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Research Article

Solution of Q-Deformed D-Dimensional Klein-Gordon Equation Kratzer Potential using Hypergeometric Method

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Abstract

The Q-deformed D-dimensional Klein Gordon equation with Kratzer potential is solved by using Hypergeometric method in the case of exact spin symmetry. The linear radial momentum of D-dimensional Klein Gordon equation is disturbed by the presence of the quadratic radial posisiton. The Klein-Gordon D-dimensional equation is reduced to one-dimensional Schrodinger like equation with variable substitution. The solution of the D-dimensional Klein-Gordon equation is determined in the form of a general equation of the Hypergeometry function using the Kratzer potential variable and the quantum deformation variable. From this equation, relativistic energy and wave function are determined. In addition, the relativistic energy equation can be used to calculate numerical energy levels for diatomic particles (CO, NO, O_2) using Matlab R2013a software. The results obtained show that the q-deformed quantum parameters, quantum numbers and dimensions affect the value of relativistic energy for zero-pin particles. The value of energy increases with increasing value of quantum number n, q-deformed parameters, and d-dimensional parameters. Of the three parameters, q-deformed parameter is the most dominant to give change in energy value; the increasing q-deformed parameter causes the energy value increases significantly compared to the d-dimensional parameter and quantum numbers n.

Keywords: *Klein-Gordon equation, quantum deformation, Kratzer potential, Hypergeometric method*

Solusi Persamaan Klein-Gordon D-Dimensi Terdeformasi Kuadrat Posisi untuk Potensial Kratzer menggunakan Metode Hypergeometri

Abstrak

Persamaan Klein-Gordon D-dimensi terdeformasi q dengan potensial Kratzer diselesaikan menggunakan metode Hypergeometri untuk kasus eksak spin simetri. Momentum linier radial dari persamaan Klein-Gordon D-dimensi terganggu karena adanya posisi radial kuadrat. Persamaan Klein-Gordon D-dimensi direduksi menjadi seperti persamaan Schrodinger satu dimensi dengan substitusi variabel. Solusi persamaan Klein-Gordon D-dimensi ditentukan dalam bentuk dari persamaan umum fungsi Hypergeometri menggunakan variabel potensial Kratzer dan variabel quantum deformasi. Dari persamaan tersebut, energi relativistik dan fungsi gelombang ditentukan. Selain itu, persamaan energi relativistik dapat digunakan untuk menghitung level-level energi secara numerik untuk partikel diatomik (CO, NO, O_2) menggunakan software Matlab R2013a. Hasil yang diperoleh menunjukkan bahwa

parameter quantum deformasi q, bilangan kuantum dan dimensi mempengaruhi nilai energi relativistik untuk partikel berspin nol. Nilai energi bertambah dengan bertambahnya nilai dari bilangan kuantum n, parameter deformasi q dan parameter dimensi D. Dari ketiga parameter tersebut, parameter deformasi q yang paling dominan memberikan perubahan nilai energi, bertambahnya parameter deformasi q menyebabkan nilai energi bertambah secara signifikan dibandingkan terhadap parameter dimensi D dan bilangan kuantum n.

Kata Kunci: *Persamaan Klein-Gordon, deformasi quantum, potensial Kratzer, metode Hypergeometri*

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I. INTRODUCTION

In quantum mechanics, the effect of relativistic should be considered when a particle moving in strong potential [1,2]. Klein-Gordon and Dirac equations describe the dynamic of particles in relativistic quantum mechanics. Klein-Gordon equation used to explain the behaviour of zero spin particle and Dirac equation for ½ spin particle [2].

Klein-Gordon equation is solvable by using some methods, such as Supersymmetric quantum mechanics (SUSY) [3], Asymptotic Iteration method (AIM) [3-5], and Nikiforov-Uvarov (NU) [5,6]. The favorite method used to solve relativistic and non-relativistic equations is Hypergeometric method [7]. This method has a specific function of second order differential equation obtained by new variable substitution [8].

Moreover, Hypergeometric method is commonly used to solve some potentials system, especially shape invariance potentials, such as Kratzer [8], Pöschl-Teller, Manning Rosen, Rosen Morse [4,10], Hulthen, and Wood-Saxon [11].

Kratzer potential is one of attractive potentials that has been applied in many fields such as atomic and molecular physics. The Kratzer potential has important role to the study about the deformed of nuclei structure and interactions between them [12]. Kratzer potential has been used to describe the behavior, structure, and motions of diatomic particles [13-15], such as H_2, O_2, I_2, CO, NO, Li_2, and LiH [16].

