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# Exploring Junior High School Students' Critical Thinking Processes in Problem Posing on Systems of Linear Equations in Two Variables Using GeoGebra

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## ABSTRACT

This study is motivated by the fact that many students still experience difficulties in critically analyzing and modifying mathematical problems, particularly in Systems of Linear Equations in Two Variables, despite the integration of technology in mathematics learning. This study aims to explore the critical thinking processes of junior high school students in posing, modifying, and validating problems involving Systems of Linear Equations in Two Variables with the support of GeoGebra. Using Ennis's (2011) theoretical framework, the study analyzes six indicators of critical thinking: focus, reasoning, inference, contextual understanding, clarity, and overview. A qualitative descriptive case study method was employed involving three eighth-grade students with different levels of mathematical ability (high, medium, and low). Data were collected through problem-posing tests, observations, and semi-structured interviews. The results indicate that the high-ability student was able to appropriately modify Systems of Linear Equations in Two Variables problems with strong mathematical reasoning and accurate interpretation of GeoGebra visualizations. The medium-ability student was able to change parameters but showed limited understanding of the implications of those changes on the solution. Meanwhile, the low-ability student struggled to modify problems, interpret representations, and validate solutions, often producing inconsistent systems. The findings highlight the important role of GeoGebra in supporting students' conceptual understanding and visualization-based reasoning while also revealing differences in critical thinking processes across mathematical ability levels. This study provides pedagogical implications for the development of technology-integrated mathematics instruction aimed at enhancing students' critical thinking and problem-posing skills.

***Keywords***: *Critical Thinking, Problem Posing, Systems of Linear Equations in Two Variables, Geogebra*

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## INTRODUCTION

Critical thinking is not merely a cognitive skill but an epistemic foundation that enables learners to navigate the complexities of the 21st century. In a world flooded with information yet impoverished in wisdom, this ability serves as a compass that guides students in distinguishing between substantive truth and intellectual illusion (Ennis, 2011). Ennis emphasizes that the essence of critical thinking lies in six integrated indicators: a sharp focus on the core issue, logically grounded reasoning, precise inference, contextual understanding of the situation, clarity in articulation, and a holistic overview. In the realm of mathematics, which is often trapped in procedural routines, critical thinking acts as a bridge toward authentic conceptual understanding, where abstract symbols are brought to life through multidimensional reasoning (Zaimah et al., 2024). Recent studies demonstrate that the integration of formal logic, spatial visualization, and metacognitive reflection forms a triad that distinguishes genuine critical thinking from mere algorithmic manipulation.

Problem posing emerges as an epistemic laboratory where students experience firsthand the process of constructing mathematical knowledge. When students design their own problems, they no longer remain passive consumers but become architects actively building understanding through conceptual experimentation (Putra & Siswono, 2021). This activity compels them to engage in structural deconstruction, analyzing relationships among variables, testing logical consistency, and predicting mathematical consequences of each modification. In this context, problem posing is not merely a pedagogical strategy but a simulation of authentic mathematical practice: formulating meaningful questions, conjecturing solutions, and reflecting on the validity of their intellectual constructions. This process transforms mathematics learning from knowledge transmission into meaning discovery.

The integration of problem posing and critical thinking creates a cognitive-metacognitive symbiosis. Each problem formulated by students becomes a mirror reflecting the depth of their understanding while simultaneously serving as a catalyst for cognitive leaps (Puspitasari et al., 2023). Empirical studies show that when problem posing is combined with dynamic visualization, there is a remarkable acceleration in students' reasoning (logical reasoning) and overview (holistic evaluation) capabilities, two critical indicators often neglected in conventional instruction. However, without proper scaffolding, this potential can backfire unguided problem posing may instead crystallize misconceptions, particularly when students fail to identify essential relationships between mathematical elements.

