

## A New Model to Calculate Stress Relaxation of Viscoelastic Material for Polyester-Wool-Spandex Yarn with Analytical Mechanics Approach

Valentinus Galih Vidia Putra<sup>1,a,\*</sup>, Juliany Ningsih Mohamad<sup>2,b</sup>, and Yusril Yusuf<sup>3,c</sup>

<sup>1</sup>Textile Engineering Department, Politeknik STTT Bandung, Bandung, 40272, Indonesia

<sup>2</sup>Physics Department, Faculty of Mathematics and Natural Sciences, Universitas Nusa Cendana, Kupang, 85228, Indonesia

<sup>3</sup>Physics Department, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Yogyakarta, 55281, Indonesia

e-mail: <sup>a</sup> [valentinus@kemenperin.go.id](mailto:valentinus@kemenperin.go.id), <sup>b</sup> [ning\\_she@staf.undana.ac.id](mailto:ning_she@staf.undana.ac.id), and <sup>c</sup> [yusril@ugm.ac.id](mailto:yusril@ugm.ac.id)

\* Corresponding Author

### Abstract

Many researchers have studied the mechanical properties of yarn in textile science because mechanical properties are the essential parameter in determining yarn quality. This research aims to make a new model and prediction of the material properties of textile yarns, especially for stress relaxation of viscoelastic textile yarn for polyester-wool-spandex yarn cases. In this research, a new approximation of the analytical mechanics model of stress relaxation using a system of four springs and a dashpot to determine viscoelastic yarn properties as polyester-wool-spandex has been studied. A yarn movement equation for viscoelastic yarn as polyester-wool-spandex having 36 yarn count number (in unit tex or g/km) has been formulated using analytical mechanics, and the model has been validated experimentally. The coefficient of determination  $R^2$  ranges from 0.82, which shows the closeness between the experimental results and the theoretical predictions. In this research, it is found that this model can be implemented to determine the viscoelastic material of yarn based on the properties of yarn as stress relaxation using the analytical mechanics approach.

**Keywords:** viscoelastic yarn; analytical mechanics; stress relaxation

## Sebuah Model Baru Perhitungan Stress Relaxation Material Viskoelastik pada Benang Poliester-Wool-Spandex dengan Pendekatan Mekanika Analitik

### Abstrak

Kajian sifat mekanik benang pada bidang tekstil saat ini telah banyak dilakukan oleh banyak peneliti dikarenakan sifat mekanik adalah parameter penting dalam penentuan kualitas benang. Tujuan dari penelitian ini adalah untuk merancang suatu model baru dan suatu prediksi sifat material benang tekstil, khususnya pada kasus pemodelan stress relaxation benang tekstil viskoelastik dengan contoh benang yang digunakan adalah benang polyester-wool-spandex. Sebuah pendekatan baru model stress relaxation menggunakan empat pegas dan sebuah dashpot melalui mekanika analitik untuk menganalisa sifat benang viskoelastik poliester-wool-spandek telah dipelajari pada penelitian ini. Sebuah persamaan gerak benang untuk benang viskoelastik seperti benang poliester-wool-spandek dengan nomor benang 36 tex

atau  $g/km$  telah dirumuskan menggunakan mekanika analitik dan divalidasi secara eksperimen. Hasil koefisien determinasi  $R^2$  berkisar 0,82 yang menunjukkan kedekatan antara hasil eksperimen dan prediksi teoritis. Pada penelitian ini, bentuk pemodelan dapat diterapkan untuk menjelaskan sifat material viskoelastik khususnya material benang yang dikaji dari sifat stress relaxation menggunakan pendekatan mekanika analitik.

**Kata Kunci:** benang viskoelastik; mekanika analitik; stress relaxation

**PACS:** 46.35.+z; 77.84.Jd

© 2021 Jurnal Penelitian Fisika dan Aplikasinya (JPFA). This work is licensed under [CC BY-NC 4.0](https://creativecommons.org/licenses/by-nc/4.0/)

**Article History:** Received: 16 January 2021

Approved with minor revision: 15 May 2021

Accepted: 28 September 2021

Published: 30 December 2021

**How to cite:** Putra VGV, et al. A New Model to Calculate Stress Relaxation of Viscoelastic Material for Polyester-Wool-Spandex Yarn with Analytical Mechanics Approach. *Jurnal Penelitian Fisika dan Aplikasinya (JPFA)*. 2021; **11**(2): 94-105. DOI: <https://doi.org/10.26740/jpfa.v11n2.p94-105>.

## I. INTRODUCTION

Several researchers have widely applied and developed the application of theoretical physics and applied mathematics, especially in applied engineering, such as in textile engineering for modelling of yarn properties [1-4]. Some researchers developed models to determine viscoelastic material properties specially used in textile [2-4]. Putra, Maruto, & Rosyid [5] and Putra, Maruto & Rosyid [6] have modelled the movement of the fibres in a textile yarn formation, especially in the realm of spinning. Putra, Maruto, and Rosyid [5] and Putra, Maruto, and Rosyid [6] have modelled and predicted yarn material properties by influencing the spinning machine input parameters. The mechanical properties of textile yarns are generally modelled as a function of time due to the viscoelastic characteristic, which combines viscous and elastic properties [7-10], namely stress relaxation and creep-behaviour. Chen [4] and Putra et al. [11] stated that the application of physics, especially in the study of textile materials model to determine the mechanical properties of stress relaxation and creep behaviour, is essential to find out a characteristic used to determine the properties of a textile viscoelastic yarn material. Pocienė,

