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Research Article

Quantum Relativistic Diesel Engine with Single Massless Fermion in 1 Dimensional Box System

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Abstract

The quantum Diesel of a single fermion in 1D box system has been explored. The Fermion particle meets the Dirac's relativistic Hamiltonian with a chosen mass worth zero. This Relativistic Diesel engine research aims to obtain Diesel engine efficiency that utilizes massless fermion particles as a working substance. This study implements a modified analogy model of the classical analogue model to quantum with the implementation of the first law of thermodynamics for quantum systems so that quantum thermodynamic processes can be defined explicitly. The exploratory results of a single quantum fermion Diesel engine of a single massless system are efficiency formulation that is suitable for the efficiency of a classic Diesel engine, but its heat capacity ratio is unique, that is 2. Based on the value of heat capacity ratio, the efficiency is higher than the classical.

Keywords: *quantum relativistic diesel engine, single massless Fermion system, Dirac equation, modified analogical model*

Mesin Diesel Kuantum Relativistik Sistem Fermion Tunggal Tak Bermassa dalam Kotak 1 Dimensi

Abstrak

Telah dieksplorasi mesin Diesel kuantum sistem fermion tunggal tak bermassa dalam kotak 1D. Partikel Fermion memenuhi Hamiltonian relativistik Dirac dengan massa dipilih bernilai nol. Penelitian mesin Diesel relativistik bertujuan mendapatkan efisiensi mesin Diesel yang memanfaatkan partikel fermion tak bermassa sebagai working substance. Penelitian ini mengimplementasikan model analogi termodifikasi yakni model analogi klasik ke kuantum dengan implementasi hukum pertama termodinamika untuk sistem kuantum sedemikian hingga proses-proses termodinamika dalam kuantum dapat didefinisikan secara eksplisit. Hasil eksplorasi dari mesin Diesel kuantum sistem fermion tunggal tak bermassa adalah formulasi efisiensi yang sesuai dengan efisiensi mesin Diesel klasik, akan tetapi rasio kapasitas panasnya bernilai unik, yakni sebesar 2. Berdasarkan nilai rasio kapasitas panas diperoleh efisiensi yang lebih tinggi dibandingkan dengan klasik.

Kata Kunci: *mesin diesel kuantum relativistik, sistem Fermion tunggal tak bermassa, persamaan Dirac, model analogi termodifikasi*

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I. INTRODUCTION

Heat engine converts heat energy into work, as likely mechanical work, chemistry work, electric work *etc.* Heat energy which is reserved from hot reservoir is not converted into work at all, thus the efficiency is not 100%.

Classically, heat engine can be modeled by a cylinder which is filled amount of ideal gas as a working substance and it undergoes some thermodynamic processes such as isothermal, isobaric, isochoric, adiabatic[1–3]. The processes ARE combined each other so it forms thermodynamic cycles like Carnot cycle, Otto cycle, Diesel cycle, Bryton cycle, Stirling cycle, etc.

Because the efficiency of the heat engine is still low [2,4–6], it necessary to development study with exploit unique properties from quantum system. As a further, studying of quantum thermodynamic is expected to be bridging between quantum theory and classical thermodynamic [5,7,8].

Increasing performance effort has been done by studying working substance of the heat engine, such as non-relativistic particle [2,4,7–9], single-ion [10], multiferroic [11,12], photon [13], fermion [3,14,15]. Because of studying working substance for heat engine there is increasing performance. Exploring of fermion which has anti-symmetry properties as working substance gives information that configuration difference effects heat engine performance [14].

The study of Quantum Carnot Engine also provides conclusion that is similar with the classical one [2,3,7,14,16]. The similarity of Quantum Carnot Engine for all system shows that it satisfies the thermodynamic first law to quantum theory.

Exploring non-Carnot cycle is to be done slightly, however it is necessary to study it especially for high-energy system. High-energy system is shown by Dirac equation as equation of motion for working substance.