Systems of linear equations in two variables provide an ideal testing ground for this symbiosis. Although systems of linear equations in two variables appear algebraically simple, they involve complex relationships among symbolic, graphical, and numerical representations, each of which provides different mathematical insights and solution interpretations (Duval, 2006; Schoenfeld, 2016; Marion et al., 2023). Ironically, when teachers attempt to "simplify" the material by overemphasizing mechanical procedures, they inadvertently deprive students of the opportunity to experience aha moments when abstract patterns suddenly come to life through an interconnected web of relationships. In fact, systems of linear equations in two variables serve as a perfect canvas for creative problem posing, where students can experiment with parameters, explore degenerative cases, and directly experience the power of mathematics in modeling reality.

GeoGebra acts as a transformative catalyst that turns these challenges into opportunities. As dynamic mathematics software, GeoGebra does not merely visualize mathematics but materializes students' thinking processes (Samura & Darhim, 2023). Each slider manipulation becomes a hypothesis experiment; each equation change becomes a test of logical consistency; and each emergent intersection point serves as empirical verification of their mental constructions. Zaimah et al. (2024) observed a crucial phenomenon: when students use GeoGebra to design problems involving systems of linear equations in two variables, spontaneously emerging questions: "What if the lines are parallel?", "What happens if all coefficients are doubled?" are no longer technical queries but vivid manifestations of Ennis's inference and overview indicators. This demonstrates that when thoughtfully designed, technology can evolve from a mere instructional aid into a true cognitive partner. Syifaurrohman & Shodikin (2025) shows that GeoGebra provides dynamic visualizations that help students connect algebraic theory with graphical representation, thereby facilitating deeper conceptual understanding on System of Linear Equations of Two Variables (SPLDV) topic. Ramidha et al. (2025) found that the integration of GeoGebra in mathematics teaching significantly improved students' critical thinking skills, as students in the experimental class demonstrated better abilities in understanding, evaluating, and reasoning about geometric rotation problems while becoming more active, engaged, and capable of applying rotational concepts meaningfully in real-life contexts.

The integration of critical thinking, problem posing, systems of linear equations in two variables, and GeoGebra creates a meaningful learning environment. Problem posing encourages students to apply critical thinking skills, while systems of linear equations provide rich mathematical content for exploration. GeoGebra supports this process by connecting abstract concepts with visual representations. Through these activities, students engage in mathematics as a process of inquiry by generating questions, testing ideas, and validating solutions.

Based on this perspective, the present study investigates how the six indicators of Ennis's critical thinking framework emerge during students' interactions with systems of linear equations in two variables using GeoGebra. As highlighted by Sinclair and Yerushalmy (2016), further research is needed to understand cognitive processes in technology-based mathematics learning environments. The findings are expected to contribute to both theory and practice by providing insights into how digital technology can support the development of students' critical mathematical thinking.

## METHODS

This study used a qualitative descriptive case study approach to explore junior high school students' critical thinking during problem posing activities on systems of linear equations in two variables using GeoGebra. Three eighth-grade students with high, medium, and low mathematical abilities were purposively selected to capture diverse thinking patterns.

Data were collected through problem posing tasks and semi-structured interviews. The tasks required students to create and evaluate mathematical problems in real-life contexts using GeoGebra as a visualization tool. After completing the tasks, students participated in interviews to explain their reasoning, strategies, and difficulties. The collected data were used to examine students' critical thinking processes and the role of GeoGebra in supporting problem posing activities.

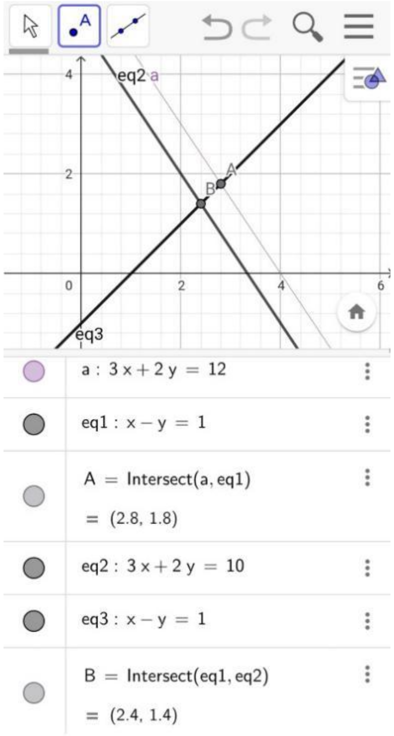
Data analysis was carried out using the Miles and Huberman model, which includes three stages: data reduction, data display, and conclusion drawing. The analysis focused on how six critical thinking indicators, according to Ennis (2011), emerged in the students' process of developing and validating their own Systems of Linear Equations in Two Variables problems.