Some researchers have investigated the application of Kratzer potential and q-deformed relativistic quantum mechanics [4,7,17]. The D-dimensional Klein-Gordon equation for Kartzer potential studied by Nugraha et al using AIM method [4] showed that relativistic energy spectra depends on diatomic molecule type, quantum number, and dimension parameters. Hassanabadi et al investigated on the solution of D-dimensional Klein-Gordon equation with Kratzer potential by using Ansatz wave function, showing that the energy spectra calculated numerically for various dimension and mass of particle was fluctuated [7].

The extension in higher dimension spaces is very important for some problems in the area of Physics. The D-dimensional system was introduced to explain the unified theory of gravitation and electromagnetic fields [20,21]. It is suspected that the extra

dimensional system is able to be used for gravitation field since it is involved in such huge universe. The D-dimensional non-relativistic and relativistic physical systems have been investigated by many authors, Poschl-Teller and Manning Rosen non-central potential [22], hyperbolic tangent [23], three anharmonics potential [24].

It us understood that the universe is too huge, therefore it is possible that there several sistems, particularly in sub-atomic system, which have dimensions more than three dimensions, such as string theory system with 11 (eleven) dimension [20].

The q-deformed quantum mechanics has been studied in non-relativistic quantum mechanics for Morse and Oscillator potential by Hassanabadi et al [25]. The q-deformed was developed based on q-derivative and q-deformed statistical physics proposed by Tsallis [25-28]. The q-deformed statistical physics was developed based on q-deformed logarithm and its inverse, q-exponential, that was derived from a slightely non-linier differential equation [29,30].

Tsallis entropy is the improved Boltszmann Gibbs theory. The Tsallis entrophy is non-extensive entropy that has been derived from q-exponential [28,30-33], given as:

$$S_q = -\frac{1-\sum_{i=1}^{n} p_i^q}{1-q} \tag{1}$$

with q is deformation parameter and $q > 0$. Then from the definitions of q-exponential and q-logarithm, q-algebra and q-calculus were derived [29,30].

The q-deformed quantum that is developed from q-derivative [29,30] is expressed as:

$$D_x^q f(x) = \lim_{y\to x} \frac{f(y)-f(x)}{x\Theta_q y} = \left(1+(1-q)\,\mathrm{x}\right)\frac{df(x)}{dx} \tag{2}$$

Then by setting $(1-q)\to q$, the q-derivative which is q-deformed calculus reduces to:

$$D_x^q f(x) = \left(1+q\,\mathrm{x}\right)\frac{df(x)}{dx} \tag{3}$$

By using the q-derivative in phase space, there is a linear momentum given as:

$$p_x^q = -i\hbar D_x^q = -i\hbar\left(1+q\,\mathrm{x}\right)\frac{d}{dx} \tag{4}$$

and the q-related Heisenberg algebra is

$$\left[\vec{x}, \vec{p}_x^{\,q}\right] = i\hbar\left(1+q\,\vec{\mathrm{x}}\right) \tag{5}$$

By modyfying the q-deformed linier momentum, another q-deformed quantum is proposed as [17]:

$$p_x^q = -i\hbar D_x^q = -i\hbar\left(1+q\,\mathrm{x}^2\right)\frac{d}{dx} \tag{6}$$

and so the q-related Heisenberg algebra is

$$\left[\vec{x}, \vec{p}_x^{\,q}\right] = i\hbar\left(1+q\,\vec{\mathrm{x}}^2\right) \tag{7}$$

Then the modified q-deformed quantum expressed in equation (6) will be applied on D-dimensional Klein Gordon equation for Kratzer potential.

Sobhani et al investigated about zero-spin (bosons) in q-deformed relativistic quantum mechanics for delta Dirac potential [17]. This q-deformed quantum that has been applied on Klein Gordon equation is postulated as a kind of q-deformed quantum [25] but it has undergone a modification like becoming what it is called as modified q-deformed quantum. The q-deformed quantum has been applied in mathematical and physics such as nuclei, theory of statistical quantum, quantum hall effect, black holes and cosmic string [17-19].

It is necessary to investigate the the application of the modified q-deformed quantum for some physical potentials. The aim of the research is to obtain the solution of the q-deformed D-dimensional Klein-Gordon equation for Kratzer potential by using

Hypergeometric method. The equations of relativistic energy and radial wave functions were obtained analytically from radial part of modified q-deformed Klein Gordon equation with Kratzer potential. The relativistic energy spectra were calculated numerically from relativistic energy equation by Matlab R2013a software, and the wave function was expressed in the Hypergeometric series.

