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MODELING THE SCARCITY OF COOKING OIL IN INDONESIA USING FRACTIONAL-ORDER DIFFERENTIAL EQUATIONS AND FUZZY PARAMETERS

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ABSTRACT

Indonesia is the largest palm oil exporter in the world. However, the latest challenge encountered by the Indonesian government is the scarcity of cooking oil in the market and the elevated price of cooking oil. Several policies have been carried out by the government. One of them is fixing the highest retail price for cooking oil. This policy has worsened the scarcity of cooking oil in the market because many distributors are keeping their cooking oil supplies. The Indonesian government's latest policy to manage the cooking oil problem is to provide subsidies for bulk cooking oil and revoke the regulation on the highest retail price for packaged cooking oil. In addition, the government also increased export levies and export duties. To study the impact of these policies, we propose a fractional-order mathematical model with fuzzy parameters. The model obtained is solved numerically. Our results show that the policy of increasing tax levies, export duties, and subsidies for bulk cooking oil will be more effective in solving the cooking oil shortage in Indonesia if the government can convince the public that oil stocks are safe and in the future there will be no shortage of cooking oil.

Keywords: Cooking oil scarcity, Fractional-order model, fuzzy parameter

1 Introduction

Palm oil has long been used by humans as food and medicine. Indonesia is one of the five largest palm oil exporting countries in the world [1]. It is estimated that in 2017, Indonesia exported 29 million tons of palm oil [2]. This number of exports places Indonesia as the largest palm oil exporter in the world. However, currently there is a scarcity and high price of cooking oil in the Indonesian market. The elevated price of cooking oil in Indonesia has been in the spotlight from the end of 2021 until the beginning of 2022 [3]. To overcome this problem, in early 2022, the Indonesian government imposed the highest retail price for cooking oil. However, this policy was revoked because cooking

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oil was increasingly difficult to find in the Indonesian market. Instead, the government increased export levies and export duties. In addition, the bulk cooking oil subsidy policy was also implemented by the government [4]. Can the two new policies of the Indonesian government solve the cooking oil problem? In this article, we develop a mathematical model to study the impact of implementing two new government policies on the dynamics of the cooking oil problem in Indonesia. To accommodate the memory effect, we proposed a fractional-order model. According to [5], in the fractional derivative, the historical memory becomes strong and weak with the change of the fractional order. In models that use fractional-order derivatives, the current condition is influenced by past events [6]. Several parameters of the proposed model are a function of the percentage increase of bulk cooking oil subsidies or export levies and export duties. The function used is a fuzzy membership function.

2 Mathematical Model

In this section, we describe the process of model formulation. The model constructed is a system of differential equations with four state variables. Suppose that there are only two types of cooking oil on the market, i.e. bulk cooking oil (B) and packaged cooking oil (P). Human groups are divided into two groups, namely prospective buyers (H_p) and consumer (H_c). Prospective buyers are groups consisting of humans who have run out of cooking oil and will buy cooking oil, while consumer are groups consisting of humans who have purchased and have stock of cooking oil for consumption.

Assume that θ is the proportion of prospective buyers who buy bulk cooking oil for their consumption. It is assumed that the oil stock held by the consumer runs out after σ^{-1} days. Therefore, the rate at which humans in the consumer group move to the prospective buyer group is σ. Also, it is assumed that each person can afford ν liter of cooking oil. The number of attempts to buy oil per person per day is denoted by ζ. Furthermore, the probability of person succeeding in getting cooking oil per attempt is denoted by κ. The dynamics of the variables that represent prospective buyers and consumer are expressed as follows

$$\begin{aligned}\frac{dH_p}{dt} &= -\kappa\zeta\nu^{-1}(\theta B+(1-\theta)P)H_p+\sigma H_c,\\ \frac{dH_c}{dt} &= \kappa\zeta\nu^{-1}(\theta B+(1-\theta)P)H_p-\sigma H_c.\end{aligned}$$

