



Intelligent tuning of LQR for Regulating Ball and Beam Balance System

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ABSTRACT

The ball and beam balance systems (BBBS) are widely utilized to test and demonstrate the performance of several control methods. However, regulating BBBS is challenging since its inherent instability and nonlinear dynamic system. In this paper, linear quadratic regulator (LQR) is proposed to regulate BBBS. To get the optimal performance, the genetic algorithm (GA) is utilized for intelligent tuning of the LQR. Furthermore, this research uses MATLAB Simulink for numerical simulation. The results reveal that the system can reach the reference for fixed reference. In addition, when the reference is changed, the response can follow the reference with τ_d , τ_r , and τ_s are 0.2644 s, 0.5018 s, and 0.6840 s, respectively and when the system is given a disturbance, the response can return to the reference with τ_d , τ_r , and τ_s are 0.2644 s, 0.3885 s, and 0.6785 s, respectively.

1. INTRODUCTION

Ball and beam balance system (BBBS) is widely applied in control engineering to evaluate and demonstrate control methods since its nonlinear dynamic system and difficulty in maintaining balance [1], [2], [3]. BBBS is inherently unstable systems, where the position of the ball can be changed indefinitely for fixed beam angle input [4],[5], [6]. Although it does not require a complex design, tools used to evaluate and control the system can be developed into other more complex systems. This system has 2 Degrees of Freedom (DOFs), where the ball is assumed to have an inertia acceleration during the movement of the beam. In BBBS, a ball is located on a beam that can be tilted at varying angles [7][8]. The BBBS needs accurate control to maintain the ball balanced at a desired place. However, to achieve a balance between stability and control method is challenging and requires an accurate control technique [9], [10].

In the literature, several techniques have been proposed for the BBBS. Ali H et al. have proposed optimization of proportional-integral-derivative (PID) using particle swarm optimization (PSO) to control BBBS. In this article, PSO was used to tune the gains of PID. The results revealed the proposed technique provides a better response than other method [11][12]. In [13][14], modelling and control of BBBS has been presented. To find the ball position, Lagrange method is used. In this research, PID has controlled the nonlinear characteristics of the system. The results showed that the proposed approach gives better performance. In [15][16], the optimal design of PID using intelligent tuning has been proposed to balance the BBBS. The intelligent tuning methods employed in this research are generic algorithms (GA) and PSO. The results showed that the proposed method provided fast responses for the BBBS. In [17], A. Umar et al. demonstrated the performance of linear quadratic regulator (LQR) for the BBBS. The results revealed that the velocity and beam angular velocity are superior at smaller Q . However, they may have limitations to achieve global optimum.

In this paper, an intelligent tuning of LQR for BBBS to achieve global optimum is proposed. To get global optimum parameter, the GA is utilized. The intelligent tuning using GA for optimal gain of LQR improves the control performance and stability of the BBBS. The proposed technique provides the robustness of LQR with the optimization capability of GA [18].

The rest of this paper is organized as follows. Section II explains the system modelling of ball and beam. In section III, the proposed control method used in this paper is explained. Section IV explains the results of the discussion and simulation. Section V is a conclusion of the paper.

2. BBBS

The BBBS as shown in **Figure 1** is unstable without active feedback. It causes the ball to keep moving up and down. The BBBS has a linear beam shape, and the ball is placed there. The ball moves on both sides of the beam without leaving the track.

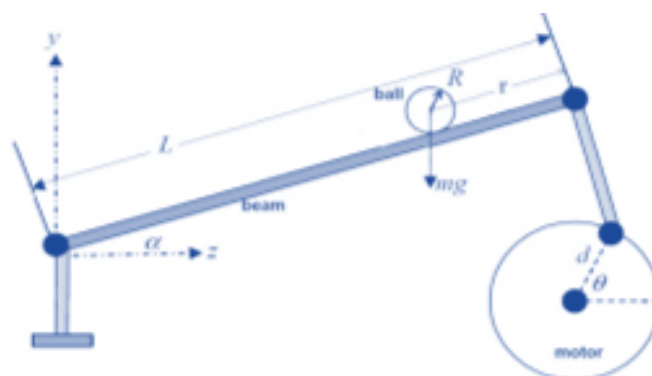


Figure 1. Model Ball and Beam Systems.