II. METHOD

A zero spin particle which is moving on scalar $S(r)$ and vector $V(r)$ potentials with mass M and relativistic energy E with $(\hbar = c = 1)$ is described by the D-dimensional Klein-Gordon equation expressed by [4,35,37]

$$\left\{\nabla_D{}^2 + \left(E - V(r)\right)^2 - \left(M + S(r)\right)^2\right\}\psi^{\ell_{D-1}=\ell}_{\ell_1\ldots\ell_{D-2}}(\mathrm{x}) = 0 \quad (8)$$

with the D-dimensional Laplacian operator ∇_D^2 is given by [20]

$$\nabla_D^2 = \frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\sum_{j=1}^{D-2}\left\{\frac{1}{\sin^2\theta_{j+1}\sin^2\theta_{j+2}\ldots\sin^2\theta_{D-1}}\right\}\left\{\frac{1}{\sin^{j-1}\theta_j}\left(\frac{\partial}{\partial\theta_j}\sin^{j-1}\theta_j\frac{\partial}{\partial\theta_j}\right)\right\} + \frac{1}{r^2}\left\{\frac{1}{\sin^{D-2}\theta_{D-1}}\left(\frac{\partial}{\partial\theta_{D-1}}\sin^{D-2}\theta_{D-1}\frac{\partial}{\partial\theta_{D-1}}\right)\right\} \quad (9)$$

In the case of exact spin symmetry, when the vector potential $V(r)$ is equal to the scalar potential $\mathrm{S}(r)$ and if we set $2V(r) \to V(r)$. Then equation (8) reduces to

$$\left\{\nabla_D{}^2 + E^2 - M^2 - (E+M)V(r)\right\}\psi^{\ell_{D-1}=\ell}_{\ell_1\ldots\ell_{D-2}}(\mathrm{x}) = 0 \quad (10)$$

By inserting Equation (9) into Equation (10) and setting

$$\psi^{\ell_{D-1}=\ell}_{\ell_1\ldots\ell_{D-2}}(\mathrm{x}) = \mathrm{R}_\ell(r)Y^{(\ell)}_{\ell_1\ldots\ell_{D-2}}(\theta_1,\theta_2,\ldots,\theta_{D-1}) \quad (11)$$

in Equation (10) and then also by applying variable separation method to Equation (10), it is obtained

$$\frac{r^2}{R_\ell r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)R_\ell + \left\{E^2 - M^2 - (E+M)V(r)\right\}r^2 = -\frac{1}{Y_{lm}}\left[\sum_{j=1}^{D-2}\left\{\frac{1}{\sin^2\theta_{j+1}\sin^2\theta_{j+2}\ldots\sin^2\theta_{D-1}}\right\}\left\{\frac{1}{\sin^{j-1}\theta_j}\left(\frac{\partial}{\partial\theta_j}\sin^{j-1}\theta_j\frac{\partial}{\partial\theta_j}\right)\right\} + \frac{1}{\sin^{D-2}\theta_{D-1}}\left(\frac{\partial}{\partial\theta_{D-1}}\sin^{D-2}\theta_{D-1}\frac{\partial}{\partial\theta_{D-1}}\right)\right]Y_{lm} = L^2_{D-2} \quad (12)$$

with L^2_{D-2} is variable separation constant. From Equation (12) it is obtained [11,26]

$$\frac{1}{r^{D-1}}\frac{\partial}{\partial r}\left(r^{D-1}\frac{\partial}{\partial r}\right)R_\ell + \left\{E^2 - M^2 - (E+M)V(r)\right\}R_\ell - \frac{L^2_{D-2}}{r^2}R_\ell = 0 \quad (13)$$

and

$$\left[\sum_{j=1}^{D-2}\left\{\frac{1}{\sin^2\theta_{j+1}\sin^2\theta_{j+2}\ldots\sin^2\theta_{D-1}}\right\}\left\{\frac{1}{\sin^{j-1}\theta_j}\left(\frac{\partial}{\partial\theta_j}\sin^{j-1}\theta_j\frac{\partial}{\partial\theta_j}\right)\right\} + \frac{1}{\sin^{D-2}\theta_{D-1}}\left(\frac{\partial}{\partial\theta_{D-1}}\sin^{D-2}\theta_{D-1}\frac{\partial}{\partial\theta_{D-1}}\right)\right]Y^{(\ell)}_{\ell_1\ldots\ell_{D-2}} = -L^2_{D-2}Y^{(\ell)}_{\ell_1\ldots\ell_{D-2}} \quad (14)$$