Suppose that Λ_b is the additional bulk cooking oil per day, while Λ_p is the additional packaged cooking oil per day. Here, we consider that bulk cooking oil and packaged cooking oil expire after μ_b^{-1} and μ_p^{-1} days, respectively. The dynamics of bulk cooking oil and packaged cooking oil are represented by the following equations.

$$\begin{aligned}\frac{dB}{dt} &= \Lambda_b-\nu\nu^{-1}\kappa\zeta\theta BH_p-\mu_b B\\ &= \Lambda_b-\kappa\zeta\theta BH_p-\mu_b B,\\ \frac{dP}{dt} &= \Lambda_p-\nu\nu^{-1}\kappa\zeta(1-\theta)PH_p-\mu_p P\\ &= \Lambda_p-\kappa\zeta(1-\theta)PH_p-\mu_p P\end{aligned}$$

As we know, cooking oil distribution issues in the national market are related to several factors, including business, politics, and government policies. In this article, we assume that several parameters are influenced by the policy of export levies and export duties as well as bulk cooking oil subsidies. These parameters are assumed to be fuzzy membership functions.

Now we will consider the parameter $\theta(\Pi)$ as a parameter depending on the percentage increase in bulk cooking oil subsidies (Π). the proportion of prospective buyers who buy bulk cooking oil is assumed to be a membership function of fuzzy number. The function is an increasing function of the percentage increase of bulk cooking oil subsidies. The greater the percentage increase of bulk cooking oil subsidies, the more prospective buyers will buy bulk cooking oil. However, the maximum value of the function is not equal to one because there are middle-upper economic groups who always use packaged cooking oil to meet their daily needs. In addition, the minimum value of this function is not equal to zero because there are lower-middle economic groups who always use bulk cooking oil. We assume that the minimum and maximum value of $\theta(\Pi)$ are θ_{min} and θ_{max}, respectively.

If the percentage increase in bulk cooking oil subsidies is relatively low, then the proportion of potential buyers who buy bulk cooking oil is about the proportion of people who always use bulk cooking oil, namely θ_{min}. On the other hand, if the percentage increase in the bulk cooking oil subsidy is high, many prospective buyers will buy bulk cooking oil with a potion equal to θ_{max} from the number of prospective buyers. The fuzzy membership function for the proportion of prospective buyers who buy bulk cooking oil is given as follows:

$$\theta(\Pi) = \begin{cases} \theta_{min}, & 0 \le \Pi \le \Pi_{min}, \\ (\theta_{max} - \theta_{min}) \frac{\Pi - \Pi_{min}}{\Pi_{max} - \Pi_{min}} + \theta_{min}, & \Pi_{min} < \Pi < \Pi_{max}, \\ \theta_{max}, & \Pi \ge \Pi_{max}, \end{cases}$$

where $\theta_{min}, \theta_{max} \in [0,1]$ and $\theta_{min} < \theta_{max}$. Figure 1 shows membership diagram of $\theta(\Pi)$.

