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RISK MEASUREMENT USING EXTENDED GINI SHORTFALL WHILE CONSIDERING RISK-AVERSION PARAMETER

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ABSTRACT

Risk is an uncertainty that may occur in the future and cause a loss. To minimize the loss, a risk measure is needed to predict future losses. Corrado Gini (1912) invented a risk measure known as Gini Shortfall (GS). GS is coherent and provide information about variability of the distribution tail. However, GS generalizes that everyone has the same tendency to take risks, when in reality they do not. Therefore, Yitzhaki (1983) developed GS into Extended Gini Shortfall (EGS). EGS is a generalization of GS by taking risk-aversion into consideration. Risk-aversion is a tendency to take minimum risk. EGS is a coherent risk measure under certain conditions and can calculate average severity and variability of losses in the distribution tail with Tail Extended Gini functional, a variability measure based on the Extended Gini functional. Furthermore, the explicit formula of EGS for exponential, Pareto, and logistic distributions and the example of EGS calculation are presented. This calculation uses monthly loss of PT Unilever Indonesia Tbk stock from November 2010 to November 2020. Assuming a constant risk-aversion parameter, EGS tends to increase with the increasing prudence level. Meanwhile, with a constant prudence level, EGS tends to decrease with the increasing risk-aversion parameter.

Key Words: Extended Gini functional; loss; Tail Extended Gini functional; variability.

1 Introduction

According to Open Compliance and Ethics Group (OCEG), risk is a measure that represents the negative effect of uncertainty when achieving an objective. The risk could be tend into loss and can lead to bankruptcy. Companies, mainly the ones in the financial industry, need a good and reliable risk measure to quantify the risks they face so they can mitigate and manage the risk.

Some common used risk measures are Value-at-Risk (VaR) and Expected Shortfall (ES). VaR is defined as the quantile of a particular loss distribution and can be interpreted as the lower bound for a percentage of the biggest loss, while ES is defined as the expectation of a loss given that the loss surpasses the VaR. ES is said to be a better risk measure than VaR because it provides information on the severity of loss on the distribution tail. However, ES still has deficiency that is it failed to explain about the variability of losses on the distribution tail.

Some well-known variability measures are variance and standard deviation. Unfortunately, variance does not represent the actual distance from an observation to the data mean because it uses a quadratic function in calculation and the standard deviation still does not fully nullify this distortion effect. Therefore, Corrado Gini (1912) introduced a new

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variability measure called the Gini functional which uses the absolute value concept instead of the quadratic function used in variance. However, the Gini functional still also have deficiency that is generalizes that everyone has the same tendency to take risks, when in reality they do not.

The tendency when face uncertainty like risk is called risk-attitude, which is a thought selected by a person in relation to uncertainties that may cause the effects of positive or negative to the objective. Risk-attitude itself is divided into three types, namely, risk-averse, risk-neutral, and risk-seeking. Risk-averse is the attitude of investors who tend to feel uncomfortable with the uncertainty and prefer less profitable but less risky options. Meanwhile, risk-neutral is the attitude of investors who are quite accepting the risks, but will not want to take more risks to try to get higher return rate. Last type is risk-seeking attitude which is the attitude of investors who dare to take high risks in the hope of also getting a high return rate.

This risk-attitude is very necessary to be considered because a company's risk-attitude will affect the general approach about how the company identifies, analyzes, evaluates, and address the risks faced. Extended Gini Shortfall (EGS) is a coherent risk measure instrument based on variability measurement with Extended Gini functional that taking risk-averse into consideration.

2 Materials and Method

2.1 Variability Measures

Variability of the data is the size of data deviation in a distribution. The value of data variability will affect the calculation of risk reserves for a company. Therefore, data variability measuring instrument is required. The variability measurement will represent how data deviates or differs from other data. A variability measure is a function v that, for random variables X and Y, may fulfill these properties [1]:

a. Standardization: $v(c) = 0$ for all $c \in R$.
   The variability of a constant is always zero.
b. Location Invariance: $v(X - c)$=$v(X)$ for all $c \in R$.
   Adding or subtracting a constant to all possible values of X will not affect its variability.
c. Positive Homogeneity: $v(\lambda X) = \lambda v(X)$ for all $\lambda > 0$.
   Multiplying all possible values of X by λ also multiplies its variability by λ.
d. Sub-additivity: $v(X + Y) \leq v(X) + v(Y)$.
   The variability of the sum of two random variables is not greater than the sum of variabilities of each individual random variable.