By setting $\mathrm{R}_\ell(r) = U(r)r^{-\frac{D-1}{2}}$ in Equation (13) and $L^2_{D-2} = L(L+D-2)$ then Equation (13) becomes

$$\frac{\partial^2 U(r)}{\partial r^2} - \frac{(D-1)(D-3)U(r)}{4r^2} - (E+M)V(r)U(r) - \frac{L(L+D-2)}{r^2}U(r) + \left\{E^2 - M^2\right\}U(r) = 0 \quad (15)$$

and Equation (15) can be simplified as

$$\frac{\partial^2 U(r)}{\partial r^2} - \frac{\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)}{r^2}U(r) + \left(E^2 - M^2 - (E+M)\mathrm{V(r)}\right)U(r) = 0 \quad (16)$$

with the effective potential $V_{ef}(r) = (E+M)V(r)$ and $V(r)$ is central potential.

Q-Deformed Quantum

In this section, the modified q-deformed quantum mechanics is applied on the radial part of D-dimensional Klein Gordon equation with Kratzer potential. The q-deformed radial momentum is expressed by [25,34]

$$p_r^q = -i\hbar D_r^q = -i\hbar\left(1+qr^2\right)\frac{\partial}{\partial r} \quad (17)$$

By using the modified q-deformed momentum in Equation (17) for radial momentum in Equation (16) and when $\hbar = 1$ then $\frac{\partial^2}{\partial r^2}$ changes to $D_r^{q^2}$ given as

$$D_r^{q^2} = (1+qr^2)^2\frac{d^2}{dr^2} + 2qr(1+qr^2)\frac{d}{dr} \quad (18)$$

By setting new variable as follows

$$x = r\sqrt{q} \rightarrow dx = dr\sqrt{q} \rightarrow dr = \frac{dx}{\sqrt{q}} \quad (19)$$

and inserting Equation (19) into Equation (18), it is obtained:

$$D_r^{q^2} = (1+x^2)^2 q\frac{d^2}{dx^2} + 2qx(1+x^2)\frac{d}{dx} \quad (20)$$

Equation (20) will be applied in D-dimensional Klein Gordon equation to investigate the effect of the q-deformed parameter to the relativistic energy and wavefuction.

Kratzer Potential

In this research Kratzer potential is used as the scalar $S(r)$ and vector $V(r)$ potentials which is expressed as [4,35,36]:

$$V(r) = -\frac{V_1}{r} + \frac{V_2}{r^2} \quad (21a)$$

and $$\mathrm{S}(r) = -\frac{S_1}{r} + \frac{S_2}{r^2} \quad (21b)$$

with $V_1 = S_1$ and $V_2 = S_2$, $V_1 = 2D_e a$ and $V_1 = 2D_e a^2$, a is the equilibrium internuclear distance [13].

Then from Equations (21), there is $V_{ef}(r)$ given as:

$$V_{ef}(r) = V_{ef}(\mathrm{x}) = (E+M)\left(-\frac{V_1}{x}\sqrt{q} + \frac{V_2}{x^2}q\right) \quad (22)$$

By inserting Equations (20) and (22) into Equation (16), it is obtained the D-dimensional modified q-deformed Klein-Gordon equation with Kratzer potential that is written as:

$$\left((1+x^2)^2 q\frac{d^2}{dx^2} + 2qx(1+x^2)\frac{d}{dx}\right)U(x) - \left(\frac{\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)}{x^2}\right)qU(x) + \left(E^2 - M^2 - (E+M)\left(-\frac{V_1}{x}\sqrt{q} + \frac{V_2}{x^2}q\right)\right)U(x) = 0 \quad (23)$$

By using variable and parameter substitutions Equation (23) is reduced into standard form of one-dimensional Schrodinger equation which is solvable by Hypergeometric method.

Hypergeometric Method

The second order differential of Hypergeometric function equation is expressed by [8,10]:

$$z(1-z)\frac{\partial^2\phi}{\partial z^2} + \left(c-(a+b+1)z\right)\frac{\partial\phi}{\partial z} - ab\phi = 0 \quad (24)$$

The Equation (24) has two points regular singular at $z = 0$ and $z = 1$. By choosing simple form, the solution of Equation (24) can be solved in series form around point $z = 0$ given as:

$$\phi = z^s\sum_{n=0}^{n} a_n z^n \quad (25)$$

The solution form of differential equation of Hypergeometric function in Equation (24) is given as Gaussian Hypergeometric function [38]

$$ {}_2F_1(a,b,c;z)=\Phi_1(z)=\sum_{n=0}\frac{(a)_n(b)_n}{n!(c)_n}z^n \quad (26) $$

with

$$ a_n=a(a+1)(a+2)(a+3)....(a+(n-1)) \quad (27a) $$

and $a_0=1$ (27b)

The energy level of system is obtained when the solution of Equation (26) has the form of polynomial rank n with condition $a=-n$ or $b=-n$ [39].