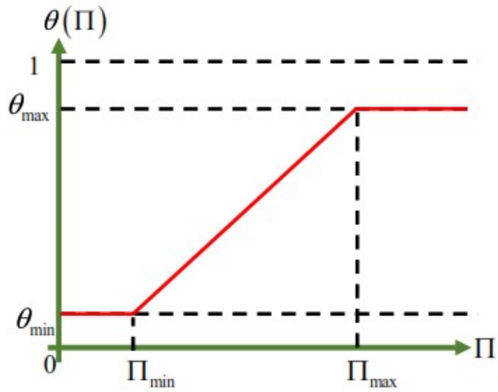


Figure 1: Membership diagram of $\theta(\Pi)$

To investigate the impact of changes in export levies and export duties, we consider $\Lambda_b(\Omega) = \rho_b(\Omega)\Lambda_b^{max}$ and $\Lambda_p(\Omega) = \rho_p(\Omega)\Lambda_p^{max}$, where $\rho_b(\Omega)$ and $\rho_p(\Omega)$ are membership functions of fuzzy number that depend on the percentage increase in export levies and export duties (Ω). We assume that the higher the percentage increase in export levies and export duties, the more bulk cooking oil and packaged cooking oil will be traded in the national market because exporters consider the large export costs that must be incurred if the cooking oil is sold overseas. As a result, the recruitment rate for bulk cooking oil and packaged cooking oil will be higher if the percentage increase in export levies and export duties is higher. We assume that the maximum value of $\rho_b(\Omega)$ and $\rho_p(\Omega)$ are 1. Consequently, the recruitment rate of bulk cooking oil and the recruitment rate of packaged cooking oil when the the percentage increase in export levies and export duties is high are Λ_b^{max} and Λ_p^{max}, respectively. Meanwhile, the minimum value of $\rho_b(\Omega)$ and $\rho_p(\Omega)$ are ρ_b^{min} and ρ_p^{min}, respectively. Therefore, when the percentage increase in export levies and export duties is low, the recruitment rate of bulk cooking oil and the recruitment rate of packaged cooking oil are $\rho_b^{min}\Lambda_b^{max}$ and $\rho_p^{min}\Lambda_p^{max}$, respectively. We assume that the minimum value of the recruitment rate for both types of cooking oil is not zero because there will always be cooking oil traded in the national market. Therefore, the definitions of $\rho_b(\Omega)$ and $\rho_p(\Omega)$ are given by

$$\rho_b(\Omega) = \begin{cases} \rho_b^{min}, & 0 \le \Omega \le \Omega_{min}, \\ \left(1 - \rho_b^{min}\right) \frac{\Omega - \Omega_{min}}{\Omega_{max} - \Omega_{min}} + \rho_b^{min}, & \Omega_{min} < \Omega < \Omega_{max}, \\ 1, & \Omega \ge \Omega_{max}, \end{cases}$$

and

$$\rho_p(\Omega) = \begin{cases} \rho_p^{min}, & 0 \leq \Omega \leq \Omega_{min}, \\ \left(1-\rho_p^{min}\right) \frac{\Omega-\Omega_{min}}{\Omega_{max}-\Omega_{min}} + \rho_p^{min}, & \Omega_{min} < \Omega < \Omega_{max}, \\ 1, & \Omega \geq \Omega_{max}, \end{cases}$$

where $\rho_b^{min}, \rho_b^{min} \in [0,1)$. The diagram for membership function $\rho_b(\Omega)$ and $\rho_p(\Omega)$ can be seen in Figure 2.

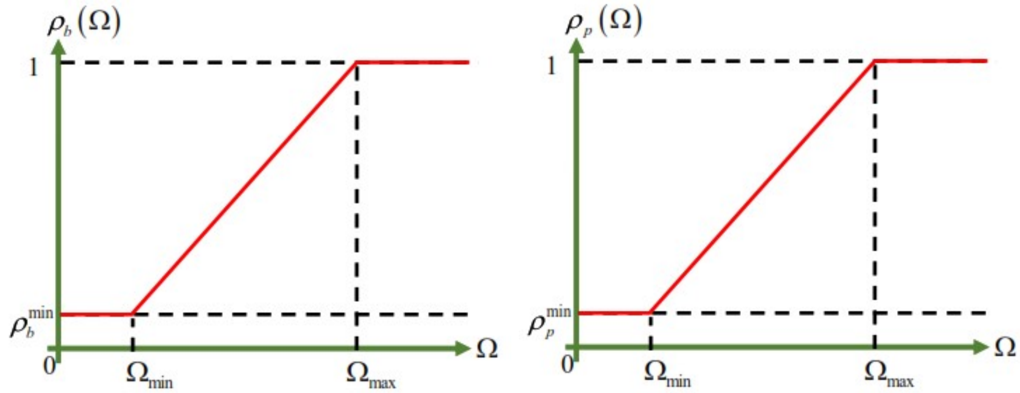


Figure 2: Membership diagram of: (a) $\rho_b(\Omega)$ and (b) $\rho_p(\Omega)$

To include the memory effect of the scarcity and high price of cooking oil problem in Indonesia, we use fractional-order derivatives in the Caputo sense. According to [7], the fractional order of the derivative (α) is index of memory, where $0 < \alpha \leq 1$. The strength of memory depends on fractional order. the smaller the order α, the greater the contribution of the first data [6]. Following [8], we obtain the fractional-order model as follows