Data validity was ensured through methodological triangulation (tasks, answers, and interviews) and member checking to guarantee the accuracy of the researcher's interpretations. Ethical considerations were taken seriously. The researcher obtained permission from the school and relevant teachers, requested written consent from the students, and ensured the confidentiality of the identity and personal data of each participant throughout and after the study.

## RESULTS AND DISCUSION

This study explored how junior high school students with different mathematical abilities demonstrated critical thinking while posing and validating problems on systems of linear equations in two variables using GeoGebra. A qualitative approach was employed involving three purposively selected students representing high (S1), medium (S2), and low (S3) mathematical ability levels.

The findings showed clear differences in students' critical thinking performance. S1 demonstrated the highest level of critical thinking (87.5%), characterized by systematic problem formulation, reflective reasoning, and effective use of GeoGebra. S2 achieved a moderate level (58.3%), while S3 showed the lowest performance (41.6%), indicating more limited critical thinking and problem-posing abilities. Overall, students' mathematical ability influenced how they applied critical thinking skills in a technology-supported learning environment. The ways in which students S1, S2, and S3 explored and utilized GeoGebra are illustrated in Figure 1, Figure 2, and Figure 3, respectively.



a : 3x + 2y = 12

eq1 : x − y = 1

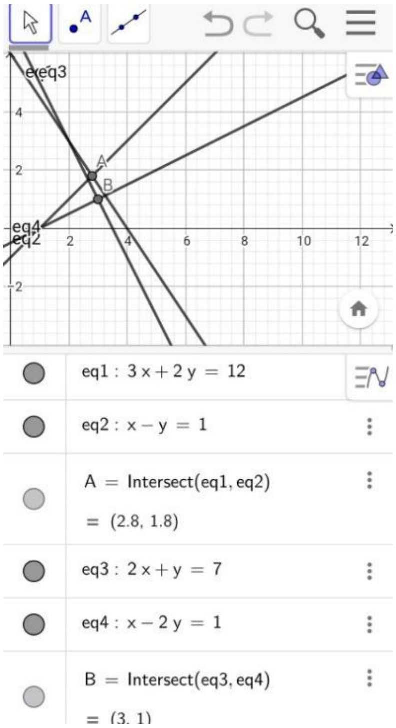
A = Intersect(a, eq1)
= (2.8, 1.8)

eq2 : 3x + 2y = 10

eq3 : x − y = 1

B = Intersect(eq1, eq2)
= (2.4, 1.4)

**Figure 1.** Geogebra exploration by S1 students



eq1 : 3x + 2y = 12

eq2 : x − y = 1

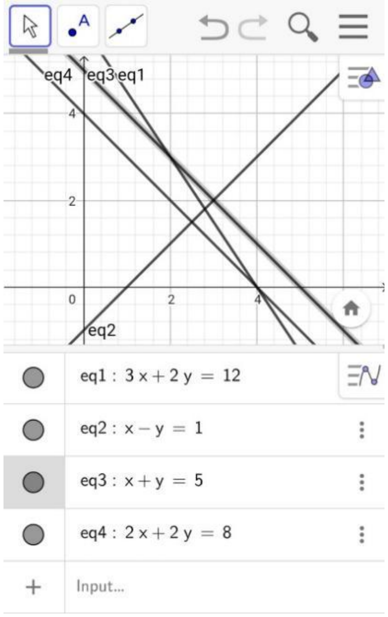
A = Intersect(eq1, eq2)
= (2.8, 1.8)

eq3 : 2x + y = 7

eq4 : x − 2y = 1

B = Intersect(eq3, eq4)
= (3, 1)