This paper presents theoretical research result of quantum Diesel engine that uses single massless fermion trapped in 1D box system that satisfies Dirac equation with zero mass as a working substance. In order to know description of isobaric process with single massless fermion, Diesel engine was chosen. On the other hand, the aim of the research is to obtain formulation of quantum Diesel engine efficiency. Next, the performance of this engine will be compared with classical system and non-relativistic quantum system. In this research, analogical model is chosen with implementation first law of thermodynamic.

II. RESEARCH METHOD

In this research, analogical model which is taking closest analogy between classical thermodynamic system and quantum mechanical system is chosen with implementation of thermodynamic first law for describing quantity changing [4,7].

Boundary condition which is taking for approaching with classical model (piston) is 1D box system (square potential well) which one of the side can shift freely (Figure 1) Meanwhile ideal gas which fill the piston is acted by single massless fermion which satisfy Dirac equation with zero mass and initial condition is ground state.

Due to apply analogical to the system, it is necessary transforming thermodynamic quantity to mechanical quantity. The transformation is shown by Table 1.

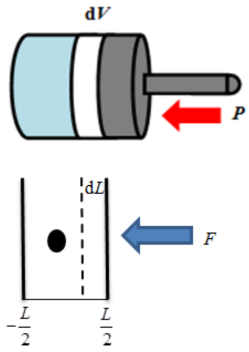

Figure 1. Analogy Between Piston with 1D Box System

Table 1. Quantum System Quantities that Correspondence with Classical Thermodynamic System

Classical Thermodynamic System Quantities	Quantum System Quantities
Pressure (*P*)	Force (*F*)
Volume (*V*)	Width of square (*L*)
Internal Energy (*U*)	Expectation of energy (*E*)
Heat (*Q*)	State probability change
Work (*W*)	Eigen value of energy change

Single Massless Fermion in 1D Box

Single massless fermion's motion equation can be described by Dirac equation with zero mass

$$c(\hat{\alpha}_x \cdot \hat{p})\psi = E\psi, \quad (1)$$

with $\hat{\alpha}$ is Dirac matric,

$\hat{\alpha}_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$, with σ_i is Pauli matrices. By applying boundary condition $j\left(x = \pm\frac{L}{2}\right) = 0$, so solution of the Dirac equation is obtained by Equation (2)

$$\psi = A\begin{pmatrix} \sin\left(\frac{n\pi\hbar c}{L}\left(x - \frac{L}{2}\right)\right) \\ 0 \\ 0 \\ -\mathrm{i}\cos\left(\frac{n\pi\hbar c}{L}\left(x - \frac{L}{2}\right)\right) \end{pmatrix}, \quad (2)$$

which is associated with eigen energy (E_n) of

$$E_n = \frac{n\pi\hbar c}{L}, \quad (3)$$

with n = 1, 2, 3,..., which is quantum number, L is width of system and A is constant.

Quantum thermodynamic process

Because of in previous works [2,7], force describing is not well defined, so in this research we will redefine the force by thermodynamic first law for quantum system which is expressed by Equation (4).

$$\mathrm{d}E = \sum_n E_n \mathrm{d}p_n + \sum_n p_n \mathrm{d}E_n, \quad (4)$$

with dE is expectation of energy changing, dE_n is Eigen value of energy changing , and dp_n is occupational probability changing. In equation (4), the expectation of energy changing represents internal energy changing. The first term and second term of equation (4) represent heat exchange and the work respectively.

Adiabatic Process

In adiabatic process, there is no exchange of heat between system and reservoir. Due to this process, the work (W) which is acted to system effects to change internal energy (U). The first law of thermodynamic is expressed by

$$\mathrm{d}U = -F\mathrm{d}L, \tag{5}$$

with F is force and dL is changing of width of the system.