III. RESULTS AND DISCUSSION

The q-deformed D-dimensional Klein-Gordon equation for Kratzer potential is reduced into the one-dimensional Schrodinger like equation through variable and parameters substitutions. By setting the variable in Equation (23) as

$$ x=\tan \mathrm{y} \rightarrow dx=\sec^2 y\,dy \quad (28) $$

By substituting equation (28) into Equation (23), it has:

$$ \frac{d^2U(\mathrm{y})}{dy^2}-\frac{\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2\right)}{\tan^2 y}U(y) +\left(\left(\frac{E^2-M^2}{q}\right)+\frac{(E+M)\mathrm{V}_1}{\sqrt{q}\tan y}\right)U(y)=0 \quad (29) $$

By using $cot^2 y=cosec^2 y-1$ equation (29) becomes

$$ \frac{d^2U(y)}{dy^2}-\left(\frac{\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2}{\sin^2 y}\right)U(y) +\frac{(E+M)\mathrm{V}_1}{\sqrt{q}}\cot \mathrm{y}\,\mathrm{U(y)}=-\left(\frac{E^2-M^2}{q}\right)U(y) -\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2\right)U(y) \quad (30) $$

Equation (24) is simplified by setting new parameters as follows

$$ \upsilon(\upsilon+1)=\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2\right) \quad (31) $$

$$ 2\mu=\frac{(E+M)\mathrm{V}_1}{\sqrt{q}} \quad (32) $$

$$ E_s^{'}=\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2+\left(\frac{E^2-M^2}{q}\right)\right) \quad (33) $$

and Equation (30) reduces to

$$ \frac{d^2}{dy^2}U(y)-\frac{\upsilon(\upsilon+1)}{\sin^2 y}U(y)+2\mu\cot y\;U(y)=-E_s^{'}U(y) \quad (34) $$

The Equation (34) is Schrodinger like equation with Rosen Morse I potential [10,30]. Therefore, Equation (34) is solvable by Hypergeometric method through variable substitution as follows

$$ \cot \mathrm{y}=i(1-2z) \;\rightarrow\; 2idz=\cos ec^2 ydy \qquad \cos ec^2 y=4z(1-z) \quad (35) $$

By substituting Equation (35) into Equation (34), it is obtained:

$$ z(1-z)\frac{d^2U}{dz^2}+(1-2z)\frac{dU}{dz} -\left(\upsilon(\upsilon+1)+\frac{\left(2\mu i+E_s^{'}\right)}{4z}-\frac{\left(-2\mu i+E_s^{'}\right)}{4(1-z)}\right)U=0 \quad (36) $$

By setting general solution of wave function equation in Equation (36) as

$$ U=z^{\alpha}(1-z)^{\beta}u(z) \quad (37) $$

and inserting Equations (37) into (36), it has:

$$ z(1-z)u''+\left((2\alpha+1)-(2\alpha+2\beta+2)z\right)u' -\left((\alpha+\beta)(\alpha+\beta+1)-\upsilon(\upsilon+1)\right)u=0 \quad (38) $$

with

$$ \alpha^2=\frac{\left(2\mu i+E_s^{'}\right)}{4} \quad (39a) $$

and $$ \beta^2=\frac{\left(-2\mu i+E_s^{'}\right)}{4} \quad (39b) $$

Equation (38) is second order differential equation of Hypergeometric function which is similar to Equation (24). Therefore, by comparing Equations (24) and (38) it is obtained

$$a=\left(\alpha+\beta-\upsilon\right) \tag{40}$$

$$b=\left(\alpha+\beta+\upsilon+1\right) \tag{41}$$

$$c=2\alpha+½ \tag{42}$$

$$a+b+1=2\alpha+2\beta+1\rightarrow a+b=2\alpha+2\beta \tag{43}$$

The energy spectra equation is obtained by setting the condition $a=-n$, then Equation (40) becomes

$$\alpha+\beta-\upsilon=-n\rightarrow\left(\alpha+\beta\right)^2=\left(\upsilon-n\right)^2 \tag{44}$$

By using Equations (33), (39), and (44), the relativistic energy equation is obtained as

$$\left(E^2-M^2\right)=q\left(\left(\frac{1+\sqrt{1+4\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2\right)}}{2}\right)-n\right)^2$$
$$-\frac{q\left(\frac{(E+M)\mathrm{V}_1}{\sqrt{q}}\right)^2}{\left(\left(\frac{1+\sqrt{1+4\left(\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)+(E+M)\mathrm{V}_2\right)}}{2}\right)-n\right)^2}$$
$$-q\left(l+\frac{D-1}{2}\right)\left(l+\frac{D-3}{2}\right)-q(E+M)\mathrm{V}_2 \tag{45}$$

Then, from Equation (45) the relativistic energy spectra of diatomic molecules will be calculated by using Matlab R2013a software together with the values of parameters of the properties of diatomic molecule from reference [24] that is shown in Table 1.