R., Vitkauskas [12], Kothari, Rajkhowa, Gupta [13], Chen [4], and Putra et al. [11] have explained that the Maxwell and Voigt-Kelvin model are the examples of some models consisting of a single spring and one dashpot arranged in series or a parallel arrangement. The two models has the inaccuracy between the predicted results (model) of the validation of the experiment to explain the general characteristic of the viscoelastic material, especially in the case of stress relaxation. Textile polymer materials usually exhibit a viscoelastic property by giving the same stress level for a specific time. Viscoelastic linear properties are usually modelled using a physics model that is a spring that follows Hook's formula from a material with elastic deformation and a dashpot that follows Newton's fluid formulation with stress proportional to strain [14, 15, 11, 4]. Maxwell's model is a viscoelastic model involving one spring and one dashpot arranged in series and has viscoelastic equation form (Equation (1) and Equation (2)) [4].

$$\frac{d\varepsilon}{dt} = \frac{1}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta} \quad (1)$$

Where the terms  $E$  and  $\eta$  are the elastic constant and viscous constant,  $\varepsilon$  is the yarn strain, and  $\sigma$  is the yarn stress. Stress relaxation,  $\sigma$ , can be formulated as Equation (2) [4, 11]

$$\sigma = \sigma_0 e^{-\frac{E}{\eta}t} \quad (2)$$

The Voigt-Kelvin model is a viscoelastic model involving one spring and one dashpot arranged in parallel and has viscoelastic equation form (Equation (3) and Equation (4)) [4, 11]

$$\sigma = E\varepsilon_e + \eta \frac{d\varepsilon_\eta}{dt} \quad (3)$$

$$\sigma = E\varepsilon \quad (4)$$

Chen [8] and Putra et al. [12] stated that the equation model of Maxwell and Voigt-Kelvin in Equation (2) and Equation (4) could not explain stress relaxation conditions under constant strain conditions because the equations were not experimentally appropriate and had the low coefficient of determination  $R^2$ . A better model was made to show a match between the theoretical model and experiments, referring to several studies from several researchers [1-4, 16, 17] regarding the Maxwell and Voigt-Kelvin models' weaknesses. However, it is necessary to have a new model to show the material properties of textile yarns, especially an approach of analytic mechanics with a stress relaxation model of viscoelastic textile yarn. In this study, we made the model with the examples of polyester-wool-spandex yarn type and yarn number 36 (tex) or 36 (g/ km) as yarn material examined in this study. Some researchers have reported that the modelling of yarn can be formulated by analytical mechanics and simulated using computer programs [18, 19, 20, 21, 22, 23, 24, 25]. Therefore, this research aims to make a better model of the material properties of textile yarns, especially for the stress relaxation model, using an example of polyester-wool-spandex yarn with an analytical mechanics

approach [26-34]. In this model, the research limitation is that it does not involve the presence of stress and strain tensors studied from the point of view of the geometric structure of the yarn and polymer molecular dynamics. The novelty of this research is a new approximation of the analytical mechanics model of stress relaxation using a system of four springs and a dashpot to determine viscoelastic yarn properties as polyester-wool-spandex. A yarn movement equation for viscoelastic yarn as polyester-wool-spandex having 36 of yarn count number (in unit tex or g/km) has been formulated using analytical mechanics and the model has been validated experimentally. The coefficient of determination  $R^2$  ranges from 0.82, which shows the closeness between the experimental results and the theoretical predictions.

## II. METHOD

### Viscoelastic model with two springs and one dashpot arranged in series and parallel

This section explains a model using two springs and one dashpot arranged in series and parallel. It can be formulated as a model with two springs with some constant values and a dashpot with a specific viscous constant value, all of which are arranged in series as in Figure 1. The model has a simple model where stresses have the same value for the whole model, while the strain is the sum of three elements.

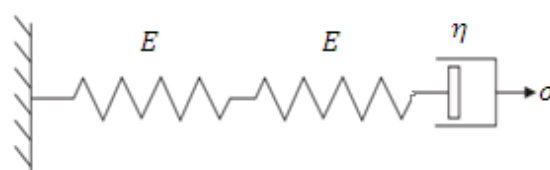


Figure 1. Two Springs and One Dashpot Model Arranged in Series

Chen [4] and Putra et al. [11] stated that in the series model, the magnitude of the force is constant, resulting in constant stress

accompanied by changes in the strain as Equation (5) and Equation (6):

$$\sum \frac{d\varepsilon}{dt} = 0 \tag{5}$$

$$\frac{d\varepsilon_s}{dt} = \frac{1}{E} \frac{d\sigma_e}{dt} + \frac{1}{E} \frac{d\sigma_e}{dt} + \frac{\sigma_\eta}{\eta} \tag{6}$$

Because of stress  $\sigma_e = \sigma_\eta = \sigma$ , hence we get Equation (7)

$$\frac{d\varepsilon}{dt} = \frac{2}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{7}$$

For stress relaxation conditions, the strain is constant, hence Equation (8) and Equation (9) are obtained

$$0 = \frac{2}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta} \tag{8}$$

$$\frac{2}{E} \frac{d\sigma}{dt} = -\frac{\sigma}{\eta} \tag{9}$$

by integrating Equation (9), we get Equation (10) and Equation (11)

$$\frac{d\sigma}{\sigma} = -\frac{E}{2\eta} dt \tag{10}$$

$$\int \frac{d\sigma}{\sigma} = -\int \frac{E}{2\eta} dt \tag{11}$$

By doing the derivation of Equation (11) it will yield Equation (12) and Equation (13)

$$\sigma = \sigma_o e^{-\frac{E}{2\eta}t} \tag{12}$$

$$\begin{aligned} \sigma &= \sigma_o e^{-\frac{E}{2\eta}t} \\ &= \sigma_o e^{-at} \end{aligned} \tag{13}$$