$$\begin{aligned} D_t^{\alpha} H_p &= -\kappa \zeta^{\alpha} \nu^{-1} (\theta(\Omega) B + (1-\theta(\Omega)) P) H_p + \sigma^{\alpha} H_c, \\ D_t^{\alpha} H_c &= \kappa \zeta^{\alpha} \nu^{-1} (\theta(\Omega) B + (1-\theta(\Omega)) P) H_p - \sigma^{\alpha} H_c, \\ D_t^{\alpha} B &= \Lambda_b (\Pi)^{\alpha} - \kappa \zeta^{\alpha} \theta(\Omega) B H_p - \mu_b^{\alpha} B, \\ D_t^{\alpha} P &= \Lambda_p (\Pi)^{\alpha} - \kappa \zeta^{\alpha} (1-\theta(\Omega)) P H_p - \mu_p^{\alpha} P. \end{aligned} \tag{1}$$

3 Numerical Simulations

Based on [9], there are $67,945,900$ households in Indonesia. Here, we assume that there is only one prospective buyer representing each household and 50% of them use bulk cooking oil. Therefore, we use $H_p(0) = 33,972,950$ and $H_c(0) = 33,972,950$. Based on [10], the amount of cooking oil that has been distributed to the national market is $415,787,000$ kg. This amount is equivalent to 461,523,570 liters of cooking oil. We assume that 75% of the total cooking oil that has been distributed is bulk cooking oil and the rest is packaged cooking oil. Therefore, we use $B(0) = 311,840,250$ and $P(0) = 103,946,750$. The numerical simulations is conducted using the parameter values given in Table 1. Due to the lack of data related to parameter values, we selected parameter values based on assumptions.

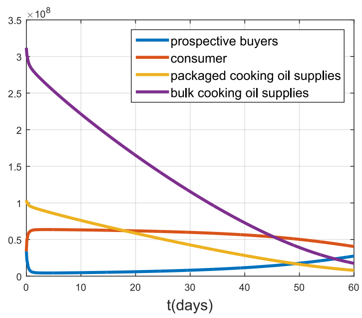
3.1 Low percentage increase in bulk cooking oil subsidies, export levies, and export duties

In this subsection, we perform numerical simulations for the scheme of low percentage increase in bulk cooking oil subsidies $(\Pi = 0.1 < \Pi_{min})$ and low percentage increase in export levies and export

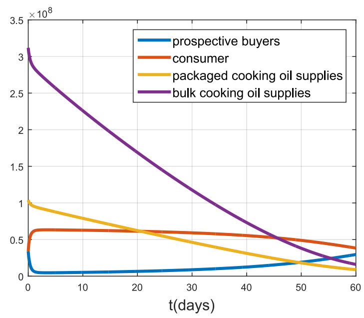
Tabel 1: Parameter Values

Parameter	Values	Units
κ	0.001	$\frac{1}{\text{attempt}}$
ζ	0.01	$\frac{\text{attempt}}{\text{person}\times\text{day}}$
σ	$\frac{1}{7}$	$\frac{1}{\text{day}}$
ν	1	$\frac{\text{liter}}{\text{person}}$
Λ_b^{max}	2500000	$\frac{\text{liter}}{\text{day}}$
Λ_p^{max}	1000000	$\frac{\text{liter}}{\text{day}}$
μ_b, μ_p	$\frac{1}{2\times 365}$	$\frac{1}{\text{day}}$
ρ_b^{min}	0.4	–
ρ_p^{min}	0.5	–
θ_{min}	0.5	–
θ_{max}	0.8	–
Π_{min}	0.3	–
Π_{max}	0.6	–
Ω_{min}	0.2	–
Ω_{max}	0.5	–