Table 1. The Properties of Spectroscopic and Mass of Reduce for Diatomic Molecules in Ground State

Parameters	CO	NO	O_2
$D_e(eV)$	10.84514471	8.043782568	5.156658828
$a(\text{Å})$	1.1282	1.1508	1.208
$\mu(amu)$	6.860586000	7.468441000	7.997457504

Table 1 shows the parameters of dissosiative energy D_e, distance of equilibrium internuclear a, and mass of diatomic molecules μ. The parameter values in Table 1 will be used to calculate the relativistic energy spectra of the diatomic molecules. The relativistic energy spectra of diatomic molecule NO that are listed in Tables 2-4 are numerically calculated from Equation (45) by using parameters in Table 1.

The values of relativistic energy for NO molecule in Table 2 are positive. The effect of the q-deformed parameter is observable when the value of q is increased by amount of 0.3, and it causes the increase of energy $\approx$ 0.15 GeV for nr=2 and $\approx$0.3 GeV for nr=3. The effect of the dimension parameter is observable for the value of q-deformed parameter is greater than 0.1 and the increase of the energy spectra is almost the same for every dimension change by one. The increase of the quantum number causes the increase of the values of energy spectra; the greater of the value of the q parameters, the greater the energy increases. But the q-deformed parameter provides the dominant effect to the increase of energy at higher quantum number n and the D-dimensional parameter has the least effect to the energy increase. From Table 2, it can also be predicted that the higher of the quantum number results the higher of the energy spectra. Therefore, the increase of quantum number will be needed for the high energy. The relativistic energy for CO molecule is shown in Table 3.

Table 2. The Relativistic Energy for Modified q-Deformed D-Dimensional Klein-Gordon Equation for Kratzer Potential of Diatomic Molecule for NO (GeV)

n	q	$E_n(NO)$ $D=3$	$E_n(NO)$ $D=4$	$E_n(NO)$ $D=5$	$E_n(NO)$ $D=6$
1	0.01	6.9581	6.9585	6.9589	6.9593
	0.04	6.9625	6.9639	6.9654	6.9668
	0.07	6.9667	6.9693	6.9718	6.9743
	0.1	6.9711	6.9747	6.9783	6.9818
	0.4	7.0139	7.0282	7.0425	7.0566
	0.7	7.0566	7.0814	7.1061	7.1307
	1	7.0990	7.1342	7.1691	7.2039
2	0.01	6.9618	6.9628	6.9639	6.9650
	0.04	6.9768	6.9812	6.9855	6.9897
	0.07	6.9918	6.9994	7.0069	7.0144
	0.1	7.0068	7.0176	7.0283	7.0389
	0.4	7.1552	7.1969	7.2385	7.2798
	0.7	7.3004	7.3720	7.4429	7.5131
	1	7.4428	7.5429	7.6417	7.7393
3	0.01	6.9668	6.9686	6.9704	6.9722
	0.04	6.9969	7.0040	7.0112	7.0183
	0.07	7.0268	7.0393	7.0517	7.0641
	0.1	7.0566	7.0744	7.0920	7.1096
	0.4	7.3482	7.4159	7.4831	7.5496
	0.7	7.6286	7.7425	7.8547	7.9653
	1	7.8992	8.0558	8.2095	8.3604

The relativistic energy for CO molecule is listed in Table 3. From Table 3, it can be seen that the increase of q-deformed parameter, quantum number n, and D-dimensional parameter cause the increase of the energy level. Similar to the molecule NO, the most dominant effect that causes the energy increases higher is the q-deformed parameter. The increase of the energy values that is caused by the q-deformed parameter higher for higher quantum number.

Table 4 shows that the relativistic energy increases by the increase of q-deformed parameter, quantum number n, and D-dimensional parameter. But among these three parameters, the deformed parameter q provides the dominant effect to the increase of the energy levels and the least dominant effect is coming from D-dimensional parameter.

Table 2, 3, and 4 show the relativistic energy for the diatomic molecules NO, CO, and O_2 with same variations of dimension, deformation and quantum number parametrs D, q, and n. It is shown that the increase of relativistic energy values occurs when q-deformed, dimension and quantum number parameters increase. However, the q-deformed parameter provides dominant effect to the increase of realativistic energy. Molecule O_2 has the higher relativistic energy values as it has higher mass value M.