Chen [4] and Putra et al. [11] explained that for the condition of creep behaviour, the stress is constant so that the Equation (14) to Equation (16) is obtained

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} \tag{14}$$

$$\varepsilon = \frac{\sigma}{\eta} t \tag{15}$$

$$\varepsilon = \frac{\sigma}{\eta} t \tag{16}$$

According to Chen [8] and Putra et al. [12] the results of Maxwell's model in Equation (9) and Equation (12) show poor results in describing the condition of viscoelastic yarn material properties for stress relaxation and creep behaviour cases. This research developed a model of two springs and one dashpot arranged in parallel to improve the accuracy of the Maxwell and Voigt-Kelvin modelling. The model shows that adding the number of springs in a series arrangement gives an equation similar to the Maxwell model, results in equation (2). Still, adding the number of springs will only reduce the elasticity. In this research, we have also shown that in the case of two springs and one dashpot model, which can be described through the springs and dashpot arrangement in a parallel circuit (Figure 2) and assuming that the strain is constant, hence we get Equation (17) and Equation (18):

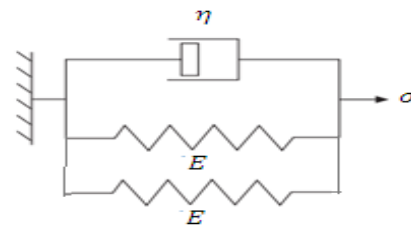


Figure 2. Two Springs and One Dashpot Model Arranged in Parallel

$$\sum \sigma = 0 \tag{17}$$

$$\begin{aligned} \sigma &= E\varepsilon_e + E\varepsilon_e + \eta \frac{d\varepsilon_\eta}{dt} \\ &= 2E\varepsilon_e + \eta \frac{d\varepsilon_\eta}{dt} \end{aligned} \tag{18}$$

In the case of stress relaxation (constant strain), Equation (19) is obtained below.

$$\sigma = 2E\varepsilon \tag{19}$$

The value of stress  $\sigma$  over time will be constant. For the case of creep behaviour, then the stress is constant so that we will get

Equation (20) and Equation (21)

$$\sigma - 2E\varepsilon = \eta \frac{d\varepsilon}{dt} \quad (20)$$

$$\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta} \left( 1 - \frac{2E}{\sigma} \varepsilon \right) \quad (21)$$

The results of solving Equation (21) above will turn out to Equation (22).

$$\frac{d\varepsilon}{\left( 1 - \frac{2E}{\sigma} \varepsilon \right)} = \frac{\sigma}{\eta} dt \quad (22)$$

By assuming that  $1 - \frac{2E}{\sigma} \varepsilon = u$ , hence  $du = -\frac{2E}{\sigma} d\varepsilon$ , therefore we get Equation (23) to

Equation (28)

$$-\frac{\sigma}{2E} \frac{du}{u} = \frac{\sigma}{\eta} dt \quad (23)$$

$$\int \frac{du}{u} = - \int \frac{2E}{\eta} dt \quad (24)$$

$$u = u_0 e^{-\frac{2E}{\eta} t} \quad (25)$$

$$1 - \frac{2E}{\sigma} \varepsilon = e^{-\frac{2E}{\eta} t} \quad (26)$$

$$\sigma - 2E\varepsilon = \sigma e^{-\frac{2E}{\eta} t} \quad (27)$$

$$\varepsilon(t) = \frac{\sigma}{2E} \left( 1 - e^{-\frac{2E}{\eta} t} \right) \quad (28)$$

The model shows that adding the number of springs in a parallel arrangement will give an equation similar to the Voigt-Kelvin model results in equation (4), but adding the number of springs will only increase the elasticity value.

### III. RESULTS AND DISCUSSION

#### Viscoelastic model using combination of four springs and one dashpot arranged in series and parallel

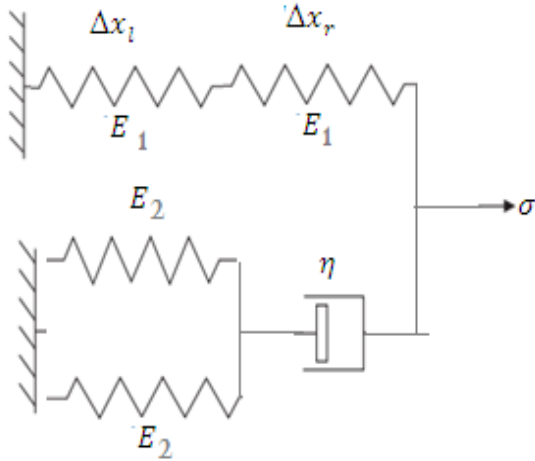
Due to the weakness of the previous models (Eq. (16) and Eq. (28)), where neither the series nor the parallel arrangement has been able to show good accuracy in predicting the viscoelastic properties of the material, hence we developed a new model. This new model has developed a series circuit and a parallel circuit with an arrangement of four springs and one dashpot. To validate the model, then we also conduct experiments. The experiments were carried out in the Yarn Evaluation Laboratory Polytechnic STTT Bandung, Ministry of Industry of the Republic of Indonesia, West Java, Indonesia. The material used in this experiment was polyester-wool-spandex yarn, whose yarn number was 36 (tex) or 36 (g / km), purchased from the traditional market in Bandung, Indonesia. The equipment used for this experiment was stress relaxation tester equipment, Yarn Evaluation Laboratory Polytechnic STTT Bandung, Ministry of Industry of the Republic of Indonesia, West Java, Indonesia as in Figure 3.