**Figure 2.** Geogebra exploration by S1 students



eq1 : 3x + 2y = 12

eq2 : x − y = 1

eq3 : x + y = 5

eq4 : 2x + 2y = 8

**Figure 3.** Geogebra exploration by S1 students

### Focus and Clarity of Thought

The analysis of students' focus and clarity of thought revealed substantial differences in the depth of mathematical understanding and attentional control demonstrated during the problem-posing process. Student S1 exhibited a high level of concentration on the essential mathematical relationships embedded within the system of linear equations in two variables. This finding reflects what Schoenfeld (2016) describes as mathematically productive thinking, in which learners selectively attend to conceptually significant elements while maintaining coherence within the mathematical structure. S1's performance also aligns with Ennis's (2011) notion of focus as a critical thinking indicator, where individuals are able to identify and concentrate on the core issue of a problem without being distracted by irrelevant features.

This intentional focus was evident when S1 strategically modified the constants of the equations while preserving the fundamental characteristics of the system. During the interview, S1 stated, *"I carefully adjusted the y-intercept to maintain a single intersection point."* This response indicates not only procedural competence but also conceptual awareness of how algebraic modifications influence graphical representations in the Cartesian plane. The student demonstrated an integrated understanding of symbolic and visual representations, which is considered an important aspect of mathematical reasoning in technology-enhanced environments (Sinclair & Yerushalmy, 2016). Furthermore, the use of GeoGebra enabled S1 to dynamically verify the consistency of the modified system, thereby supporting reflective thinking and conceptual validation. Previous studies have shown that dynamic mathematics software can enhance students' visualization skills, conceptual understanding, and reflective reasoning during algebra learning activities (Fuchs & Hohenwarter, 2004; Arbain & Shukor, 2015).

In contrast, student S3 demonstrated what may be characterized as "surface engagement," where attention was primarily directed toward superficial manipulations rather than underlying mathematical relationships. Instead of considering how changes in coefficients or constants affected the overall structure of the system, S3 tended to modify equations randomly without systematic reasoning. This finding is consistent with studies by Cai and Hwang (2020), which indicate that students with limited conceptual understanding often struggle to connect symbolic manipulations with underlying mathematical meaning, particularly in tasks involving multiple representations and problem posing activities. As a consequence, S3 frequently produced inconsistent systems and demonstrated limited ability to explain the mathematical implications of the modifications made.

These differences suggest that clarity of thought in mathematical problem posing is strongly associated with students' ability to maintain conceptual focus, coordinate symbolic and graphical representations, and evaluate the consequences of mathematical transformations. The findings further support the argument that digital tools such as GeoGebra can facilitate deeper conceptual engagement when accompanied by adequate scaffolding and critical thinking support.

### Mathematical Reasoning Processes

The reasoning dimension demonstrated substantial differences in how students constructed, justified, and validated mathematical ideas during the problem-posing process. Student S1 exhibited sophisticated mathematical reasoning characterized by the integration of multiple layers of logic, including graphical interpretation, algebraic consistency, and contextual applicability. This finding reflects what Ennis (2011) identifies as reasoned judgment, where individuals are able to formulate conclusions based on logical evidence and coherent argumentation. S1 not only modified the equations correctly but also explained how the changes influenced the graphical behavior of the system in GeoGebra. The student consistently evaluated whether the modified equations would maintain a single solution, parallel relationship, or coincident lines, demonstrating a strong connection between symbolic manipulation and conceptual understanding.

This multilayered reasoning process is consistent with the framework proposed by Calma & Davies (2025), who emphasize that critical thinking in academic work involves the ability to construct logical explanations, evaluate evidence systematically, and justify decisions using coherent reasoning structures. S1's responses also reflected metacognitive awareness, as the student frequently revisited and verified the correctness of the generated problems through graphical visualization and algebraic substitution. Such reflective verification indicates higher-order mathematical reasoning, which is considered essential in technology-supported mathematics learning environments (Schoenfeld, 2016).