Table 3. The Relativistic Energy for Modified q-Deformed D-Dimensional Klein-Gordon Equation for Kratzer Potential of Diatomic Molecule CO (GeV)

n	q	$E_n(CO)$ $D=3$	$E_n(CO)$ $D=4$	$E_n(CO)$ $D=5$	$E_n(CO)$ $D=6$
1	0.01	6.3920	6.3925	6.3928	6.3933
	0.04	6.3967	6.3983	6.3999	6.4015
	0.07	6.4014	6.4042	6.4069	6.4097
	0.1	6.4061	6.4100	6.4139	6.4178
	0.4	6.4527	6.4683	6.4837	6.4992
	0.7	6.4991	6.5259	6.5527	6.5794
	1	6.5451	6.5832	6.6211	6.6587
2	0.01	6.3960	6.3972	6.3984	6.3996
	0.04	6.4124	6.4171	6.4217	6.4265
	0.07	6.4287	6.4369	6.4451	6.4532
	0.1	6.4451	6.4567	6.4683	6.4799
	0.4	6.6059	6.6512	6.6962	6.7408
	0.7	6.7631	6.8403	6.9166	6.9921
	1	6.9166	7.0242	7.1302	7.2346
3	0.01	6.4015	6.4035	6.4054	6.4074
	0.04	6.4342	6.4420	6.4497	6.4575
	0.07	6.4667	6.4803	6.4938	6.5073
	0.1	6.4992	6.5184	6.5375	6.5566
	0.4	6.8146	6.8876	6.9598	7.0313
	0.7	7.1161	7.2380	7.3579	7.4759
	1	7.4054	7.5729	7.7356	7.8955

The result of relativistic energy values similar to the research conducted by Nugraha [4] and Ikhdair and Sever [42]. But the relativistic energy values have negative value in [4] and positive values in this research. On the other hand, the relativistic values have positive and negative values [42]. In this research, the q-deformed factor that influence the relativistic energy result.

Overall, the relativistic energy value is influenced by the depth of potential. The depth of diatomic particle potential is indicated by distance of equilibrium internuclear. From the Table 1, the increase of distance of equilibrium internuclear r of CO, NO, and O_2 is necessary to be at the low energy. However, the relativistic energy in this research is the increase of energy by the increasing of distance of equilibrium internuclear of CO, NO, and O_2.

Table 4. The Relativistic Energy for Modified q-Deformed D-Dimensional Klein-Gordon Equation for Kratzer Potential of Diatomic Molecule O_2 (GeV)

n	q	$E_n(O_2)$ $D=3$	$E_n(O_2)$ $D=4$	$E_n(O_2)$ $D=5$	$E_n(O_2)$ $D=6$
1	0.01	7.4509	7.4512	7.4516	7.4519
	0.04	7.4549	7.4563	7.4576	7.4589
	0.07	7.4589	7.4613	7.4636	7.4660
	0.1	7.4629	7.4663	7.4697	7.4730
	0.4	7.5031	7.5164	7.5297	7.5429
	0.7	7.5429	7.5661	7.5892	7.6122
	1	7.5826	7.6155	7.6483	7.6809
2	0.01	7.4543	7.4553	7.4563	7.4573
	0.04	7.4683	7.4724	7.4764	7.4804
	0.07	7.4824	7.4894	7.4964	7.5034
	0.1	7.4964	7.5064	7.5164	7.5264
	0.4	7.6352	7.6744	7.7134	7.7522
	0.7	7.7715	7.8387	7.9055	7.9716
	1	7.9054	7.9997	8.0929	8.1851
3	0.01	7.4589	7.4606	7.4623	7.4640
	0.04	7.4871	7.4937	7.5004	7.5071
	0.07	7.5151	7.5267	7.5383	7.5499
	0.1	7.5429	7.5595	7.5761	7.5925
	0.4	7.8164	7.8801	7.9433	8.0060
	0.7	8.0806	8.1882	8.2944	8.3992
	1	8.3364	8.4851	8.6311	8.7747

Furthermore, by inserting equations (45) into (39), we have α and β parameters equations as

$$\alpha^2=\frac{\left(2\mu i+(\upsilon-n)^2-\frac{\mu^2}{(\upsilon-n)^2}\right)}{4} \tag{46}$$

$$\text{and } \beta^2=\frac{\left(-2\mu i+(\upsilon-n)^2-\frac{\mu^2}{(\upsilon-n)^2}\right)}{4} \tag{47}$$