Figure 3. Stress Relaxation Tester

In this model, the polyester-wool-spandex viscoelastic material and yarn

number 36 (tex) are modelled using a system of four springs and one dashpot arranged as in Figure 4.



**Figure 4. Form of Analytical Mechanical Modelling of Viscoelastic Polyester-Wool-Spandex Yarn Material with yarn number 36 (tex)**

In this model, the constant spring values are  $E_1$  and  $E_2$ , while the viscous coefficient is  $\eta$ . The explanation of this polyester-wool-spandex yarn system and yarn number 36 (tex) can be completed using Newton's third law, namely by reviewing the relationship of each spring as Equation (29) to Equation (32)

$$\sum F = 0 \quad (29)$$

$$F_{ext} - F_{spring} = 0 \quad (30)$$

$$F - k\Delta x_r = 0 \quad (31)$$

$$F = k\Delta x_r \quad (32)$$

Looking at the connection point of two  $E_1$  springs, we get Equation (33) to Equation (37)

$$\sum F = 0 \quad (33)$$

$$F_r - F_l = 0 \quad (34)$$

$$F_r = F_l \quad (35)$$

$$E\Delta x_r = E\Delta x_l \quad (36)$$

$$\Delta x_r = \Delta x_l \quad (37)$$

Using Equation (37), the total shift of the entire series will yield Equation (38)

$$\Delta x = \Delta x_r + \Delta x_l = 2\Delta x_l \quad (38)$$

To find the combined effective spring coefficient, we get Equation (39) to Equation (42).

$$\sum F = 0 \quad (39)$$

$$F_{ext} - F_{total} = 0 \quad (40)$$

$$F = F_{total} \quad (41)$$

$$F = E_{eff}\Delta x \quad (42)$$

When substituting  $F = k\Delta x_l$  to  $F = k_{eff}\Delta x$ , we get Equation (43) to Equation (45).

$$E\Delta x_l = E_{eff}\Delta x \quad (43)$$

$$E_{eff} = \frac{E\Delta x_l}{\Delta x} = \frac{E\Delta x_l}{2\Delta x_l} = \frac{E}{2} \quad (44)$$

$$\begin{aligned} &= \frac{E^2}{2E} \\ \frac{1}{E_{eff}} &= \frac{2E}{E^2} = \frac{1}{E} + \frac{1}{E} \end{aligned} \quad (45)$$

Thus, we get a general formula for a spring arranged in series, as in Equation (46).

$$\frac{1}{E_{eff}} = \frac{1}{E_1} + \frac{1}{E_1} = \frac{1}{E} + \frac{1}{E} = \frac{2}{E} \quad (46)$$

As for the parallel spring arrangement  $E_2$ , we get  $E_2^* = 2E$  and for a case when  $E_1 = E_2 = E$ . The application of polyester-wool-spandex modelling and yarn number 36 (tex) and the prediction of the study of viscoelastic properties in the textile field is a study to find out the viscoelastic material properties of fibre, yarn or fabric, for example, in a viz. yarn. Polyester-wool-spandex has a yarn number of 36 tex or 36 g / km. The results of experimental data for the process of stress relaxation (constant strain) of polyester-wool-spandex yarn and yarn number 36 (tex) are as in Table 1.



**Table 1. Stress Relaxation Polyester-Wool-Spandex Yarn with Yarn Number 36 (tex)**

No	Time (10 <sup>3</sup> s)	stress (cN/ tex)
1	0	6.1
2	0.5	5.3
3	1	5.0
4	1.5	5.0
5	2.0	4.9
6	2.5	4.9
7	3.0	4.8
8	3.5	4.8

The derivation of analytical equations of polyester-wool-spandex, which has yarn number 36 (tex) for the series circuit in Figure 4, can be completed first. Due to the series arrangement, the stress value is constant, so  $\sigma_e = \sigma_v = \sigma_2$  and  $\varepsilon_s = \varepsilon_e + \varepsilon_v$  and we get Equation (47) to Equation (50):

$$\sum \left( \frac{d\varepsilon}{dt} \right) = 0 \tag{47}$$

$$\frac{d\varepsilon_s}{dt} = \frac{d\varepsilon_e}{dt} + \frac{d\varepsilon_v}{dt} \tag{48}$$

$$\begin{aligned} \frac{d\varepsilon_s}{dt} &= \frac{1}{E_2^*} \frac{d\sigma_e}{dt} + \frac{\sigma_v}{\eta} \\ &= \frac{1}{E_2^*} \frac{d\sigma_2}{dt} + \frac{\sigma_2}{\eta} \end{aligned} \tag{49}$$

$$\begin{aligned} \frac{d\sigma_2}{dt} &= E_2^* \frac{d\varepsilon_s}{dt} - E_2^* \frac{\sigma_2}{\eta} \end{aligned} \tag{50}$$