In contrast, student S2 demonstrated what may be described as "transitional reasoning." Although S2 showed an emerging awareness of mathematical principles, the reasoning process was not consistently systematic. The student could identify certain relationships between coefficients and graphical representations but often failed to fully explain the mathematical consequences of the modifications made. For example, S2 recognized that changing a coefficient could alter the intersection point of the lines, yet the explanation lacked analytical depth regarding why such changes occurred mathematically. This finding suggests that S2 possessed partial conceptual understanding but still relied heavily on procedural intuition rather than fully developed logical reasoning.

Meanwhile, student S3's reasoning remained at what can be characterized as a "pre-structural" level, consistent with the SOLO taxonomy framework proposed by Biggs & Collis (1982). S3 frequently relied on trial-and-error strategies and random equation modifications without considering the mathematical relationships among variables, coefficients, and graphical outcomes. As a result, the student often produced inconsistent systems and demonstrated difficulty in explaining the validity of the generated problems. This limited reasoning process indicates insufficient integration between procedural knowledge and conceptual understanding, a challenge commonly identified among students with lower mathematical proficiency (Cai & Hwang, 2020).

Overall, the findings indicate that mathematical reasoning in problem posing is not limited to the ability to manipulate equations procedurally, but also involves reflective evaluation, conceptual interpretation, and logical justification of mathematical relationships. Furthermore, the integration of GeoGebra appears to support reasoning development by enabling students to dynamically visualize the consequences of their mathematical decisions, thereby facilitating deeper analytical thinking and conceptual verification.

### Inference and Conclusion-Drawing

The inference-making processes observed in this study provided significant evidence of differing levels of conceptual understanding and critical thinking development among the participants. Student S1 consistently demonstrated what may be characterized as "validated inference," in which conclusions were formulated only after systematic verification through both graphical and algebraic approaches. This pattern reflects advanced inferential reasoning, where mathematical conclusions are not accepted intuitively but are critically examined using multiple representations. According to Ennis (2011), inference is a central component of critical thinking that involves drawing reasonable conclusions from available evidence while evaluating the reliability and consistency of the reasoning process.

S1's inferential process was particularly evident when the student used GeoGebra to compare graphical intersections with algebraic substitution results before finalizing conclusions about the nature of the system. The student repeatedly checked whether the generated equations produced a single solution, infinitely many solutions, or no solution at all. This dual-verification strategy demonstrates a high degree of metacognitive awareness, as S1 actively monitored and evaluated the validity of the reasoning process. Such reflective thinking aligns with the findings of Schoenfeld (2016), who argues that effective mathematical thinking involves continuous self-regulation, monitoring, and evaluation during problem solving and mathematical exploration. Furthermore, the integration of graphical and symbolic verification indicates strong representational fluency, which is considered essential for deep algebraic understanding (Duval, 2006).

Student S2 showed partial inferential skills. Although the student generally reached correct conclusions, these were often based on visual observations from GeoGebra without thorough algebraic verification. For example, S2 identified intersecting lines correctly but did not consistently confirm the intersection points using algebraic methods. Previous studies have shown that students at intermediate levels of mathematical proficiency frequently depend on procedural confirmation rather than integrated conceptual reasoning (Cai & Hwang, 2020).

In contrast, student S3 had difficulty drawing accurate conclusions from graphical representations. The student frequently misinterpreted graphs, especially parallel lines, and struggled to connect visual patterns with their corresponding algebraic properties. As a result, conclusions were often based on guesses rather than logical reasoning. This finding is consistent with research by Duval (2006), which emphasizes that difficulties in coordinating multiple mathematical representations can significantly hinder conceptual understanding and reasoning. Instead of using graphical evidence as a basis for logical conclusions, S3 tended to rely on guesses and unsystematic assumptions, resulting in invalid or inconsistent conclusions.

The findings indicate that drawing valid conclusions in mathematical problem posing requires both procedural knowledge and the ability to connect symbolic and graphical representations. GeoGebra can support this process by helping students visualize mathematical relationships, but its effectiveness depends on students' conceptual understanding and critical evaluation skills.