2.1.1 Variance and Standard Deviation

Variability measures that usually used are variance and standard deviation, which can be defined as [2]:

$$\begin{aligned} \mathrm{Var}(X) &= E[(X - E[X])^2 \\ &= E[X^2] - (E[X])^2 \end{aligned} \tag{1}$$

and

$$\sigma_X = \sqrt{\mathrm{Var}(X)}. \tag{2}$$

However, variance is still use quadratic function that make data which are far from the mean contribute more to the variance disproportionally and may cause some problems. Also, variance is not a coherent variability measure because it does not fulfill positive-homogeneity and sub-additivity [1]. Standard deviation is a coherent variability measure [1] and already use root to minimize the effect of quadratic form in variance, but it does not rectify the problem either.

Gini Functional and Tail-Gini Functional

In 1912, Corrado Gini said that variability of any random variable should not be based on the center of the corresponding distribution. Thus, he noted an alternative variability measure called Gini functional, defined as [1]:

$$\mathrm{Gini}(X) = 4\mathrm{Cov}[X, U_X] \quad (3)$$

or

$$\mathrm{Gini}(X) = E[|X^* - X^{**}|]. \quad (4)$$

where X^* and X^{**} are two independent and identically distributed (i.i.d.) random variables with the same pdf as X, $f_X(x)$. Gini functional is a coherent risk-measure because it fulfills the fourth criteria [1].

Gini functional is a general variability measure for a random variable. However, it is important to have a variability measure that focused on the distribution tail. The reason is because it is vital in risk management to minimize probability of big losses that may happen in the future. Therefore, there exist Tail-Gini functional that is a tail variability measure based on Gini functional. With prudence level $p \in [0,1)$, Tail-Gini functional is defined as [1]:

$$\mathrm{TGini}_p(X) = E\left[|X^* - X^{**}| \mid X^* > \pi_p,\, X^{**} > \pi_p\right]. \quad (5)$$

Tail-Gini functional is not a coherent variability measure because it fails to satisfy sub-additivity [1].

2.2 Risk Measures

Besides variability measure, there is risk measurement as a function $\rho(X)$ that maps the loss random variable X to the set of real numbers [3]. $\rho(X)$ is used to quantify risk and can be interpret as amount or reserve that company needs to protect their company from risks that may cause harm in the future. Consider the set of random variables such that X and Y are two numbers of the set, then cX with c are the constant that bigger than zero and $X + Y$ are also in the set. A risk measure $\rho(X)$ is said to be coherent if it fulfills these criteria for any random variables X and Y [4]:

a. Sub-additivity: $\rho(X + Y) \le \rho(X) + \rho(Y)$.
   The risk measure for the sum of two losses is not greater than the sum of risks of each individual loss. For example, smaller companies can do a merger without the worry of facing a greater risk. Other applications is for stock investing, portfolio diversification (investing in more than one particular stock) will decrease the risk an investor faces.
b. Monotonicity: If $X \le Y$ for all possible outcomes, then $\rho(X) \le \rho(Y)$.
   If a loss random variable always has greater losses than the other, then it also always faces a greater risk. In applications, if two investors invest in the same stock, then the investor who invests more will face a greater risk than the other.
c. Positive homogeneity: For any positive constant c, $\rho(cX) = c\rho(X)$.
   If losses are multiplied by a constant c, than the risk a company faces will also be multiplied by c. In applications, if an investor doubles his ownership in a particular stock, then the risk he faces is also doubled.
d. Translation invariance: For any positive constant c, $\rho(X + c) = \rho(X) + c$.
   There are no additional risk when add a risk-free investment. In applications, additional investments that are protected by hedging techniques are risk-free and will not cause additional risk.

2.2.1 Value-at-Risk (VaR) and Expected Shortfall (ES)

Common used risk measure are Value-at-Risk (VaR) and Expected Shortfall (ES), which can be defined as [1]:

$$\mathrm{VaR}_p(X) = \inf\{x \in \mathbb{R} : F_X(x) \geq p\} \tag{6}$$

and

$$\begin{aligned} \mathrm{ES}_p(X) &= E\left(X | X > \pi_p\right) \\ &= \frac{1}{1-p}\int_{\pi_p}^{\infty} x f(x) dx. \end{aligned} \tag{7}$$