In quantum, this process is marked by changing of Eigen value of energy (E_n) and the occupational number of probability (p_n) remains constant [17]. Thus based on equation (5) the changing of expectation value of energy is

$$\mathrm{d}E = \sum_n p_n \mathrm{d}E_n \tag{6}$$

Isobaric Process

In along of Isobaric process, the pressure which is acted on the system is remains constant. The internal energy changing is because of heat exchange and work that is acted on the system. That statement can be expressed by

$$\mathrm{d}U = \delta Q + \delta W \tag{7}$$

In quantum system especially for 1 Dimensional case, pressure is represented by force. So in this process the initial force is equal to force along the system [4].

Isochoric Process

In isochoric process, pressure and temperature is changing, but volume of system remains constant. Therefore, no work is in a system and heat exchange is used to changing internal energy of system.

In quantum, heat exchange is shown by changing of occupational number of probability (dp_n), so the quantum representation for expectation value of energy changing (dE) is

$$\mathrm{d}E = \sum_n E_n \mathrm{d}p_n \tag{8}$$

III. RESULTS AND DISCUSSION

The modified analogical method, as stated before, was used into single massless fermion as the working substance. The changes of the energies by using the first law of thermodynamic describe some thermodynamic processes according to the working substance. The quantum Diesel engine efficiency was evaluated in this paper.

Quantum Thermodynamic Processes with Single Massless Fermion in 1 Dimensional Box System

In this point, we explain thermodynamic processes for single massless fermion system. As far as known, the Eigen value of single massless fermion is shown in equation (3), thus the expectation value of energy for this system is

$$\begin{aligned} E &= \sum_n p_n E_n \\ &= \sum_n p_n \frac{n\pi\hbar c}{L}, \end{aligned} \tag{9}$$

with p_n is occupational number of probability.

On other hand, based on equation (5) and (6), to expand or compress the system along to thermodynamic processes, it is necessary the average force $F(L)$ of

$$F(L) = \sum_n p_n n \frac{\pi\hbar c}{L^2} \tag{10}$$

From equation (9) and (10), the energy and force depend on the occupational number of probability.

Adiabatic Process

Consider we have system that is on ground state and the width of system is L_1 and is adiabatic compression process. Due to there is no heat exchange between system and reservoir, the occupational number of probability remains constant ($dp_n = 0$). However, the width of system is decreasing and the Eigen value of energy is definitely increasing.

Based on equation (10), the work that is used to compress the system to L_2 is

$$W_{1\to 2} = \frac{\pi\hbar c}{L_2}\left(\frac{L_2}{L_1} - 1\right) \tag{11}$$

On the other hand, if the system undergoes adiabatic expansion, then it needs a work to change the width of system from L_3 to L_4 of

$$W_{3\to 4} = \frac{\pi\hbar c}{L_2}\left(\frac{L_3}{L_2} - \frac{L_3^2}{L_2 L_1}\right) \tag{12}$$

Isobaric Process

In previous section, we have used analogy of pressure with force, so that the force which is acted to system remains constant along to this process [4]. From equation (4), the total of occupational number is

$$\sum_n p_n n = \left(\frac{L}{L_i}\right)^2, \tag{13}$$

with L is the width of system along the process, L_i is initial width of system along the process. Based on equation (10) and (13), the average force to make system expands to L is

$$F(L) = \frac{\pi\hbar c}{L_i^2} \tag{14}$$

From equation (14), the force just depends on initial width of the system.

Consider the system is expanding from L_2 to L_3, based on equation (14) the work that is acted by the system is

$$W_{2\to 3} = \frac{\pi\hbar c}{L_2}\left(\frac{L_3}{L_2} - 1\right) \tag{15}$$

On the other hand, based on equation (9) and (11) the heat is absorbed by

$$Q_H = \frac{\pi\hbar c}{L_2}\left(2\frac{L_3}{L_2} - 2\right) \tag{16}$$

We have some physical quantities as formulated in Equations (13), (14), (15), (16) and are different with the associate quantities for previous researches with different working substance [4, 9, 16].