## Contextual and Situational Understanding

The analysis of students' contextual and situational understanding revealed important differences in how learners connected symbolic manipulations to broader mathematical meanings during the problem-posing process. Student S1 demonstrated a strong ability to contextualize mathematical modifications within coherent conceptual frameworks, reflecting what may be described as "mathematical situatedness." This concept refers to learners' capacity to interpret algebraic operations not merely as procedural actions, but as meaningful transformations connected to graphical representations, solution characteristics, and underlying mathematical structures. According to Vygotsky (1978), meaningful learning occurs when students are able to construct understanding through contextualized cognitive activity rather than isolated procedural execution. In this study, S1 consistently showed awareness of how changes in coefficients and constants influenced the relationships between lines in the coordinate plane.

This contextual awareness became particularly evident when S1 modified equation parameters while simultaneously predicting their graphical consequences in GeoGebra. The student demonstrated an understanding that altering the slope or intercept of a line would affect the nature of the system, including whether the equations would intersect, overlap, or remain parallel. Such reasoning indicates the integration of symbolic, graphical, and conceptual knowledge, which is considered a key characteristic of meaningful algebraic understanding (Duval, 2006). Furthermore, S1's ability to connect mathematical procedures with conceptual outcomes aligns with the notion of relational understanding proposed by Skemp (1976), where learners understand not only how procedures work but also why they work within broader mathematical contexts. Setiyowati and Shodikin (2022) demonstrated that students with higher mathematical abilities were better able to understand and analyze problems. These students could identify the known and developed information in the problem by systematically writing it down and were able to provide accurate and appropriate solutions.

Student S2 demonstrated emerging contextual awareness, although the understanding remained partially developed. S2 was generally able to recognize that modifications in equations influenced graphical outcomes; however, the explanations provided were often incomplete and lacked deeper conceptual elaboration. For example, the student could identify that changing coefficients altered the intersection point of the lines but struggled to explain the mathematical relationships underlying these changes systematically. This finding suggests that S2 had begun to develop connections between symbolic manipulation and graphical interpretation, yet the reasoning process remained procedural rather than fully conceptual. Similar patterns have been identified in previous studies showing that intermediate-achieving students frequently demonstrate partial representational understanding when working in technology-enhanced algebra environments (Cai & Hwang, 2020).

In contrast, student S3 exhibited what may be characterized as "contextual isolation," namely an inability to connect symbolic manipulations to their broader mathematical meaning. S3 frequently changed coefficients arbitrarily without considering the graphical implications or structural consistency of the resulting system. This disconnect became particularly apparent when the student produced equations that unintentionally generated parallel lines or inconsistent systems without recognizing the mathematical consequences of these transformations. Such findings support Duval's (2006) argument that difficulties in coordinating multiple mathematical representations can significantly hinder conceptual understanding and meaningful reasoning in algebra learning.

Overall, the findings indicate that contextual and situational understanding plays a crucial role in students' ability to engage in meaningful mathematical problem posing. The use of GeoGebra appears to facilitate this understanding by enabling students to dynamically observe the effects of algebraic transformations on graphical representations. However, the effectiveness of technology integration depends strongly on students' conceptual readiness and their ability to interpret mathematical representations critically and contextually.

## Clarity and Precision of Expression

The analysis of students' communication patterns revealed that clarity and precision of expression served as significant indicators of the depth of mathematical understanding and critical thinking demonstrated during the problem-posing activities. Student S1 consistently exhibited what may be described as "mathematical precision," characterized by the accurate use of mathematical terminology, coherent explanation structures, and logically sequenced reasoning. According to the National Council of Teachers of Mathematics (2014), effective mathematical communication involves the ability to express mathematical ideas clearly, justify reasoning logically, and communicate relationships among mathematical representations precisely. In this study, S1 demonstrated these competencies by systematically explaining how modifications to coefficients and constants affected the graphical behavior of the system of linear equations in two variables.