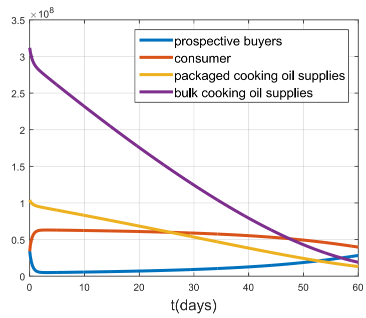
duties ($\Omega = 0.1 < \Omega_{min}$). The results of the numerical simulation can be seen in Figure Figure 3 and Table 2



(a) $\alpha = 0.9$



(b) $\alpha = 0.95$



(c) $\alpha = 1$

Figure 3: Solution curve of system (1) for $\Pi < \Pi_{min}$ and $\Omega < \Omega_{min}$

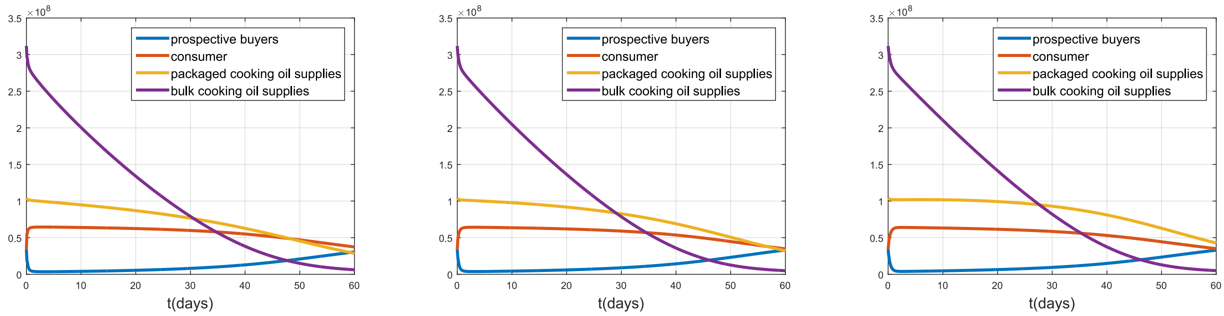
Figure 3 and Table 2 show that for $\alpha = 0.9$, the total stock of cooking oil in the market is 25524000 liters while the number of prospective buyers is 27449000. This indicates a shortage of cooking oil. The same condition also occurs for $\alpha = 0.95$, where the number of prospective buyers is 29578000 while the total stock of cooking oil in the market is 24990000. On the other hand, when $\alpha = 1$, the amount of cooking oil is more than the number of potential buyers. Thus, the condition of a low percentage increase in bulk cooking oil subsidies, export levies, and export duties resulted in a shortage of oil when the memory effect is considered.

Tabel 2: Numerical solutions for $\Pi < \Pi_{min}$ and $\Omega < \Omega_{min}$

α	$H_p(60)$	$H_c(60)$	$B(60)$	$P(60)$
0.9	27449000	40484000	7957000	17567000
0.95	29578000	38361000	8926000	16064000
1	28261000	39681000	13182000	18964000

3.2 High percentage increase in bulk cooking oil subsidies and low percentage increase in export levies, and export duties

In this subsection, we perform numerical simulations for the scheme of high percentage increase in bulk cooking oil subsidies ($\Pi = 0.7 > \Pi_{max}$) and low percentage increase in export levies and export duties ($\Omega = 0.1 < \Omega_{min}$). The results of the numerical simulation can be seen in Figure Figure 4 and Table 3



(a) $\alpha = 0.9$ (b) $\alpha = 0.95$ (c) $\alpha = 1$

Figure 4: Solution curve of system (1) for $\Pi > \Pi_{max}$ and $\Omega < \Omega_{min}$