For parallel circuits, the strain value is the same, so  $\varepsilon_s = \varepsilon_1 = \varepsilon$  hence we get Equation (51) to Equation (52):

$$\sum \sigma = 0 \tag{51}$$

$$\sigma = \sigma_1 + \sigma_s \tag{52}$$

If it is lowered once with time, it is obtained Equation (53) to Equation (61):

$$\sum \frac{d\sigma}{dt} = 0 \tag{53}$$

$$\frac{d\sigma}{dt} = \frac{d\sigma_1}{dt} + \frac{d\sigma_s}{dt} \tag{54}$$

$$\begin{aligned} \frac{d\sigma}{dt} &= E_{eff} \frac{d\varepsilon_1}{dt} + E_2^* \frac{d\varepsilon_s}{dt} \\ &\quad - E_2^* \frac{\sigma_2}{\eta} \end{aligned} \tag{55}$$

$$\frac{d\sigma}{dt} = (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} - E_2^* \frac{\sigma_2}{\eta} \tag{56}$$

$$\begin{aligned} \frac{d\sigma}{dt} &= (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} \\ &\quad - E_2^* \frac{(\sigma - \sigma_1)}{\eta} \end{aligned} \tag{57}$$

$$\begin{aligned} \frac{d\sigma}{dt} &= (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} - \frac{E_2^*}{\eta} \sigma \\ &\quad + \frac{E_2^*}{\eta} \sigma_1 \\ &= (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} \\ &\quad - \frac{E_2^*}{\eta} \sigma \\ &\quad + \frac{E_2^*}{\eta} E_{eff} \varepsilon \end{aligned} \tag{58}$$

$$\begin{aligned} \frac{d\sigma}{dt} &= (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} - \frac{E_2^*}{\eta} \sigma \\ &\quad + \frac{E_2^*}{\eta} E_{eff} \varepsilon \end{aligned} \tag{59}$$

$$\begin{aligned} \frac{d\sigma}{dt} + \frac{E_2^*}{\eta} \sigma &= (E_{eff} + E_2^*) \frac{d\varepsilon}{dt} \\ &\quad + \frac{E_2^* E_{eff}}{\eta} \varepsilon \end{aligned} \tag{60}$$

$$\begin{aligned} \frac{\eta}{E_2^*} \frac{d\sigma}{dt} + \sigma &= \frac{(E_{eff} + E_2^*) \eta}{E_2^*} \frac{d\varepsilon}{dt} \\ &\quad + E_{eff} \varepsilon \end{aligned} \tag{61}$$

For  $E_{eff}$  and  $E_2^*$  are equal, we get Equation (62) and Equation (63)

$$\frac{\eta}{E} \frac{d\sigma}{dt} + \sigma = 2\eta \frac{d\varepsilon}{dt} + E\varepsilon \tag{62}$$

$$\frac{\eta}{E} \frac{d\sigma}{dt} + \sigma = E\varepsilon + 2\eta \frac{d\varepsilon}{dt} \tag{63}$$

The results of the completion for stress relaxation conditions or fixed strains are in accordance with Equation (64).

$$\frac{\eta}{E} \frac{d\sigma}{dt} + \sigma = E\varepsilon \tag{64}$$

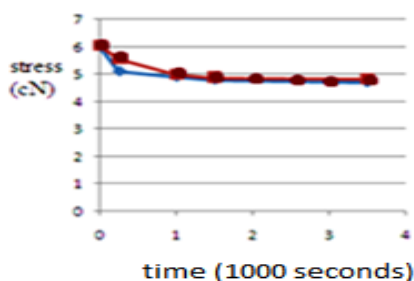
With the value of stress can be seen in

Equation (65) and Equation (66).

$$\frac{d\sigma}{dt} + \frac{E}{\eta}\sigma = \frac{E^2}{\eta}\varepsilon \quad (65)$$

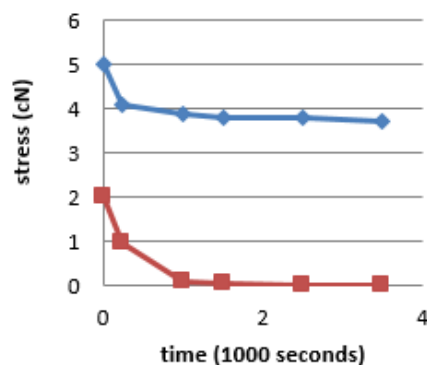
$$\sigma = \sigma_0 e^{-\frac{E}{\eta}t} + E\varepsilon \quad (66)$$

Using the data fitting method, we can develop a polyester-wool-spandex curve with yarn number 36 (tex) as in Equation (62) presented in Figure 5.



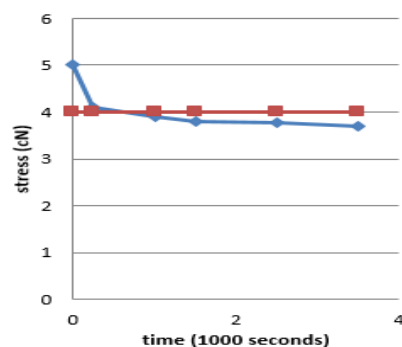
**Figure 5. Stress Relaxation Condition Equation (62) Stress Curve (cN) to Time in Seconds, (Red Line for Experimental, Blue Line for Theory)**

The application of theoretical physics and applied mathematics, especially in applied engineering, such as in textile engineering, has been widely used and carried out by several researchers [1-16]. Putra, Maruto & Rosyid [5] and Putra, Maruto & Rosyid [6] have modelled the movement of the fibres in a textile yarn formation, especially in the spinning of yarn. Chen [8] and Putra et al. [12] stated that the equation model of Maxwell and Voigt-Kelvin in Equation (2) and Equation (4) could not explain stress relaxation conditions under constant strain conditions because the equations were not experimentally appropriate and had the low coefficient of determination  $R^2$ . According to some researchers [2-4], the results of one spring and one dashpot arranged in the series model in Equation (2) showed poor results in describing the condition of viscoelastic yarn material properties for stress relaxation cases, which can be described as in Figure 6.