The system uses a linear potentiometer sensor to know the position of the ball. When the servo motor rotates the lever arm, angle θ will change the angle of the beam with α , this is because of gravity the ball rolled along the beam [19]. From the BBBS, we get a mathematical model which comprises a servo motor transfer function (TF) and a ball model rolling on a beam. The expression of the servo motor which has an angle and voltage position will be written as follows [20].

$$G(s) = \frac{\theta(s)}{V(s)} = \frac{K}{sT+s} \quad (1)$$

where K donates the DC motor gain and T represents the motor time constant. The formula for a ball linear acceleration along the beam will be written as follows.

$$\left(\frac{J}{R} + m\right) \ddot{r} + mg \sin \alpha - m\dot{r}\dot{\alpha}^2 = 0 \quad (2)$$

where J is a solid ball inertia moment with a small α . The formula of the system can be written as follows.

$$W(s) = \frac{r(s)}{a(s)} = \frac{mg}{\left\{\left(\frac{J}{R^2} + m\right)s^2\right\}} \quad (3)$$

The connection between the beam angle α and displacement motor angle θ is as follows.

$$\alpha = \frac{d}{L} \theta \quad (4)$$

The complete TF equation for ball moving to the beam path in relation to the ball position $r(s)$ and the rotation angle $\theta(s)$ will be written as follows.

$$W'(s) = \frac{r(s)}{\theta(s)} = \frac{mgd}{\left(L\left(\frac{J}{R^2} + m\right)s^2\right)} \quad (5)$$

3. OPTIMAL METHOD

3.1 Linear Quadratic Regulator

The state space system formula is presented as follows.

$$\dot{x} = Ax + Bu \quad (6)$$

Reducing the performance index to a minimum value to attain the system's acceptable ultimate performance is the ideal design for the LQR control approach. In the LQR control design, the performance index (J) is:

$$u(t) = -Kx(t) \tag{7}$$

To obtain the feedback gain matrix, two primary equations need to be considered. The equation is K.:

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \tag{8}$$

the feedback matrix K in the Algebraic Riccati Equation (ARE) is as follows.

$$K = T^{-1}(T^T)^{-1}B^T P = R^{-1}B^T P \tag{9}$$

Then, Eq. 10 is substituted into Eq. 7.

$$\dot{x} = Ax - BKx = (A - BK)x \tag{10}$$

System control may be obtained using the MATLAB software from the state 6 matrix equation. The Q matrix is the identity matrix. The dimensions of the R are determined by the number of entries in B.

3.2 GA

GA is a method of problem-solving that is most appropriate and comparable to genetic processes that naturally occur, also known as crossovers and mutations, and is founded on Darwinian principles of reproduction and survival. GA are a method for optimization and mathematical problems. This programming is designed to increase the genetic population of humans over several generations to identify the optimum solution to a problem. Each member of a population can serves as a representation of a solution in the genetic algorithm for any given issue, such as controlling or altering an individual with match value.

The purpose of genetic programming is to be able to create programs automatically on computer programs, so that the computer can solve problems. Genetic algorithms function within populations of diverse computer programs. This software starts with a previous dial flow of millions of randomly created programs with preprogrammed content. Next, use the breeding concept, which creates new program populations. Darwin survival of the fittest principle, which is analogous to genetic processes that occur in crossovers and mutations, is used to run the nursery domain-independently. The purpose of crossover operations is to provide syntactical sound descent programs. Based on the idea of evolution, genetic algorithm programming is a search method that can sustain the population in use.

The learning efficiency of Holland's Genetic Algorithm is nearly ideal when using GA. This technique frequently results in computer programs that are used to solve specific problems. The following actions can be taken when using GA.