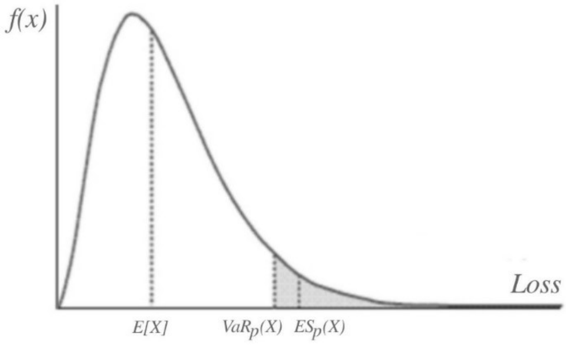


FIGURE 1. Illustration of VaR and ES

These two risks measure can be illustrated as in Figure 1. The grey area in Figure 1 is also called the distribution tail and defined as an area that is bigger than VaR. The average of the grey area is called ES. From this illustration, it can be seen that VaR just can perform the minimum value of the distribution tail, while ES can explain the average value of the distribution tail. Also, ES is a coherent risk measure, not like VaR that fail to fulfill sub-additivity [5]. Therefore, it can be concluded that ES give a better overview of loss in the distribution tail.

2.2.2 Gini Shortfall (GS)

Unfortunately, ES failed to explain the variability in the tail distribution. The deficiency of ES is covered by Gini Shortfall (GS). GS is a risk measure that used a linear combination of ES as a tail central tendency measure and TGini as a tail variability measure. GS for a loss random variable X with prudence level $p \in (0,1)$ is defined as [1]:

$$\mathrm{GS}_p^{\lambda}(X) = \mathrm{ES}_p(X) + \lambda \, \mathrm{TGini}_p(X), \tag{8}$$

where $\lambda \geq 0$ is the loading parameter that explain the proportion of TGini in GS. GS is coherent risk measure and can provide information about variability of the distribution tail. However, GS generalizes that everyone has the same tendency to take risks, when in reality they do not.

2.3 Risk Attitude

The tendency or chosen response when face a risk is called risk attitude [6]. There are three types of risk attitude that can be illustrated as in Figure 1. First is risk-averse, the type of people that will choose the one with the minimum risk although the benefit will also become less. The second type is risk-neutral, the type of people that will choose something based on the expected benefit they will get. Last is risk-loving, the type of people who will have tendency to choose the one with the most risk with the hope of getting more benefit. Risk attitude can be represented with parameter r. The greater the parameter r, the more risk-averse someone will be [7].

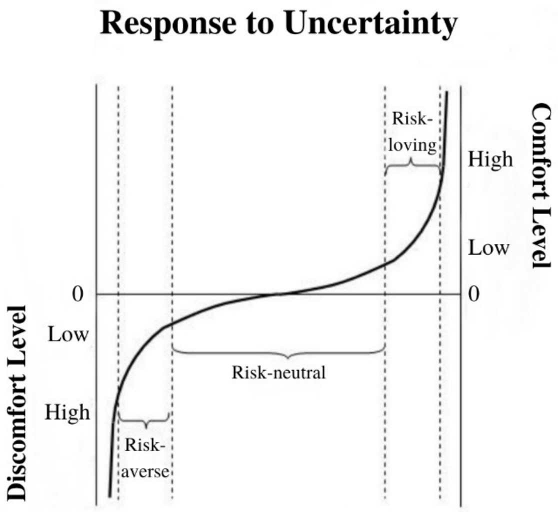


FIGURE 2. Risk Attitude Spectrum

2.4 Extended Gini functional

In 1983, Yitzhaki proposed a new variability measure that taking risk-aversion into consideration, known as Extended Gini functional. For a random variable X, the Extended Gini functional is defined as [7]:

$$\mathrm{EGini}_r(X) = -2r\mathrm{Cov}\left[X, \left(1 - F_X(X)\right)^{r-1}\right], r > 1, \tag{9}$$

where r is the risk-aversion parameter. Definition (9) can also be represented in single integral form as:

$$\mathrm{EGini}_r(X) = 2\int_0^1 F_X^{-1}(u)(1 + g_r(u))du. \tag{10}$$

Extended Gini functional is a coherent variability measure since it satisfies standardization, location invariance, positive homogeneity, and sub-additivity [7].