**Figure 6. Stress Relaxation Condition Equation (62) Stress Curve (cN) to Time (in seconds) for Two Springs and One Dashpot Model (Red Line for Experimental, Blue Line for Theory)**

The model of two springs and one dashpot arranged in the parallel model in Equation (19) cannot explain stress relaxation conditions under constant strain conditions, as described in Figure 7.



**Figure 7. Stress Relaxation Condition Equation (62) Stress Curve (cN) to Time (in seconds) for Two Springs and One Dashpot Arranged in the Parallel Model (Red Line for Experimental, Blue Line for Theory)**

Based on the previous model and some research [4, 11, 13, 10], we developed a better model to show a match between a theoretical model with experiments. In this study, viscoelastic linear mechanical properties of polyester-wool-spandex yarn and yarn number 36 (tex) are usually modelled using a physics model that is a spring that follows the Hook's formula from a material that has



elastic deformation and also a dashpot that follows the formula Newtonian mechanics. Based on the results of this study, it was found that the viscoelastic model form of polyester-wool-spandex yarn and yarn number 36 (tex) using Equation (66) can be used to predict the stress relaxation conditions of a model and predict the study of viscoelastic properties of polyester-wool-spandex yarn and yarn number 36 (tex). In this study, the results of the coefficient of determination  $R^2$  range from 0.82, which shows the closeness between the experimental results and the theoretical predictions. The nature of stress relaxation is an important property to explain the nature of a material such as viscoelastic yarn. The Maxwell model and the Voigt-Kelvin model, which consists of a single spring and a dashpot arranged in series or parallel, show results in less representative of experimental data. The understanding of the two models is the inaccuracy between the predicted results (model) against the validation of the experiment. In this study, it has been studied that viz. yarn. Polyester-wool-spandex, which has a yarn number of 36 tex or 36 g/km, is an example of viscoelastic material in textile material. The equation of the viscoelastic model for making the curve in Figure 5 can be written as Equation (67) and Equation (68):

$$\frac{\eta}{E_2^*} \frac{d\sigma}{dt} + \sigma = \frac{(E_{eff} + E_2^*)\eta}{E_2^*} \frac{d\varepsilon}{dt} \quad (67.a)$$

$$+ E_{eff}\varepsilon \quad (67.b)$$

$$\frac{\eta}{2E} \frac{d\sigma}{dt} + \sigma = 1.25\eta \frac{d\varepsilon}{dt} + \frac{E}{2}\varepsilon$$

$$\sigma = \sigma_0 e^{-\frac{E}{\eta}t} + E\varepsilon \quad (68)$$

with a condition  $E_{eff} = \frac{E}{2}$ ,  $E_2^* = 2E$  and for a case when  $E_1 = E_2 = E$  and for  $E_{eff}$  and  $E_2^*$  are equal. Based on the model results and validation of experiments, it was found that  $\frac{E}{\eta} = 2$  with  $E = 2\eta$ . Equation (68) shows that the time stress for a constant force will have

an inverse relationship with the decaying exponential curve of the viscoelastic material of polyester-wool-spandex yarn and yarn number 36 (tex). Table 2 displays some equations of viscoelastic models from some researchers.

**Table 2. Some Kinds of Viscoelastic Models**

Name of Authors	Equations of model
Maxwell [3]	$\frac{d\varepsilon}{dt} = \frac{1}{E} \frac{d\sigma}{dt} + \frac{\sigma}{\eta}$
Voigt [3]	$\sigma = E\varepsilon_e + \eta \frac{d\varepsilon_\eta}{dt}$
Rosyidan, et al [9]	$\frac{d\sigma}{dt} + \left[ \frac{E_2 E_1}{E_1 + E_2} \right] \frac{\sigma}{\eta_1}$ $= \left[ \frac{E_2 E_1}{E_1 + E_2} \right] \frac{d\varepsilon}{dt}$ $+ \left[ \frac{E_2 E_1}{E_1 + E_2} \right] \frac{E_3 \varepsilon}{\eta_1}$ $+ \frac{1}{E_3} \frac{d\varepsilon_3}{dt}$
Ghosh et. al[31]	$\frac{d\sigma}{dt} = \frac{d\varepsilon}{dt} (E_1 + E_2) - E_1 A \sinh(\sigma - E_2 \varepsilon) \alpha$
Hanief [34]	$\frac{2\eta}{E_2} \frac{d\sigma}{dt} + \sigma = \frac{(E_1 + E_2)\eta}{E_2} \frac{d\varepsilon}{dt} + E_1 \varepsilon$
Putra [10]	$\frac{\eta}{E} \frac{d\sigma}{dt} + \sigma = 2\eta \frac{d\varepsilon}{dt} + E\varepsilon$

Textile yarn, such as polyester-wool-spandex viscoelastic yarn with yarn number 36 (tex), has elastic properties or viscous properties and a combination of the two properties. For example, if a pull or constant stress is applied to yarn, there will be a short strain, and then a slow strain will occur, as in the description of the polyester-wool-spandex viscoelastic yarn material with yarn number 36 (tex). In this study, polyester-wool-spandex yarn properties, which have viscous and elastic properties, are called viscoelastic material and have been determined by Equation (67) and Equation (68). The coefficient of determination  $R^2$  range from 0.82, which shows the closeness between the

experimental results and the theoretical predictions as well as a better model than some previous models [8], [12], [28]. However, the research limitation of this model is that it does not involve the presence of stress and strain tensors studied from the point of view of the geometric structure of the yarn and polymer molecular dynamics resulting in the coefficient  $R^2$ . It has a yield of not more than 0.9. The implication of this research in materials science, especially in textile science, is that it is possible to test the viscoelastic and elastic properties of solid materials, particularly yarn. Considering its benefits or significance for the development of science and technology, this research has developed a model that can predict the material properties of viscoelastic yarn more accurately to be used by researchers in textile science in designing a textile product that requires good viscoelastic and mechanical properties.

