Evaluation Theoretical Efficiency Selection Sort By Big-O Notation Analysis

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Abstract—Algorithm complexity analysis is a fundamental aspect in computer science education and research, providing a critical framework for evaluating computational efficiency. This study presents a comprehensive theoretical evaluation of the Selection Sort algorithm using Big-O notation analysis to determine formal complexity bounds. The research aims to rigorously assess the time complexity of Selection Sort across best-case, average-case, and worst-case scenarios through asymptotic analysis methodology. The theoretical framework employs Big-O, Big-Theta, and Big-Omega notations alongside mathematical proof techniques including summation analysis and formal verification methods. A systematic operation-counting methodology is applied to derive precise complexity characterizations for each algorithmic phase. The analysis shows that Selection Sort exhibits uniform quadratic time complexity $O(n^2)$ under all input conditions, unlike other sorting algorithms whose performance varies based on input characteristics. Mathematical evidence confirms that the algorithm performs exactly $n(n-1)/2$ comparisons regardless of the initial data arrangement, thereby establishing a strict boundary for theoretical complexity. These2 findings provide a complete mathematical basis for evaluating Selection Sort complexity, making a significant contribution to algorithm analysis literature and educational methodologies. Despite its consistent performance predictability, the quadratic complexity limits its scalability for large datasets. This theoretical evaluation serves as a comprehensive reference for algorithm selection decisions and complexity analysis instruction.

Keywords—Selection Sort, asymptotic analysis, computational complexity, algorithm evaluation

I. INTRODUCTION

Algorithms are key elements inccomputer science that function to arrange items in a specific order, either ascending or descending. Some common sorting algorithms include Bubble Sort, Selection Sort, Insertion Sort, Merge Sort, Quick Sort, and Heap Sort. Each algorithm own characteristics typical in matter complexity time, memory usage [18][43], and efficiency in different data contexts. The complexity of sorting algorithms can be classified based on best case, average case, and worst case. For example, Quick Sort has an average complexity of $O(n \log n)$, but in the worst case it can reach $O(n2)$. On the other hand, Merge Sort always has a complexity of $O(n \log n)$ in all cases, although it requires a considerable amount of extra space. Selection Sort, although simple, has a time complexity of $O(n2)$ for all types of cases [16].

Algorithm analysis has undergone significant development And become aspect important in theory computers. With the increasing need to process large amounts of data in real-time, the study of algorithm efficiency has become increasingly necessary. The analysis not only focuses on execution time, but also considers aspects such as stability, space efficiency, adaptability, and parallel capabilities. These innovations have bring up various method analysis advanced such as amortized analysis, probabilistic analysis, And parameterized complexity [44][19].

Research in the field of sorting algorithms tends to be grouped into several structural approaches, including

- Divide and Conquer Methods: For example Quick Sort and Merge Sort.
- Iterative Selection Based Methods: Such as Selection Sort and Bubble Sort.
- Data Structure Based Methods: Such as Heap Sort which utilizes a heap structure, or the radix and bucket algorithm which uses arrays and hashes [45].

A. The background of the importance of algorithm analysis in computer science

Algorithm analysis is very important in the field of computer science as it helps in selecting the best solution for a problem. a problem. Algorithm is a series steps or procedures used to solve problems in an organized manner, and in the programming process, algorithms relate to the logic of determining what program to create or write. In software development, selecting the right algorithm has a significant impact on system performance, especially for large-scale applications such as data processing, compression, and artificial intelligence [20]. Without sufficient analysis, software is at risk of experiencing performance bottlenecks or inefficient use of resources. Algorithms can understood as a series step or calculation Which required For finish problems, especially the processes followed by the computer. A good algorithm is one that uses the right tools for the purpose, while choosing the wrong algorithm is like cooking a dish that does not follow the recipe and tool And material Which required, so that the result inefficient or inappropriate, because cooking also has procedures and rules that must be followed [46].

B. Algorithm in Knowledge Computer

Algorithm analysis is very important in the field of computer science as it helps in selecting the best solution for a problem. a problem. Algorithm is a series steps or procedures used to solve problems in an organized manner, and in the programming process, algorithms relate to the logic for determining the program to be created or written.[21] In software development, selecting the right algorithm very impact on system performance, especially for large-scale applications such as data processing, compression, and artificial intelligence. Without sufficient analysis, software risks experiencing performance bottlenecks or inefficient use of resources. An algorithm can be understood as a series of steps or calculations Which required For finish problems, especially the processes followed by the computer. A good algorithm is one that uses the right tools for the purpose, while choosing the wrong algorithm is like cooking a dish that does not follow the recipe and tool And material Which required, so that the result inefficient or inappropriate, because cooking also has procedures and rules that must be followed [47].

C. Role Notation Big O in Efficiency Evaluation

Application of Big O Notation to evaluate the efficiency of algorithms. Notation O Big, Which Also known as Landau Notation or Notation Asymptotic, is symbol mathematical term used to describe the characteristics of an asymptotic function. The purpose of this is to understand behavior A function on mark input extremes, whether very large or very small, in a simple but precise manner so that they can be compared with other functions.