1. Generate an initial population.
2. Perform repetition until the termination criteria in accordance with the way:
 - a. Run the code in the population and provide a match value utilizing an appropriate size.
 - b. Generate a new population from the program by selecting populations with probabilities based on match levels.
 - Darwinian reproduction, by reproducing existing programs by copying into new populations.

- Crossover, by combining parts of 2 new programs that already exist randomly using a crossover operation to get two new programs.
- Mutation, by creating a new computer program from a mutated piece of random program.

The escape led to the identification of the program using the method. These findings suggest that it is the most effective way to address the issue.

4. RESULTS AND DISCUSSION

To be able to simulate the calculation of the equation system derived from plant ball and beam, researchers use statistical software using MATLAB and Simulink. Before performing the simulation should include the required parameters in the system (**Table 1**).

Table 1. BBBS Parameters

Parameters	Value
Radius of the ball (R)	0,01 m
The ball mass (M)	0,027 Kg
Moment inertia of the ball (J)	$\frac{2}{5} mR^2$
Length of the beam (L)	0,4 m
Gravity (G)	9,81 m/sec ²
DC motor gain (K)	0,7/rev/sec/volts
Time constant motor (T)	0,0141 sec
Distance between center of the gear and joint of the lever arm (D)	0,04 m

To determine state space model, can enter the values of equation (5) and equation (1) to get the following values.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0 \quad 0] x$$

The value of a on the matrix is -71.43 and the value of b is 89. After determining the system parameter values of the ball and beam model, these parameters can be entered into the MATLAB Simulink to get the response value. In determining the response of this system for the first step do not use a controller or can use an open loop circuit. This study uses a set point value of 1 to determine the value of BBBS response.

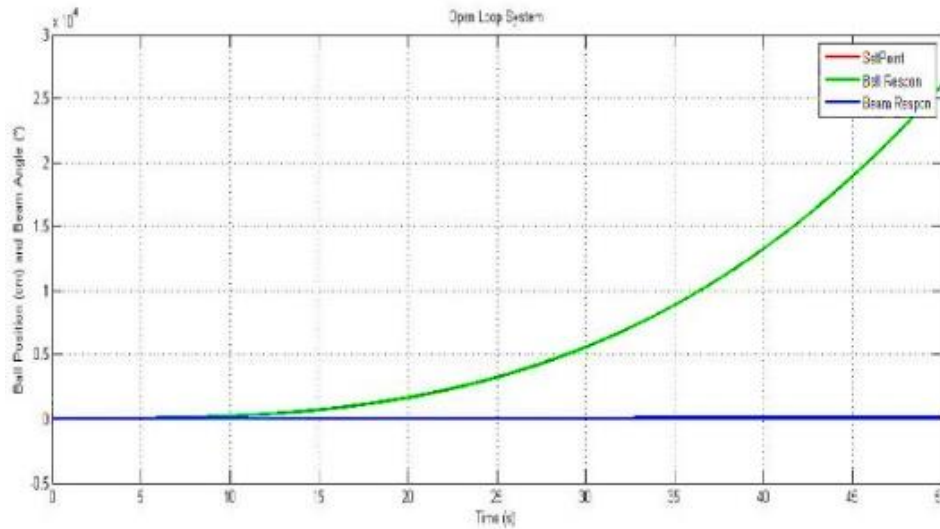


Figure 2. Ball and Beam response in open loop conditions

Figure 2 is a response image from BBBS using an open loop circuit without using a controller. Can be seen for the response value of the ball can't follow the reference because there is no control system. In using the LQR technique, Q and R often occur by giving random values or using trial-error methods. But in this study the Q matrix is obtained using the help of genetic programming tuning. The calculation process using this tuning is shown in **Figure 2**.

The first phase taken is to get the quantity of parameter values such as the amount of populations, chromosomes and generations. After entering the value of the parameter, the function value used as a Q value will begin to pass through the selection, crossover and mutase stages. At the selection stage is the process of selecting individuals from the population that is already available. Once selected, it will enter the crossover stage where this process is the process of merging data or marriage between different individuals. During the marriage process is completed new individuals or new data will be mutated to produce good offspring or produce data and Q values that have been optimized.