#### IV. CONCLUSION

This research has discussed some models of viscoelastic material in polyester-wool-spandex yarn. Based on the results of this study, a concordance was obtained between theoretical prediction results and experimental validation with a better magnitude of  $R^2$ . The potential of the findings from this research for future research is to obtain a more detailed and accurate mathematical model formulation in terms of geometric mechanics and molecular structure of polymer yarns.

#### ACKNOWLEDGMENT

The author would like to thank the Department of Textile Engineering Polytechnic STTT Bandung and the Physics Laboratory of Universitas Nusa Cendana and Universitas Gadjah Mada for supporting this research activity so that this research can be completed.

#### REFERENCES

- [1] Chailleux E and Davies P. Modelling the Non-Linear Viscoelastic and Viscoplastic Behaviour of Aramid Fibre Yarns. *Mechanics of Time-Dependent Materials*. 2003; **7**(3): 291-303. DOI: <http://dx.doi.org/10.1023/B:MTDM.0000007199.01142.f8>.
- [2] Chailleux E and Davies P. Non-Linear Viscoelastic: Viscoplastic Model for the Behaviour of Polyester Fibres, *Mechanics of Time-Dependent Materials*. 2005; **9**: 147-160. DOI: <https://doi.org/10.1007/s11043-005-1082-0>.
- [3] Chen X. *Modeling and Predicting Textile Behaviour*, New York: CRC Press; 2010.
- [4] Putra VGV, Maruto G, and Rosyid MF. New theoretical modeling for predicting yarn angle on OE yarn influenced by fibre movement on torus coordinate based on classical mechanics approach. *Indian Journal of Fibre and Textile Research*. 2017; **42**(3): 359-363. Available from: <http://op.niscair.res.in/index.php/IJFTR/article/view/8685>.
- [5] Putra VGV, Rosyid MF, and Maruto G. A Simulation Model of Twist Influenced by Fibre Movement Inside Yarn on Solenoid Coordinate. *Global Journal of Pure and Applied Mathematics*. 2016; **12**(1): 405-412. Available from: [https://www.ripublication.com/gjpam16/gjpamv12n1\\_37.pdf](https://www.ripublication.com/gjpam16/gjpamv12n1_37.pdf).
- [6] Liu H, Tao XM, Choi KF, and Xu BG. Analysis of the Relaxation Modulus of Spun Yarns. *Textile Research Journal*. 2010; **80**(5): 403-410. DOI: <https://doi.org/10.1177/0040517509342315>.
- [7] Van Miltenburg JGM. Stress Relaxation and Tensile Modulus of Polymeric Fibers. *Textile Research Journal*. 1991; **61**: 363-369. DOI: <https://doi.org/10.1177/004051759106100608>.
- [8] Ward L and Sweeney J. *Mechanical Properties of Solid Polymers*. 2nd edition.

- New York: John Wiley; 1983.
- [9] Rosyidan C, Maulani M, Samura L, and Putra VGV. Modification of the Maxwell model for calculation of stress relaxation and creep behavior for polyester yarns. *Journal of Physics: Conference Series*. 2019; **1402**(6): 066018. DOI: <https://doi.org/10.1088/1742-6596/1402/6/066018>.
- [10] Putra VGV, Wijayono A, Irwan, Martina T, and Rosyidan C. Suatu Studi Pemodelan Viskoelastik Material Benang Viz 100% Wool 36 Tex (G/Km) Menggunakan Pendekatan Fisika. *Wahana Fisika*. 2019; **4**(1): 27-34. DOI: <https://doi.org/10.17509/wafi.v4i1.15500>.
- [11] Pocienė R and Vitkauskas A. Inverse Stress Relaxation and Viscoelastic Recovery of Multifilament Textile Yarns in Different Test Cycles. *Materials Science*. 2005; **11**(1): 68-72. Available from: <https://matsc.ktu.lt/index.php/MatSc/article/view/26490>.
- [12] Kothari VK, Rajkhowa R, and Gupta VB. Stress Relaxation and Inverse Stress Relaxation in Silk Fibers. *Journal of Applied Polymer Science*. 2001; **82**(5): 1147-1154. DOI: <https://doi.org/10.1002/app.1949>.
- [13] Hearle JWS, Gupta BS, and Merchant VB. Migration of Fibres in Yarns Part I: Characterization and Idealization of Migration Behaviour. *Textile Research Journal*. 1965; **35**(4): 329-334. DOI: <https://doi.org/10.1177/004051756503500407>.
- [14] Hearle JWS and Gupta BS. Migration of Fibres in Yarns Part III: A Study of Migration of Staple Rayon Yarn. *Textile Research Journal*. 1965; **35**(9): 788-795. DOI: <https://doi.org/10.1177/004051756503500902>.
- [15] Hearle JG, Grosberg P, and Backer S. *Structural Mechanics of Fibres, Yarns and Fabrics*. New York: Wiley-Interscience; 1969.
- [16] Asayesh A, Jeddi AAA, and Ghadimi P. Modeling the fatigue behavior of plain woven fabrics constructed from textured polyester yarn. *Textile Research Journal*. 2009; **79**(13): 1213-1222. DOI: <https://doi.org/10.1177/0040517508099393>.
- [17] Gao XP, Sun YZ, Meng Z, and Sun ZJ. On the transversal vibration of pile-yarn with time-dependent tension in tufting process. *International Conference on Applied Mechanics and Mechanical Engineering*. 2010; **29-32**: 1517-1523. DOI: <https://doi.org/10.4028/www.scientific.net/AMM.29-32.1517>.
- [18] Serwatka A, Bruniaux P, and Frydrych I. Modeling the Stress-strain Curve of Textile Products. Part 2 – Simulation of Real Stress-strain Curves. *Fibre and Textiles in Eastern Europe*. 2007; **15**(3): 60–62. Available from: <http://fibtext.lodz.pl/article447.html>.
- [19] Hernawati R and Putra VF. Predicting the Actual Strength of Open-End Spun Yarn Using Mechanical Model. *Applied Mechanics and Materials*. 2015; **780**: 69-74. DOI: <https://doi.org/10.4028/www.scientific.net/AMM.780.69>.
- [20] Demidov AV, Makarov AG, and Stalevich AM. A Version of Modeling of Nonlinear Hereditary Viscoelasticity of Polymer Materials. *Mechanics of Solids*. 2009; **44**(1): 122-130. DOI: <https://doi.org/10.3103/S0025654409010130>.
- [21] Zhang WM. Practical expressions of relaxation modulus and creep compliance. *Natural Science Journal of Xiangtan University*. 1999; **21**(3): 26–28.
- [22] Shi F. Modeling Stretching-Relaxation Properties. *FIBRES & TEXTILES in Eastern Europe*. 2013; **21**(2): 51-55. Available from: <http://fibtext.lodz.pl/numerhtml/dep/article905.html>.
- [23] Putra VGV and Rohmah S. A New Model of Open End Yarn Twist Using Torus Coordinate Based on Dynamical Mechanics.