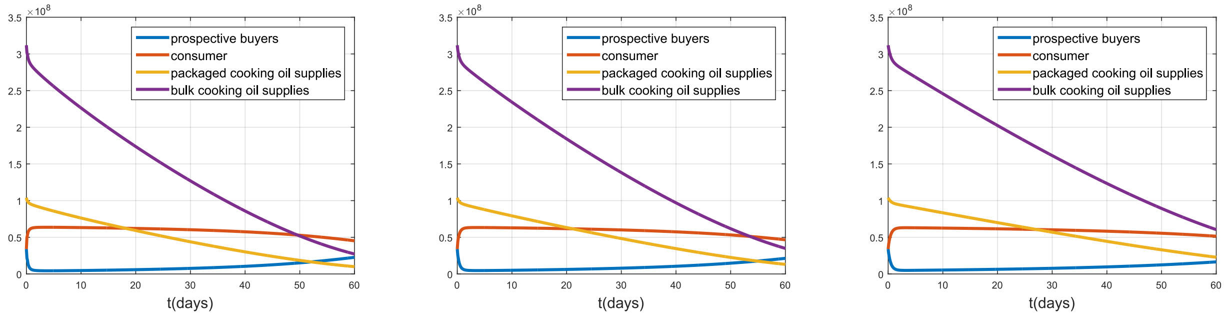
Figure 4 and Table 3 show that for $\alpha = 0.9$, the total stock of cooking oil in the market is 34797000 liters while the number of prospective buyers is 30602000. If $\alpha = 0.95$, the number of prospective buyers is 33219000 while the total stock of cooking oil in the market is 37115000. Furthermore, when $\alpha = 1$, the difference between the amount of oil in the market and the number of prospective buyers is greater than the difference between the amount of cooking oil in the market and the number of prospective buyers for the case of $\alpha = .9$ and $\alpha = 0.95$. This shows the positive impact of policies that can provide confidence to the public regarding cooking oil stocks that meet national needs.

Tabel 3: Numerical solutions for $\Pi > \Pi_{max}$ and $\Omega < \Omega_{min}$

α	$H_p(60)$	$H_c(60)$	$B(60)$	$P(60)$
0.9	30602000	37333000	28617000	6180000
0.95	33219000	34721000	32170000	4944000
1	32913000	35030000	42678000	5046000

3.3 Low percentage increase in bulk cooking oil subsidies and high percentage increase in export levies, and export duties

In this subsection, we perform numerical simulations for the scheme of low percentage increase in bulk cooking oil subsidies ($\Pi = 0.1 < \Pi_{min}$) and high percentage increase in export levies and export duties ($\Omega = 0.6 > \Omega_{max}$). The results of the numerical simulation can be seen in Figure Figure 5 and Table 4



(a) $\alpha = 0.9$ (b) $\alpha = 0.95$ (c) $\alpha = 1$

Figure 5: Solution curve of system (1) for $\Pi < \Pi_{min}$ and $\Omega > \Omega_{max}$

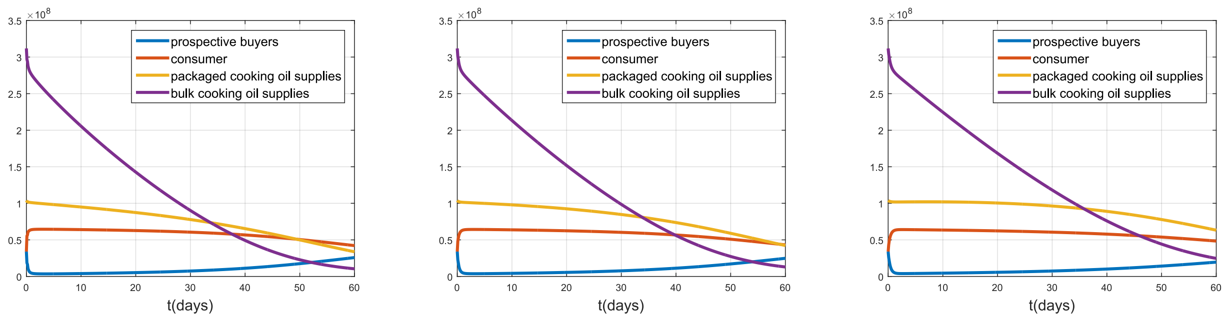
Figure 5 and Table 4 show that for $\alpha = 0.9$, the total stock of cooking oil in the market is 37241000 liters while the number of prospective buyers is only 22692000. It is clear that the difference between the number of oil stocks in the market and the number of potential buyers is 14549000. If $\alpha = 0.95$, the number of prospective buyers is 21323000 while the total stock of cooking oil in the market is 47962000. The difference between the two is 26639000. Furthermore, the largest difference between the number of cooking oil stocks in the market and the number of prospective buyers is 66833000 which occurs if $\alpha = 1$. This indicates that the weaker the memory, the greater the difference between the number of oil stocks on the market and the number of prospective buyers.