Isochoric Process

In the previous section, we have already explained that in isochoric process, the width of system remains constant, so that the changing of internal energy is only contributed from heat exchange. Because of heat exchange, entropy of the system is changing and there is transition of energy level that is presented by changing of occupational number probability. Consider the system that is in energy level E_n is down to ground state, the decreasing of state is followed by releasing the heat of

$$Q_L = \frac{\pi\hbar c}{L_2}\left(\frac{L_2}{L_1} - \left(\frac{L_3}{L_2}\right)^2 \frac{L_2}{L_1}\right), \tag{17}$$

with $L_2 < L_1$ and $L_3 > L_2$.

Quantum Diesel Engine with Single Massless Fermion in 1 Dimensional Box System

Diesel cycle is consisting adiabatic compression, isobaric expansion, adiabatic expansion, and isochoric (Figure 2). Diesel cycle is starting by adiabatic compression that changes width of the system L_1 to be L_2, and the occupational probability remains constant thus the system is still on ground state. After that, the system undergoes isobaric expansion and the width is changing L_2 to be L_3. Due to this process, the system absorbs heat from hot reservoir and the occupational probability increases. After the heat flows to system, the system undergoes adiabatic expansion and the width is changing L_3 to be L_4 but occupational probability remains constant.

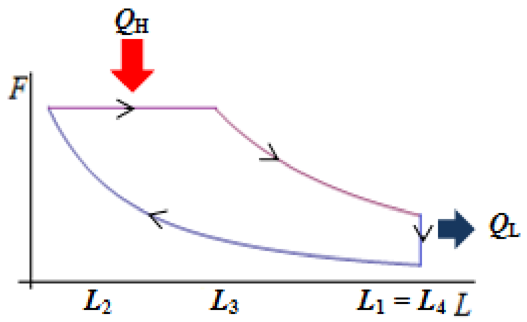

Figure 2. *F-L* Diagram for Quantum Diesel Cycle with Single Massless Fermion System

Based on equation (11), (12), and (15), total work for quantum Diesel cycle with single massless fermion is

$$W_{\text{total}} = \frac{\pi\hbar c}{L_2}\left(2\frac{L_3}{L_2} - 2 - \frac{L_2}{L_1}\left(\left(\frac{L_3}{L_2}\right)^2 - 1\right)\right) \quad (18)$$

Next, we will calculate the efficiency of quantum Diesel engine with single massless fermion in 1D box system. Efficiency is ratio between work in 1 cycle (W_{total}) and heat which is absorbed from hot reservoir (Q_H). Based on equation (16) and (17), the efficiency of this engine is shown in Equation (19).

$$\eta_D^{QR} = 1 - \frac{L_2}{2L_1}\frac{\left(\left(\frac{L_3}{L_2}\right)^2 - 1\right)}{\left(\frac{L_3}{L_2} - 1\right)} \quad (19)$$

This efficiency is correspondence with non-relativistic system [4,16]. This result confirms the previous study of heat engine [3,4,9,14,15,16] that there is correspondence quantum heat engine with classical engine, which the classical Diesel engine efficiency is

$$\eta_D^k = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1}\left(\frac{1}{\gamma}\frac{(V_3/V_2)^\gamma - 1}{(V_3/V_2) - 1}\right) \quad (20)$$

Based on equation (19) and (20), it has been obtained that ratio of heat capacity (γ) for this system is 2. It shows that there is increasing performance of Diesel engine. In previous research, it has obtained that the heat capacity ratio for non-relativistic is 3 [16]. Thus, there is effect of a working substance on a Diesel engine performance. The effect can be shown by the heat capacity ratio (γ), which is a response of material to temperature.

IV. CONCLUSION

The exploration of Quantum Diesel Engine by applying single massless fermion in 1 dimensional box system has been done. The total occupational number along to the isobaric process for single massless fermion system depends on ratio of the system's width square L^2 and initial width of system L_i^2. As a result, there is increasing performance of Diesel engine. The increasing that is presented by heat capacity ratio is 2, which this value is bigger than the classical system one. It shows that working substance effects on the performance of Diesel Engine. However, it is necessary to explore the ensemble of system as a consolidating.

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