- Journal of Physics: Conference Series*. 2019; **1381**(1): 012051. DOI: <https://doi.org/10.1088/1742-6596/1381/1/012051>.
- [24] Mandhyana PK, Nachane RP, Banerjee S, Pawar BR, and Koli HS. Nonlinear Maxwell modelling of inverse relaxation in yarns and fabrics. *Indian Journal of Fibre & Textile Research*. 2017; **42**(1): 168-174. Available from: [http://nopr.niscair.res.in/bitstream/123456789/42066/1/IJFTR%2042\(2\)%20168-174.pdf](http://nopr.niscair.res.in/bitstream/123456789/42066/1/IJFTR%2042(2)%20168-174.pdf)
- [25] Li Y and Wu M. Uniaxial Creep Property and Viscoelastic Plastic Modelling of Ethylene Tetrafluoroethylene (ETFE) Foil. *Mechanics of Time-Dependent Materials*. 2015; **19**(1): 21–34. DOI: <https://doi.org/10.1007/s11043-014-9248-2>.
- [26] Nachane RP and Sundaram V. Analysis or Relaxation Phenomena in Textile Fibres Part II: Inverse Relaxation. *The Journal of the Textile Institute*. 1995; **86**(1): 20-32. DOI: <https://doi.org/10.1080/00405009508631307>.
- [27] Nakashima T and Sakai Y. Visco-Elastic Behavior of a Filament Which Undergoes Impulsive Tension at its End. *Journal of the Textile Machinery Society of Japan*. 1975; **28**(1): 1-6. Available from: [https://www.jstage.jst.go.jp/article/jte1955/2/1/22\\_1\\_1/pdf](https://www.jstage.jst.go.jp/article/jte1955/2/1/22_1_1/pdf).
- [28] Shim VPW, Lim CT, and Foo KJ. Dynamic mechanical properties of fabric armour. *International Journal of Impact Engineering*. 2001; **25**(1): 1-15. DOI: [https://doi.org/10.1016/S0734-743X\(00\)00038-5](https://doi.org/10.1016/S0734-743X(00)00038-5).
- [29] Zhang Y, Xu S, Zhang Q, and Zhou Y. Experimental and Theoretical Research on the Stress-Relaxation Behaviors of PTFE Coated Fabrics under Different Temperatures. *Advances in Materials Science and Engineering*. 2015; **2015**(7): 319473. DOI: <http://dx.doi.org/10.1155/2015/319473>.
- [30] Ward IM and Hadley DW. *An Introduction to the Mechanical Properties of Solid Polymer*. Chichester: Wiley; 1993.
- [31] Ghosh A, Das S, and Benerjee D. Simulation Of Yarn Stress Relaxation And Creep Behaviors Using Genetic Algorithm. *Indian Journal of Fibre and Textile Research*. 2013; **38**(1): 375-379. Available from: <http://nopr.niscair.res.in/bitstream/123456789/24980/1/IJFTR%2038%284%29%20375-379.pdf>.
- [32] Bandyopadhyay S, Ghosh A, and Ali SY. Tensile Fatigue, Stress Relaxation, and Creep Behaviour of Worsted Core Spun Yarns. *Journal of Applied Polymer Science*. 2011; **121**(4): 2123-2126. DOI: <https://doi.org/10.1002/app.33513>.
- [33] Asayesh A and Jeddi AAA. Modeling the creep behaviour of plain woven fabrics constructed from textured polyester yarn. *Textile Research Journal*. 2010; **80**(7): 642-650. DOI: <https://doi.org/10.1177/0040517509343816>.
- [34] Hanief A. *Studi Pemodelan Viskoelastik Material Benang Wool-Lycra 36 Tex Menggunakan Empat Pegas dan Satu Dashpot*. Bandung: Politeknik STTT Bandung; 2020.