Tabel 4: Numerical solutions for $\Pi < \Pi_{min}$ and $\Omega > \Omega_{max}$

α	$H_p(60)$	$H_c(60)$	$B(60)$	$P(60)$
0.9	22692000	45244000	9992000	27249000
0.95	21323000	46618000	13107000	34855000
1	16485000	51458000	22846000	60472000

3.4 High percentage increase in bulk cooking oil subsidies, export levies, and export duties

In this subsection, we perform numerical simulations for the scheme of high percentage increase in bulk cooking oil subsidies ($\Pi = 0.7 > \Pi_{max}$) and low percentage increase in export levies and export duties ($\Omega = 0.6 > \Omega_{max}$). The results of the numerical simulation can be seen in Figure Figure 6 and Table 5



(a) $\alpha = 0.9$ (b) $\alpha = 0.95$ (c) $\alpha = 1$

Figure 6: Solution curve of system (1) for $\Pi > \Pi_{max}$ and $\Omega > \Omega_{max}$

Figure 6 and Table 5 show that in conditions of a high percentage increase in bulk cooking oil subsidies, export levies, and export duties, cooking oil scarcity has been successfully handled. For the three alpha values, the amount of cooking oil stock in the market is much larger than the number of potential buyers. In addition, the weaker the memory (α is getting closer to 1), the fewer the number of potential buyers and the higher the stock of cooking oil. These results indicate that to address the problem of scarcity of cooking oil, the government should increase the percentage increase in bulk cooking oil subsidies, export levies, and export duties. In addition, to increase its effectiveness, the government should implement policies that can assure the public that oil stocks are sufficient and that oil shortages will not occur again in the future.

Tabel 5: Numerical solutions for $\Pi > \Pi_{max}$ and $\Omega > \Omega_{max}$

α	$H_p(60)$	$H_c(60)$	$B(60)$	$P(60)$
0.9	25777000	42159000	33736000	10431000
0.95	24783000	43158000	42265000	12932000
1	19554000	48389000	63258000	24631000

From the four numerical simulation schemes, we identified that the weaker the memory (the order of the derivatives is getting closer to 1), the greater the difference between the number of cooking oil stocks in the market and the number of potential buyers. In this case, the amount of cooking oil stock is more than the number of potential buyers. These results show the positive impact that occurs when people's concerns and fears about the scarcity of oil can be reduced. The government should implement policies that can provide confidence to the public that the stock of cooking oil is sufficient to meet national needs. In addition, the government should allocate a budget for bulk cooking oil subsidies. It is advisable to increase export levies and export duties so that oil distributors prefer to distribute cooking oil to the national market.

4 Conclusion

In this paper, we propose a fractional-order mathematical model with fuzzy parameters to study the impact of memory, export levies, export duties, and bulk oil subsidies on oil scarcity in Indonesia. The results show that the government should be able to determine the appropriate percentage increase in export levies, export duties, and bulk oil subsidies. In addition, the government should implement policies that can convince the public that in the future there will be no shortage of cooking oil. The limitation of this study is that the parameter values are selected based on assumptions. More data and reliable parameters will enhance the outcomes of this study.

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