S1's explanations reflected strong conceptual coherence and logical organization. When describing changes made in GeoGebra, the student used appropriate mathematical vocabulary such as "intersection point," "parallel lines," "slope," and "substitution verification" accurately and consistently. Furthermore, the explanations followed a structured reasoning sequence beginning with the modification of equations, followed by graphical interpretation, and concluding with validation of the resulting system. This pattern indicates not only procedural fluency but also conceptual understanding and reflective reasoning. Such mathematical communication aligns with the findings of Schoenfeld (2016), who emphasizes that mathematically proficient learners are able to articulate reasoning processes explicitly and connect procedural actions with conceptual meanings.

In contrast, student S2 demonstrated communication that was generally understandable but occasionally reflected what may be termed "colloquial mathematical discourse." Although S2 could explain certain mathematical procedures and identify graphical relationships, the explanations often lacked precision and formal mathematical structure. The student tended to use informal expressions or incomplete reasoning when describing why particular equation modifications produced specific graphical outcomes. This finding suggests that S2 possessed emerging mathematical communication skills but had not yet fully internalized formal mathematical language and argumentative structure. Previous research has shown that intermediate-level students frequently rely on intuitive or conversational explanations when they have not yet developed strong conceptual confidence in mathematical reasoning (Moschkovich, 2010).

Meanwhile, student S3's expressions frequently contained "conceptual gaps," referring to missing or incomplete links within the chain of mathematical reasoning. The student often provided fragmented explanations that failed to clearly connect symbolic manipulations, graphical interpretations, and mathematical conclusions. For example, S3 could mention that "the graph changed" after modifying coefficients but was unable to explain how or why the changes occurred mathematically. These communication difficulties indicate limited conceptual understanding and insufficient coordination between procedural knowledge and mathematical meaning. According to Duval (2006), weaknesses in coordinating multiple mathematical representations can significantly affect students' ability to communicate mathematical ideas coherently and meaningfully.

Overall, the findings suggest that clarity and precision of expression are closely related to students' conceptual understanding, representational fluency, and critical thinking abilities. The integration of GeoGebra appears to support mathematical communication by providing visual representations that help students explain abstract algebraic relationships more concretely. However, the quality of students' explanations depends substantially on their ability to connect visual, symbolic, and conceptual representations within a coherent reasoning framework.

### Systems Thinking and Holistic Perspective

The overview dimension revealed substantial differences in students' capacities for systems thinking and holistic mathematical understanding during the problem-posing activities. Student S1 demonstrated what may be characterized as "structural perception," namely the ability to recognize, preserve, and evaluate the integrity of the system of linear equations in two variables throughout the transformation process. Rather than viewing equations as isolated symbolic expressions, S1 understood the system as an interconnected mathematical structure in which modifications to one component inevitably influenced the relationships among coefficients, graphical representations, and solution characteristics. According to Ennis (2011), the overview dimension of critical thinking involves reflective reconsideration of the entire reasoning process to ensure coherence, consistency, and logical validity. In this study, S1 consistently demonstrated this reflective capacity by evaluating whether modifications maintained the intended mathematical properties of the system.

This structural awareness became particularly evident when S1 used GeoGebra to dynamically analyze the consequences of equation transformations. The student carefully examined how altering slopes, intercepts, or coefficients affected the overall behavior of the graphical system, including the number and nature of intersection points. S1 not only identified these changes visually but also explained their algebraic implications systematically. Such reasoning reflects systems thinking, which involves understanding mathematical objects as interconnected structures rather than disconnected procedural elements (Saat et al., 2024). Furthermore, the student's ability to maintain coherence between symbolic manipulation and graphical interpretation aligns with relational mathematical understanding as described by Skemp (1976).

Student S2 showed a partial understanding of the relationships among equations, graphs, and solutions. The student could recognize some effects of modifying equations but often failed to consider the overall impact on the system. As a result, S2 sometimes made correct changes without fully checking whether the new system met the intended conditions.