From the results of the genetic tuning algorithm calculations Q values are generated using the following parameters (**Table 2**).

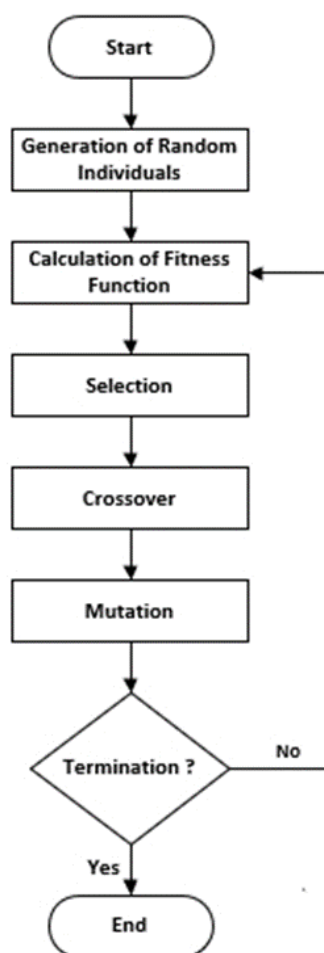


Figure 3. Flowchart of Genetic Algorithm

Table 2. Parameters Used for Genetic Algorithm Tuning

Population Size	Chromosome Size
100	10

$$Q = \begin{bmatrix} \alpha & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 \\ 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & \theta \end{bmatrix}$$

where α is 0.8175, β is 0.7357, γ is 0.1582, and θ is 0.7916. After the Q matrix is obtained, then find for the K matrix using the LQR formula in Simulink with the value of $R = [0.817]$ and K matrix is obtained as follows.

$$K = [1.0000 \quad 2.9082 \quad 2.9974 \quad 0.3379]$$

After determining all parameters to get the Q value, it can determine the response value of BBBS utilizing the LQR-GA controller. In this study, using 3 times of experiments on the system, the first using a controller system by adding a controller, the second set the reference value is changing, and the last gives disturbance to the system.

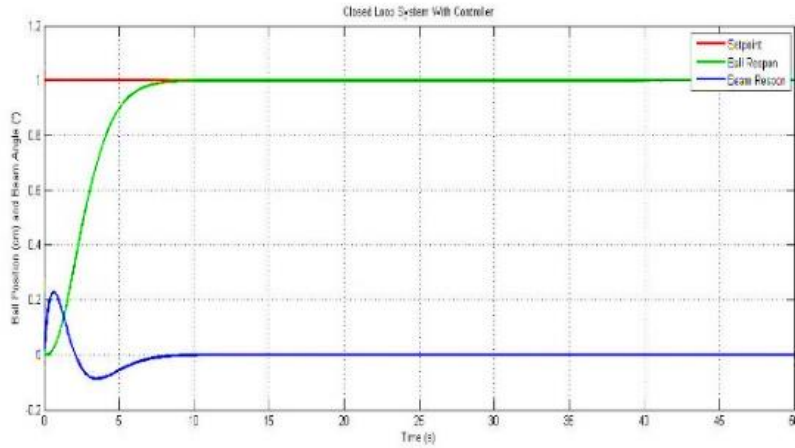


Figure 4. Ball and Beam Closed Loop Response

In **Figure 4** shows the BBBS response system applying a controller. After the addition of the controller, the ball response can follow the reference. In other words, in his real state the ball is in a balanced location on the Beam path according to the reference value used. The reference value used in the closed loop system is 1 cm for beam response and angle 1° for ball response. From the observed system responses, the parameters that have been required for this system are delayed time which is the time for the system to initially respond to the input. The rise time reflects a measure of the time the response takes to move from a specified lower percentage (often 10%) to a higher percentage (often 90%) of the reference; and the settling time, which is a measure of the time it takes for the system to settle within a particular percentage of the reference. The complete response is presented in **Table 3**.