2.5 Tail Extended Gini functional

Extended Gini functional is a general measure of variability for a random variable while considering risk-aversion parameter. As previously mentioned, it is important to have variability measure that have focused on the tail distribution. Therefore, there is a tail variability measure based on the Extended Gini functional, called Tail Extended Gini functional. For a random variable X at the $100p\%$ level, Tail Extended Gini functional can be defined as [7]:

$$\mathrm{TEGini}_{r,p}(X) = \frac{-2r}{1-p}\mathrm{Cov}\left[X, \left(1 - F_X(X)\right)^{r-1} | X > x_p\right], r > 1, p \in [0,1]. \tag{11}$$

Definition (11) can also be represented in single integral form as:

$$\mathrm{TEGini}_{r,p}(X) = \frac{2}{(1-p)^2}\int_p^1 F_X^{-1}(u)(-r(1-u)^{r-1} + (1-p)^{r-1})du. \tag{12}$$

When $p = 0$, then $\mathrm{TEGini}_{r,0}(X)$ is the same measure as $\mathrm{TGini}_r(X)$. Also, when $r = 2$, then $\mathrm{TEGini}_{2,p}(X)$ is the same measure as $\mathrm{TGini}_p(X)$.

Tail Extended Gini functional satisfies standardization, location invariance, and positive homogeneity, but fails to satisfy sub-additivity. In other words, there exists at least a pair of random variables X and Y or a risk-aversion parameter r such that $\mathrm{TEGini}_{r,p}(X+Y) > \mathrm{TEGini}_{r,p}(X) + \mathrm{TEGini}_{r,p}(Y)$. As a result, Tail Extended Gni functional is not a coherent variability measure.

2.6 Extended Gini Shortfall (EGS)

As previously stated, Gini Shortfall (GS) is said to be more comprehensive risk measure than VaR and ES because it provides information on the average severity and the variability in the distribution tail. However, GS generalizes that everyone's risk-attitude are same, when in reality they do not. Therefore, there is a generalized form of GS that taking risk-aversion into consideration, known as Extended Gini Shortfall (EGS).

EGS is a risk measure that combines the concepts of ES as a measure of average severity in the distribution tail and TEGini as a tail variability measure that consider riak-aversion parameter into a linear combination. The EGS for a loss random variable X with prudence level $p \in (0,1)$ is defined as [7]:

$$\mathrm{EGS}_{r,p}^{\lambda}(X) = \mathrm{ES}_p(X) + \lambda\, \mathrm{TEGini}_{r,p}(X), \tag{13}$$

Where $\lambda \geq 0$ is the loading parameter that explain the proportion of TEGini in EGS. Definition (13) can also be represented in single integral form as:

$$\mathrm{EGS}_{r,p}^{\lambda}(X) = \int_0^1 F_X^{-1}(u)\phi_{r,p}^{\lambda}(u)du \tag{14}$$

with

$$\phi_{r,p}^{\lambda}(u) = \frac{1}{(1-p)^2}\big(1-p + 2\lambda(-r(1-u)^{r-1} + (1-p)^{r-1})\big)I_{[p,\ 1]}(u) \tag{15}$$

where $u \in [0, 1]$ and $\mathbb{I}_{[p,1]}(u)$ is the indicator function of the interval $[p, 1]$:

$$\mathbb{I}_{[p,1]}(u) = \begin{cases} 1\,, & u \geq p \\ 0\,, & u < p. \end{cases} \tag{16}$$

As previously stated, the Expected Shortfall (ES) is a coherent risk measure, but the Tail Extended Gini functional (TEGini) is not a coherent variability measure. Therefore, a large value of λ can affect the coherence of EGS because the TEGini will dominate the EGS, and thus the monotonicity and subadditivity of EGS will not hold. As a risk measure, EGS will always satisfy positive homogeneity and translation invariance for any λ. Meanwhile for monotonicity and sub-additivity, both criteria are only fulfilled when $\lambda \in \left[0,\ \frac{1}{2(r-1)(1-p)^{r-2}}\right]$.

2.7 EGS Explicit Formula

After discussed about the definition and the properties that EGS satisfied, the explicit formula of some continuous distribution will be derived. There are three types of distribution that commonly used for modelling loss on a portfolio that are exponential distribution, Pareto distribution, and logistic distribution.