In contrast, student S3 displayed a fragmented understanding. The student frequently changed coefficients or constants without considering their effects on the relationships between equations and graphs. This often led to inconsistent systems or incorrect graphical results that S3 could not adequately explain. This fragmented understanding reflects difficulties in coordinating multiple mathematical representations and maintaining coherence across symbolic and visual domains. According to Duval (2006), students who fail to coordinate mathematical representations effectively often struggle to develop holistic conceptual understanding in algebra learning.

Overall, the findings suggest that systems thinking is important for effective problem posing and critical thinking. Students with stronger structural understanding were better able to predict the effects of mathematical changes, maintain consistency across representations, and evaluate the validity of their problems. GeoGebra supported this process by helping students visualize relationships, although its effectiveness depended on students' conceptual understanding and reflective reasoning abilities.

## Pedagogical Implications and Theoretical Contributions

These findings significantly enhance our understanding of how technology-mediated learning environments foster critical thinking in mathematics. First, this study supports what Putra & Siswono (2021) highlight as the importance of scaffolding in GeoGebra-assisted problem posing, students with varying levels of ability benefit from differentiated instructional strategies when engaging with mathematical software. High-achieving students thrive in exploratory environments that promote independent reasoning, while lower-achieving students require structured guidance to effectively utilize technological tools.

Second, this research extends Ennis's (2011) critical thinking framework by demonstrating how its indicators, such as inference and overview, manifest uniquely within digital problem-posing contexts. Echoing the observations of Sinclair & Yerushalmy (2016), the study suggests that technology can amplify certain cognitive processes (e.g., visualization and inference), but may obscure others (e.g., reasoning clarity) without intentional pedagogical support.

Third, the six-dimensional analytical framework developed here offers educators a practical rubric for assessing critical thinking within technology-enhanced mathematics instruction. This addresses the gap identified by Zaimah et al. (2024), who emphasize the need for comprehensive assessment models that capture the full scope of students' critical thinking processes when using dynamic digital tools like GeoGebra.

### Limitations and Future Research Directions

While providing valuable insights, the study acknowledges several constraints that open productive avenues for future investigation. The small sample size, though appropriate for qualitative depth, limits the generalizability of findings; thus, larger-scale studies could apply the developed framework to identify normative patterns across broader populations. Additionally, the relatively brief intervention period may not have captured the full developmental trajectory of students' critical thinking skills, suggesting the value of longitudinal research to observe how these indicators evolve with sustained technology integration. Furthermore, since this study focused on a specific mathematical content area, future research could explore the transferability of findings across different domains of mathematics. This research makes significant theoretical contributions by bridging critical thinking theory, technology integration frameworks, and mathematics education research. The findings also have immediate practical applications for curriculum design, particularly in supporting the implementation of Indonesia's Kurikulum Merdeka, by offering evidence-based strategies for developing critical thinking through judicious use of educational technology. Future studies might further investigate how these insights apply in diverse cultural and educational contexts, and how they can inform the design of adaptive learning systems that respond to students' evolving critical thinking capacities.

## CONCLUSION

This study revealed that the critical thinking abilities of junior high school students in posing problems related to systems of linear equations in two variables using GeoGebra varied according to their level of mathematical proficiency. High-ability students (S1) were able to modify the system of equations accurately, supported by strong mathematical reasoning and optimal use of GeoGebra for visualization and solution validation. Students with moderate ability (S2) demonstrated a basic understanding of SLETV but still encountered difficulties in validating the modifications they made. Meanwhile, low-ability students (S3) tended to make modifications without sufficient conceptual consideration, even producing inconsistent systems. These findings highlight the effectiveness of GeoGebra as an instructional tool that facilitates conceptual understanding of SLETV while also identifying challenges faced by students with different ability levels. However, this study is limited by its small number of participants and narrow scope, indicating the need for further research with larger samples and more varied problem contexts. Moving forward, it is essential to develop more inclusive instructional strategies that consider students' ability levels and to promote a more systematic integration of problem-posing approaches with digital technologies such as GeoGebra to enhance students' mathematical critical thinking development.

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