Table 3. Parameter From Ball and Beam Closed Loop System

Method	τ	τ_d	τ_r	τ_s
LQR-GA	0.3191s	0.2644s	0.5018s	0.6840s

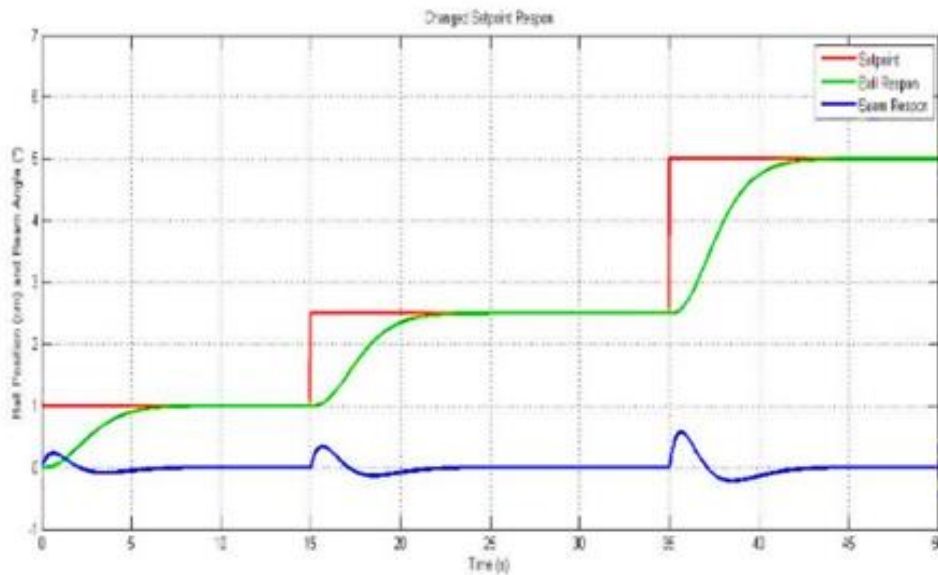


Figure 5. Ball and Beam Closed Loop Response

Figure 5 shows the response of BBBS due to reference changes. The aim of this test is to determine the performance of the proposed method and whether we can follow the reference changes. From the figure, it can be shown that the response always follows the reference change. The response of the suggested method with the disturbances is depicted in **Figure 6**. It can be shown that the response is always back to the reference when there is a disturbance. The complete data of the system with the disturbance is presented in **Table 4**.