2.7.1 Exponential Distribution

The first distribution is exponential distribution which can be used for loss that have positive value such as insurance claims. The explicit formula of this distribution will be derived with two methods from formula (13) and (14). The result will be compared in the end. Let X has the exponential distribution with the cdf [3]:

$$F_X(x) = 1 - e^{-\frac{x}{\theta}}. \tag{17}$$

From formula (17), the inverse of cdf could be derived, which is:

$$F_X^{-1}(x) = -\theta \ln(1-x). \tag{18}$$

The ES with prudence level p for X is:

$$\mathrm{ES}_p(X) = \theta[1 - \ln(1-p)]. \tag{19}$$

For the first method, the TEGini formula is needed. Using formula (12), TEGini for X can be derived as:

$$\mathrm{TEGini}_{r,p}(X) = \frac{2}{(1-p)^2}\int_p^1 -\theta\ln(1-u)\left(-r(1-u)^{r-1} + (1-p)^{r-1}\right)du.$$

Skipping the integration steps, the result will be:

$$\mathrm{TEGini}_{r,p}(X) = 2\theta(1-p)^{r-2}\left(1-\frac{1}{r}\right). \tag{20}$$

Furthermore, the linear combination of EGS can then be constructed:

$$\begin{aligned}\mathrm{EGS}_{r,p}^{\lambda}(X) &= \mathrm{ES}_p(X) + \lambda\,\mathrm{TEGini}_{r,p}(X)\\ &= \theta\left[[1-\ln(1-p)] + 2\lambda(1-p)^{r-2}\left(1-\frac{1}{r}\right)\right].\end{aligned} \tag{21}$$

For the second method, EGS for X will be derived using the single integral form:

$$\begin{aligned}\mathrm{EGS}_{r,p}^{\lambda}(X) &= \int_0^1 F_X^{-1}(u)\phi_{r,p}^{\lambda}(u)du\\ &= \int_p^1 F_X^{-1}(u)\frac{1}{(1-p)^2}\left(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\right)du\\ &= \int_p^1 -\theta\ln(1-u)\frac{1}{(1-p)^2}\left(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\right)du\end{aligned}$$

Skipping the integration steps, the result will be:

$$\mathrm{EGS}_{r,p}^{\lambda}(X) = \theta\left[[1-\ln(1-p)] + 2\lambda(1-p)^{r-2}\left(1-\frac{1}{r}\right)\right]. \tag{22}$$

It can be seen from (21) and (22) that both methods produced the same explicit EGS formula for a loss random variable with exponential distribution.

2.7.2 Pareto Distribution

Pareto distribution can be used for loss that have positive value and have many extreme values. The explicit formula of this distribution will also be derived with two methods from formula (13) and (14). The result will be compared in the end. Let X has the Pareto distribution with the parameters α and θ, then its cdf is [3]:

$$F_X(x) = F_X(x) = 1-\left(\frac{\theta}{x}\right)^{\alpha}, x > \theta. \tag{23}$$

From formula (23), the inverse of cdf could be derived, which is:

$$F_X^{-1}(x) = \frac{\theta}{(1-x)^{1/\alpha}}. \tag{24}$$

The ES with prudence level p for X is:

$$\mathrm{ES}_p(X) = \frac{\alpha\theta(1-p)^{-1/\alpha}}{\alpha-1}, \alpha > 1. \tag{25}$$

For the first method, the TEGini formula is needed. Using formula (12), TEGini for X can be derived as:

$$\mathrm{TEGini}_{r,p}(X) = \frac{2}{(1-p)^2}\int_p^1 \frac{\theta}{(1-u)^{1/\alpha}}\left(-r(1-u)^{r-1} + (1-p)^{r-1}\right)du.$$

Skipping the integration steps, the result will be:

$$\mathrm{TEGini}_{r,p}(X) = \frac{2\alpha\theta}{\alpha-1}(1-p)^{r-\frac{1}{\alpha}-2}\left(\frac{r-1}{\alpha r-1}\right). \tag{26}$$

Furthermore, the linear combination of EGS can then be constructed:

$$\begin{aligned}\mathrm{EGS}^{\lambda}_{r,p}(X) &= \mathrm{ES}_p(X) + \lambda\, \mathrm{TEGini}_{r,p}(X) \\ &= \frac{\alpha\theta(1-p)^{-1/\alpha}}{\alpha-1}\left[1 + 2\lambda(1-p)^{r-2}\left(\frac{r-1}{\alpha r-1}\right)\right]. \end{aligned} \tag{27}$$

For the second method, EGS for X will be derived using the single integral form:

$$\begin{aligned}\mathrm{EGS}^{\lambda}_{r,p}(X) &= \int_0^1 F_X^{-1}(u)\phi^{\lambda}_{r,p}(u)du \\ &= \int_p^1 F_X^{-1}(u)\frac{1}{(1-p)^2}\left(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\right)du \\ &= \int_p^1 \frac{\theta}{(1-u)^{1/\alpha}}\frac{1}{(1-p)^2}\left(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\right)du.\end{aligned}$$

Skipping the integration steps, the result will be:

$$\mathrm{EGS}^{\lambda}_{r,p}(X) = \frac{\alpha\theta(1-p)^{-1/\alpha}}{\alpha-1}\left[1 + 2\lambda(1-p)^{r-2}\left(\frac{r-1}{\alpha r-1}\right)\right]. \tag{28}$$

It can be seen from (27) and (28) that both methods also produced the same explicit EGS formula for a loss random variable with Pareto distribution.

