table 10, the return period of state 1, 2, and 3 will be $\mu_1 = 2.5$ days, $\mu_2 = 2.5$ days, and $\mu_3 = 5$ days respectively. Hence, for the long term, the number of Omicron daily new cases, with its value is in state 1 and 2, is most likely to appear about 2.5 days per time on average, with the probability of 0.4. It will be in state 3 every 5 days, with the probability of 0.2. By this finding, the government is expected to be able to provide the suitable way in handling the Omicron cases, and the people of Indonesia still have to maintain the health protocol in order to help government in reducing the daily new cases, even though it is predicted to be in the low number for the long run.

5 Conclusion

In this study, a modified WMC model is proposed to predict Omicron daily new case in Indonesia on 26 February 2022 which belongs to state 1. The simulation results of the modified WMC model shows that the different values of the maximum step M do not affect the predicting results. In the long term, the state of the amount of Omicron daily new cases is most likely to occur about every 2.5 days which belongs to state 1 and 2. For the next studies, several things can be modified, for instance, by giving another weighting scheme to the WMC and also changing the data classification method.

References

[1] S. Singh, B. M. Sundram, K. Rajendran, K. B. Law, T. Aris, H. Ibrahim, S. C. Dass, and B. S. Gill, ‘Forecasting daily confirmed COVID-19 cases in Malaysia using ARIMA models’, *J. Infect. Dev. Ctries.*, vol. 14, pp. 971–976, 2020.

[2] D. H. Lee, Y. S. Kim, Y. Y. Koh, K. Y. Song, and I. H. Chang, ‘Forecasting COVID-19 confirmed cases using empirical data analysis in Korea’, *Healthcare*, vol. 9, 2021.

[3] R. Yagoub and H. Eledum, ‘Modeling of the COVID-19 cases in Gulf cooperation council countries using ARIMA and MA-ARIMA Models’, *J. Probab. Stat.*, vol. 2021, 2021.

[4] M. Rendana and W. M. R. Idris, ‘New COVID-19 variant (B.1.1.7): Forecasting the occasion of virus and the related meteorological factors’, *J. Infect. Public Health*, vol. 14, pp. 1320–1327, 2021.

[5] S. Singh, C. Chowdhury, A. K. Panja, and S. Neogy, ‘Time Series Analysis of COVID-

19 Data to Study the Effect of Lockdown and Unlock in India', *J. Inst. Eng. India Ser. B*, vol. 102, pp. 1275–1281, 2021.
[6] R. Schlickeiser and M. Kröger, 'Forecast of Omicron Wave Time Evolution, *COVID 2022*, vol. 2, pp. 216–229, 2022.
[7] R. A. Kafi, Y. R. Safitri, Y. Widyaningsih, and B. D. Handari, 'Comparison of weighted Markov chain and fuzzy time series Markov chain in forecasting stock closing price of company X', *AIP Conf. Proc.*, vol. 2168, 2019.
[8] X. Yang, X. Ma, N. Kang, and M. Maihemuti, 'Probability interval prediction of wind power based on KDE method with rough sets and weighted Markov chain', *IEEE Access*, vol. 6, pp. 51556–51565, 2018.
[9] Y. Gui and H. Choi, 'Prediction of precipitation based on sliding grey theory and weighted Markov chain', *2018 4th International Conference on Computational Intelligence & Communication Technology (CICT)*, pp. 1–5, 2018.
[10] M. Yan, S. Li, C. A. Chan, Y. Shen, and Y. Yu, 'Mobility prediction using a weighted Markov model based on mobile user classification', *Sensors*, vol. 21, 2021.
[11] I. P. S. Siregar, R. Widyasari, and N. H. Prasetya, 'A weighted Markov chain application to predict consumer price index when facing pandemic COVID-19', *Journal of Innovation and Technology in Mathematics and Mathematics Education*, vol. 1, pp. 41–46, 2021.
[12] S. M. Ross, *Introduction to Probability Models 12th Edition*, New York: Elsevier, 2019.
[13] C. Hennig and C. Viroli, 'Quantile-based classifiers', *Biometrika*, vol. 103, pp. 435–446, 2016.
[14] Databoks, *Update Omicron: Total di Indonesia*. 2022. https://databoks.katadata.co.id/datapublish/2022/02/25/update-omicron--total-di-indonesia-ada-6453-kasus-jumat-25-februari-2022. Accessed: February 25, 2022