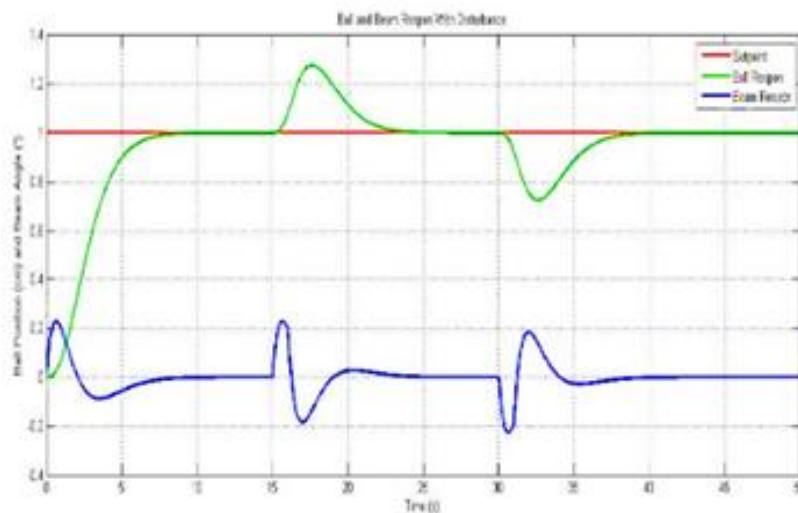


Figure 6. Ball and Beam Closed with disturbances

Table 4. Parameter From Ball and Beam with disturbance

Method	τ	τ_d	τ_r	τ_s
LQR-GA	0.316 s	0.264s	0.3885s	0.6785s

5. CONCLUSION

This paper proposes the simulation of BBBS utilizing a combination of LQR and GA. The GA method is employed to optimize the Q value, while the LQR serves as the primary controller to calculate the K gain. Simulation results validate that the proposed method achieves superior response in a closed-loop system with a constant reference, yielding a steady-state error of 0.3191 s, a delay time of 0.2644 s, a rise time of 0.5018 s, and a settling time of 0.6840 s. Additionally, when the reference value is changed, the system response effectively tracks the new reference. Furthermore, under disturbance conditions, the system quickly recovers and returns to reference, confirming the robustness of the proposed method.

6. AUTHORS' NOTE

The authors declare that there is no conflict of interest regarding the publication of this article. The authors confirmed that the paper was free of plagiarism.

7. AUTHOR'S CONTRIBUTION

Triawan Nugroho: Conceptualization, Methodology, Writing Original Draft, Investigation; Fahmi M. Mochtar: Formal Analysis, Writing Original Draft, Supervision; Muhammad B. A. Musthofa: Data Curation, Review & Editing, Investigation; Muhammad M. Syeichu: Software Review & Editing, Investigation.

8. REFERENCES

- [1] O. Silahdar, Ö. Atan, F. Kutlu, and O. Castillo, "Beam and Ball Plant System Controlling Using Intuitionistic Fuzzy Control," in *IFIP Advances in Information and Communication Technology*, 2021. doi: 10.1007/978-3-030-74826-5_22.
- [2] R. Firmansyah *et al.*, "DC Motor Speed Control using Particle Swarm Optimization based on Labview," *JEEE-U (Journal of Electrical and Electronic Engineering-UMSIDA)*, vol. 8, no. 2, pp. 111–121, Oct. 2024, doi: 10.21070/jeeeu.v8i2.1701.
- [3] R. Firmansyah, M. A. M. Ramli, and Endryansyah, "Voltage Regulation of Buck Converter in DC microgrid using Energy Valley Optimizer," in *2023 6th International Conference on Vocational Education and Electrical Engineering: Integrating Scalable Digital Connectivity, Intelligence Systems, and Green Technology for Education and Sustainable Community Development, ICVEE 2023 - Proceeding*, 2023. doi: 10.1109/ICVEE59738.2023.10348334.
- [4] C.-L. Shih, J.-H. Hsu, and C.-J. Chang, "Visual Feedback Balance Control of a Robot Manipulator and Ball-Beam System," *Journal of Computer and Communications*, vol. 05, no. 09, 2017, doi: 10.4236/jcc.2017.59002.
- [5] R. Firmansyah and M. A. M. Ramli, "A new adaptive droop control strategy for improved power sharing accuracy and voltage restoration in a DC microgrid," *Ain Shams Engineering Journal*, vol. 15, no. 9, p. 102899, Sep. 2024, doi: 10.1016/J.ASEJ.2024.102899.