2.7.3 Logistic Distribution

The last distribution is logistic distribution which can be used for loss that have negative value such as stock price because the logistic distribution defined on all real numbers. The explicit formula of this distribution will also be derived with two methods from formula (13) and (14). The result will be compared in the end. Let X has the logistic distribution with the parameters μ and s, then its cdf is [8]:

$$F_X(x) = \frac{1}{1+e^{-(x-\mu)/s}}. \tag{29}$$

From formula (28), the inverse of cdf could be derived, which is:

$$F_X^{-1}(x) = \mu + s\ln\left(\frac{x}{1-x}\right). \tag{30}$$

The ES with prudence level p for X is:

$$\mathrm{ES}_p(X) = \mu - s\frac{p\ln p + (1-p)\ln(1-p)}{1-p}. \tag{31}$$

For the first method, the TEGini formula is needed. Using formula (12), TEGini for X can be derived as:

$$\mathrm{TEGini}_{r,p}(X) = \frac{2}{(1-p)^2}\int_p^1 \left(\mu + s\ln\left(\frac{u}{1-u}\right)\right)(-r(1-u)^{r-1}+(1-p)^{r-1})du.$$

Skipping the integration steps, the result will be:

$$\begin{aligned}\mathrm{TEGini}_{r,p}(X) = \frac{-2s}{(1-p)^2}\Bigg[&(1-p)^{r-1}\left[p\ln p + \frac{1-p}{r}\right] \\ &+ r\lim_{a\to1}\left[\int_p^a (1-u)^{r-1}\ln u\, du\right]\Bigg].\end{aligned} \tag{32}$$

Furthermore, the linear combination of EGS can then be constructed:

$$\begin{aligned}\mathrm{EGS}^{\lambda}_{r,p}(X) &= \mathrm{ES}_p(X) + \lambda\, \mathrm{TEGini}_{r,p}(X) \\ &= \mu \\ &\quad - \frac{s}{(1-p)}\Bigg[p\ln p\,[1+2\lambda(1-p)^{r-2}] + (1-p)\,ln(1-p) \\ &\quad + \frac{2\lambda(1-p)^{r-1}}{r} + \frac{2\lambda r}{1-p}\lim_{a\to1}\left[\int_p^a (1-u)^{r-1}\,ln\, u\, du\right]\Bigg].\end{aligned} \tag{33}$$

For the second method, EGS for X will be derived using the single integral form:

$$\begin{aligned}
\mathrm{EGS}_{r,p}^{\lambda}(X) &= \int_0^1 F_X^{-1}(u)\phi_{r,p}^{\lambda}(u)du \\
&= \int_p^1 F_X^{-1}(u)\frac{1}{(1-p)^2}\big(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\big)du \\
&= \int_p^1 \left(\mu + s\ln\left(\frac{u}{1-u}\right)\right)\frac{1}{(1-p)^2}\big(1-p+2\lambda(-r(1-u)^{r-1}+(1-p)^{r-1})\big)du
\end{aligned}$$

Skipping the integration steps, the result will be:

$$\begin{aligned}
\mathrm{EGS}_{r,p}^{\lambda}(X) = \mu \\
&-\frac{s}{(1-p)}\Bigg[p\ln p\,[1+2\lambda(1-p)^{r-2}] + (1-p)\,ln(1-p) \\
&+\frac{2\lambda(1-p)^{r-1}}{r} + \frac{2\lambda r}{1-p}\lim_{a\to 1}\left[\int_p^a (1-u)^{r-1}\,ln\,u\,du\right]\Bigg]. \qquad (34)
\end{aligned}$$

It can be seen from (33) and (34) that both methods also produced the same explicit EGS formula for a loss random variable with logistic distribution.

3 Result and Discussion

1.1 Data Description

The data used for this simulation is the monthly closing price of PT Unilever Indonesia Tbk stock (code UNVR in Indonesia Stock Exchange) from November 2010 to November 2020, or consists of 121 observations. These data is obtained from the website Investing.com. The monthly loss is then obtained by subtracting the previous month closing price from the observed month closing price. Therefore, positive result of loss indicates a decrement of monthly price and negative result of loss indicates an increment of monthly price.