By inserting Equations (40, 41, and 42) and (46 and 47) into Equation (37), the wave function equation for the modified q-deformed D-dimensional Klein-Gordon equation for Kratzer potential given as

$$U(y)=\left(\frac{1+i\,\text{coty}}{2}\right)^{\sqrt{\left(2\mu i+(\upsilon-n)^2-\frac{\mu^2}{(\upsilon-n)^2}\right)}{4}}\times \left(\frac{1-i\,\text{coty}}{2}\right)^{\sqrt{\left(-2\mu i+(\upsilon-n)^2-\frac{\mu^2}{(\upsilon-n)^2}\right)}{4}}\times C'(-1)^n\left(2\alpha+\frac{1}{2}\right)\times {}_2F_1\left(-\text{n},2\alpha+2\beta+n,2\alpha+\frac{1}{2};\left(\frac{1+i\,\text{coty}}{2}\right)\right) \tag{43}$$

with υ and μ parameters are in equations (31) and (32), respectively.

Meanwhile, by using various quantum number, wave function equations are listed in Table 5. The wave function equation depends on the quantum number n = 0, 1, and 2.

Table 5. The Wave Function Equation for modified q-deformed D-Dimensional Klein-Gordon Equation for Kratzer Potential for n = 0, 1, 2 and $\cot y = \sqrt{q}/r$

n	$F_n(x)$
0	$U_0(y)=\left(\frac{1+i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(2\mu i+\upsilon^2-\frac{\mu^2}{\upsilon^2}\right)}/4}\left(\frac{1-i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(-2\mu i+\upsilon^2-\frac{\mu^2}{\upsilon^2}\right)}/4}$
1	$U_1(\mathrm{y})=\left(\frac{1+i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(2\mu i+(\upsilon-1)^2-\frac{\mu^2}{(\upsilon-1)^2}\right)}/4}\left(\frac{1-i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(-2\mu i+(\upsilon-1)^2-\frac{\mu^2}{(\upsilon-1)^2}\right)}/4}$ $C'\left(2\alpha+\frac{1}{2}\right)\left(1-\frac{(2\alpha+2\beta+2)}{\left(2\alpha+\frac{1}{2}\right)}\left(\frac{1+i\,\mathrm{coty}}{2}\right)\right)$
2	$U_2(\mathrm{y})=\left(\frac{1+i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(2\mu i+(\upsilon-3)^2-\frac{\mu^2}{(\upsilon-3)^2}\right)}/4}\left(\frac{1-i\,\mathrm{coty}}{2}\right)^{\sqrt{\left(-2\mu i+(\upsilon-3)^2-\frac{\mu^2}{(\upsilon-3)^2}\right)}/4} C'\left(2\alpha+\frac{1}{2}\right)\left(2\alpha+\frac{3}{2}\right)$ $\left(1-\frac{2(2\alpha+2\beta+3)}{\left(2\alpha+\frac{1}{2}\right)}\left(\frac{1+i\,\mathrm{coty}}{2}\right)+\frac{(2\alpha+2\beta+3)(2\alpha+2\beta+4)}{\left(2\alpha+\frac{1}{2}\right)\left(2\alpha+\frac{3}{2}\right)}\left(\frac{1+i\,\mathrm{coty}}{2}\right)^2\right)$

From Table 5, it can be concluded that the increase of quantum number n causes the increase of the Hypergeometric series as part of the wave functions. Since the energy values increases by increase of the quantum number n, therefore the power of wave function also increases.

Since the energy value is proportional to q value, while α and β have similar to absolute value of their operator, therefore the dominant effect to the wave function is the Hypergeometric series.

It shows the wave function equations for radial quantum number, n = 1, 2, and 3, associated with the radial ground state, first and second excited states. It can be seen that the wave function equation is influenced by μ, υ, α, and β parameters in equations (31), (32), (46), and (47), respectively. The increase of parameters values causes the increase of wave function value.

IV. CONCLUSION

The modified q-deformed D-dimensional Klein-Gordon equation for Kratzer potential is solved by Hypergeometric method. The q-deformed linear momentum that is called as the q-deformed quantum was derived from q-calculus and q-algebra that related to non-extensive statistical physics in the form of Tsallis entropy. The modified q-deformed quantum is the modification form of q-deformed momentum.The relativistic energy and wave function equations are produced from the radial part of q-deformed D-dimensional Klein Gordon equation. The relativistic energy spectra are positive. The relativistic energy values for diatomic particles CO, NO, and O_2 increase by the increase of the values of quantum number, q-deformed, and dimension parameters. The q-deformed parameter has the most dominant effect to the increase of the relativistic energy values.

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