- [6] M. M. A. Seedahmed *et al.*, “Optimal sizing of grid-connected photovoltaic system for a large commercial load in Saudi Arabia,” *Alexandria Engineering Journal*, vol. 61, no. 8, 2022, doi: 10.1016/j.aej.2021.12.013.
- [7] E. Li, Z. Z. Liang, Z. G. Hou, and M. Tan, “Energy-based balance control approach to the ball and beam system,” *Int J Control*, vol. 82, no. 6, 2009, doi: 10.1080/00207170802061269.
- [8] S. Yao, X. Liu, Y. Zhang, and Z. Cui, “Research on Solving Nonlinear Problem of Ball and Beam System by Introducing Detail-Reward Function,” *Symmetry (Basel)*, vol. 14, no. 9, 2022, doi: 10.3390/sym14091883.
- [9] M. Banu Sundareswari, G. Then Mozhi, and K. Dhanalakshmi, “Intelligent Tuning of PID Controller to Balance the Shape Memory Wire Actuated Ball and Beam System,” *Physical Mesomechanics*, vol. 23, no. 6, 2020, doi: 10.1134/S1029959920060181.
- [10] R. Firmansyah and R. Irmawanto, “Comparison Study of PI Controller Tuning Method to Regulate the DC Motor Speed,” in *ICRACOS 2021 - 2021 3rd International Conference on Research and Academic Community Services: Sustainable Innovation in Research and Community Services for Better Quality of Life towards Society 5*, 2021. doi: 10.1109/ICRACOS53680.2021.9702007.
- [11] H. Ali, A. Albagul, and A. Algitta, “OPTIMIZATION OF PID PARAMETERS BASED ON PARTICLE SWARM OPTIMIZATION FOR BALL AND BEAM SYSTEM,” *International Journal of Engineering Technologies and Management Research*, vol. 5, no. 9, pp. 59–69, Mar. 2020, doi: 10.29121/ijetmr.v5.i9.2018.289.
- [12] R. Firmansyah and P. P. S. Saputra, “Design of Model Predictive Control to stabilize Two-Stage Inverted Pendulum,” in *Proceeding - 2020 3rd International Conference on Vocational Education and Electrical Engineering: Strengthening the framework of Society 5.0 through Innovations in Education, Electrical, Engineering and Informatics Engineering, ICVEE 2020*, Institute of Electrical and Electronics Engineers Inc., Oct. 2020. doi: 10.1109/ICVEE50212.2020.9243211.
- [13] P. V. M. Maalini, G. Prabhakar, and S. Selvaperumal, “Modelling and control of ball and beam system using PID controller,” in *Proceedings of 2016 International Conference on Advanced Communication Control and Computing Technologies, ICACCCT 2016*, Institute of Electrical and Electronics Engineers Inc., Jan. 2017, pp. 322–326. doi: 10.1109/ICACCCT.2016.7831655.
- [14] R. F. Muktiadji, M. A. M. Ramli, and A. H. Milyani, “Twin-Delayed Deep Deterministic Policy Gradient Algorithm to Control a Boost Converter in a DC Microgrid,” *Electronics (Basel)*, vol. 13, no. 2, 2024, doi: 10.3390/electronics13020433.
- [15] M. Banu Sundareswari, G. Then Mozhi, and K. Dhanalakshmi, “Intelligent Tuning of PID Controller to Balance the Shape Memory Wire Actuated Ball and Beam System,” *Physical Mesomechanics*, vol. 23, no. 6, pp. 621–630, Nov. 2020, doi: 10.1134/S1029959920060181.
- [16] R. F. Muktiadji, M. A. M. Ramli, M. M. A. Seedahmed, R. Usvarman, and Endryansyah, “Power Sharing Control and Voltage Restoration in DC Microgrid Using PI Fuzzy,” in *2022 Fifth International Conference on Vocational Education and*

Electrical Engineering (ICVEE), 2022, pp. 130–135. doi:
10.1109/ICVEE57061.2022.9930441.

- [17] A. Umar *et al.*, “Performance Comparison of the Ball and Beam System using Linear Quadratic Regulator Controller,” in *PID Control for Linear and Nonlinear Industrial Processes*, 2023. doi: 10.5772/intechopen.110513.
- [18] R. F. Muktiadji *et al.*, “Control of Boost Converter Using Observer-Based Backstepping Sliding Mode Control for DC Microgrid,” *Front Energy Res*, vol. 10, Mar. 2022, doi: 10.3389/fenrg.2022.828978.
- [19] R. F. Muktiadji and J. Omotayo Oladigbolu, “Design of Dynamic Voltage Restorer Utilizing Adaptive Control to Mitigate Voltage Sag,” in *2021 Fourth International Conference on Vocational Education and Electrical Engineering (ICVEE)*, IEEE, Oct. 2021, pp. 1–5. doi: 10.1109/ICVEE54186.2021.9649714.
- [20] H. W. Tzeng and S. K. Hung, “Design of ball-beam balance control system using neural-fuzzy algorithm,” in *IEEE International Conference on Fuzzy Systems*, 2009. doi: 10.1109/FUZZY.2009.5277083.