TABLE 1. Descriptive Statistics for UNVR Monthly Loss

Descriptive Statistics	Value
Minimum	-1,565.00
Maximum	1,125.00
Mean	-35.00
Median	-30.00
Skewness	-0.33
Variance	194,868.30
Standard Deviation	441.44

Based on Table 1, it can be seen that the mean and median are not far apart and the skewness is close to 0. Therefore, it indicates that this monthly loss data may have symmetric distribution. Also note that the maximum and minimum values of the data is quite far from the mean and median which might indicate that the distribution may be a heavy-tailed one. These properties are fulfilled by the logistic distribution, so a test needs to be done in order to check if the data actually follows logistic distribution.

First, assume the data follows logistic distribution. The maximum likelihood method can determine the best fitted parameter for this data if it follows logistic distribution. These parameters are then used to do a Kolmogorov goodness-of-fit test to check if the above assumption is true and the data follows logistic distribution. With R version 3.5.2, it is concluded that the data follows logistic distribution with parameters $\hat{\mu} = -28.94046$ and $\hat{s} = 234.1633$.

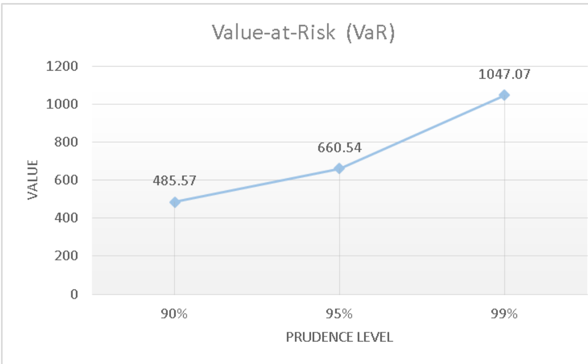
1.2 Risk Measurement

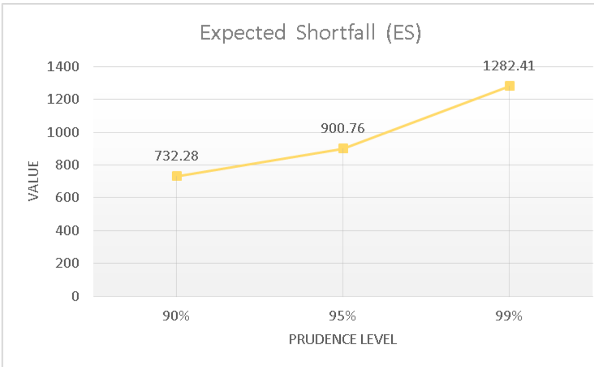
After determining the distribution that fits the data, the explicit formulas for logistic distribution obtained in the previous section can be used to calculate various variability and risk measures. Measurement will be done using a loading parameter, three different prudence levels, and five different risk-aversion parameters. The result of this calculation is summarized in Table 2.

TABLE 2. Risk Measurement Calculation Summary

Measure		Prudence Level		
		90%	**95%**	**99%**
VaR		485.57	660.54	1,047.07
ES		732.28	900.76	1,282.41
TEGini	r=2	242.38	238.17	234.95
	r=3	32.46	15.91	3.13
	r=4	3.661	0.896	0.035
	r=5	0.39123	0.04783	0.00016
	r=6	0.04081	0.00249	0
EGS	r=2	792.88	960.30	1,341.15
	r=3	772.86	940.54	1,321.59
	r=4	762.80	930.63	1,311.62
	r=5	756.74	924.68	1,292.31
	r=6	752.69	920.69	1,282.41

As seen on Figure 3, it is obvious that a higher prudence level used will result in higher VaR and ES values. This is because VaR represents the lower bound for the biggest $100(1-p)\%$ of losses while ES represents the average of those losses exceeding the VaR. The lower bound for 1% of the biggest losses is of course greater than the lower bound for 10% of the biggest losses, and the same thing applies for the average of those big losses.

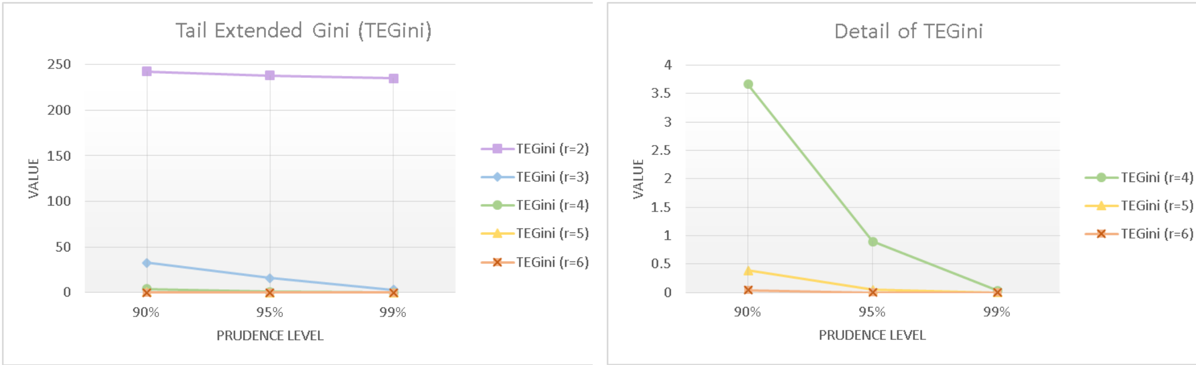




(a) (b)

FIGURE 3. (a) VaR on Various Prudence Level and (b) ES on Various Prudence Level

A different trend can be seen on the TEGini at Figure 4, where a higher prudence level results in a smaller value of TEGini. This is because the TEGini represents the variability in the biggest $100(1-p)\%$ of losses. The variability for 1% of the biggest losses is obviously greater than the variability of 10% of the biggest losses.



(a) (b)

FIGURE 4. (a) TEGini on Various Prudence Level and (b) The detail of TEGini

As an example, the numbers for the 90% prudence level and risk aversion parameter equals to 2 will be used. The value of $\mathrm{VaR}_{0.90}(X)$ is Rp485.57, which means for every 10 observed months, there may be 1 month in which the price of UNVR stocks decline by more than Rp485.57. In the other 9 months, it is predicted that price will not go down further than this number. $\mathrm{ES}_{0.90}(X)$ is the average of losses in months where losses exceed the VaR of Rp485.57. In this case, the average loss in those months is Rp732.28. Meanwhile, the value of $\mathrm{TEGini}_{2;0.90}(X)$ represents the average deviation of each loss in the distribution tail from the tail mean or the ES. On average, losses in the 90% distribution tail with risk aversion parameter equals to 2 deviates as much as Rp242.38 from the tail mean which is Rp732.28.

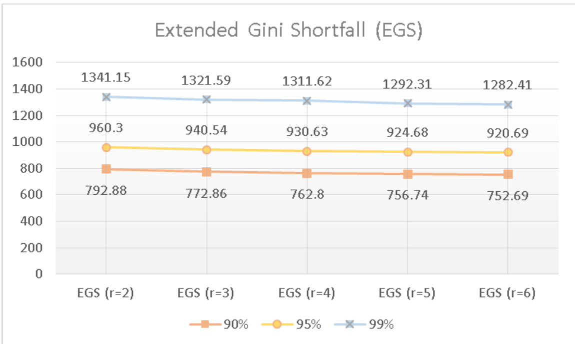


FIGURE 5. EGS on Various Risk-aversion Parameter and Various Prudence Level

Besides that, we can see the tendency of EGS value from Figure 5. With assumption of a constant risk-aversion parameter, EGS tends to increase with the increasing prudence level. Meanwhile, with assumption of a constant prudence level, EGS tends to decrease with the increasing risk-aversion parameter. As an example, for prudence level $p = 99\%$ and risk aversion parameter $r = 2$, the value of EGS is Rp792.88. It means investors or buyer of the UNVR stocks need to provide reserve of Rp792.88 for secure their asset from loss possibility in the future.

4 Conclusion

The Extended Gini Shortfall (EGS) is a more comprehensive risk measure compared to the VaR, ES, and GS because it takes into account the variability of losses in the distribution tail and takes risk-aversion parameter into consideration. This is due to the fact that EGS is constructed through a linear combination of the ES as a tail central tendency measure and the Tail Extended Gini functional as a tail variability measure that is considering the risk-aversion parameter. EGS is also a coherent risk measure under certain conditions which might prove to be a useful property for companies and investors in making business and investing decisions. Explicit formulas of EGS for exponential, Pareto, and logistic distribution have been derived in this paper and can be used to calculate risks that follow the related distribution. The example of EGS calculation have also been presented in this paper and leads into some tendencies. Assuming a constant risk-aversion parameter, EGS tends to increase with the increasing prudence level. Meanwhile, with a constant prudence level, EGS tends to decrease with the increasing risk-aversion parameter.

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