Besides That, symbol O functioning For show limit above from the asymptotic behavior of a distance or measure of a simpler function. There are also other symbols such as O and T that represent boundaries top, bottom, and average. Its implementation divided to in two field: in mathematics, this notation is used to specify the characteristics of the remaining terms in a truncated infinite region, especially in analysis series asymptotic. In In computer science, this notation is used to study the complexity of an algorithm [23].

In general, big O notation is used to express asymptotic limits. However, these asymptotic limits are more often and accurately expressed by the symbol T (big theta), as will be explained further below. This notation was first introduced in Germany by a number theorist, Paul Bachmann, in 1894 in the second edition of his book entitled Analytische Zahlentheorie, which the first edition published in 1892 did not cover the theme of big O notation. This notation became more famous thanks to the contribution of another German number theorist, Edmund Landau, so it is sometimes also called Landau notation. Big O comes from the English term "order of" and was originally the symbol Omicron big, but Then adopted with letter Latin has a similar form, namely a capital letter “O”, and not the number zero (0) [48].

D. Election Algorithm Selection Sort as Case Studies

We conducted a study on the application of the Selection Sort algorithm to manage inventory data in this article. This algorithm was chosen because it is easy to use and shows effectiveness on small to medium sized datasets. The way it works is by taking the smallest element from the unsorted section and exchanging it with the first element in that section. This process is repeated until the entire dataset succeed sorted. Study This carry out the implementation algorithm Selection Sort on system inventory management using Python programming language. The results of the study show that this algorithm increases efficiency in sorting data, making it easier for users to find the information they need. need, as well as reduce time needed to manage inventory. In addition, this system provides convenience for employees and store managers in accessing product availability information more easily. fast And easy, so that speed up decision making in inventory management. Overall, the application of the Selection Sort algorithm in inventory management inventory goods has proven effective in optimizing data management processes [24]. With thus, study This play a role in developing a better and more efficient inventory management system.

E. Research Gap: Lack of Comprehensive Theoretical Analysis

Understanding research gaps is crucial for researchers. By identifying areas in which knowledge remains limited, researchers can better focus their

studies and ensure that their work contributes meaningfully to existing literature [25]. This awareness not only promotes innovation by highlighting unanswered questions, but also enables the development of more rigorous methodologies, improves the validity of research findings, and broadens the scientific landscape by addressing underexplored areas. Studying research gaps also serves to clarify which areas require further investigation [26].

While Selection Sort has been widely discussed in the literature, particularly in classical algorithm textbooks such as that of Sedgewick and Wayne [25], the treatment often remains at the level of basic implementation and general performance analysis. However, to date, limited attention has been given to in-depth theoretical exploration or contextual adaptability of this algorithm.To clarify the distinction between prior work and the present study, a structured comparison is provided in Table 1:

Table 1. the distinction between prior work and the present study

Aspect	**Sedwick & Wayne (2011)**	**This Study**
Primary Focus	Provides an overview of sorting algorithms, including Selection Sort, with an emphasis on implementation and general performance analysis	Offers an in-depth theoretical examination of Selection Sort, focusing on structural properties and algorithmic complexity.
Theoritical Depth	Limited to introductory-level explanations aimed at practical understanding.	Develops a comprehensive and systematic theoretical framework.
Comparative Perspective	Focuses primarily on asymptotic analysis (Big-O) and empirical performance metrics.	I ncorporates both technical (Big-O) and non-technical aspects such as implementation simplicity and determinism.
Application Context	Does not address algorithm adaptation in specialized domains such as hybrid systems or embedded computing.	Explores the adaptability of Selection Sort in hybrid sorting models and embedded system scenarios.
Research Contribution	Lacks exploration of recent trends or underexplored areas related to Selection Sort.	Identifies and addresses research gaps by mapping current trends and proposing novel perspectives in sorting algorithm studies.

This comparative overview highlights a significant gap in the existing literature: the lack of a comprehensive theoretical and contextual analysis of the Selection Sort algorithm. This study aims to fill that gap by not only analyzing the algorithm's internal structure and complexity, but also by exploring its relevance and adaptability in modern computational settings. In doing so, the research contributes to both the theoretical advancement and practical application of sorting algorithm studies.

II. Literature Review

A. *The Basics Notation Big- O*

Formal Definitions of Big-O, Big-Theta, Big Omega.The mathematical foundation of algorithm analysis is very relies on asymptotic notation, which provides a convenient framework for characterizing computational complexity [1]. Big-O notation, formally introduced by Bachmann and popularized by Knuth, represents an upper bound on algorithmic complexity [4]. According to Cormen et al., Big- O notation is formally defined as: for a given function $g\ (\ n)$, $O(g\ (\ n\ )\ )$ represents the set of functions $f\ (\ n\ )$ so that there is constant positive c And n_0 in where $0 \leq f\ (\ n\ ) \leq c - g\ (\ n\ )$ for all $n \geq n\ 0$ [1].

Complementary notation provides a complete asymptotic characterization. Big-Omega (Ω) notation describe the boundaries lower, in where $\Omega\ (g\ (\ n\ )\ ) - \{\ f\ (n\ ) : \}$ There is constant positive c And $n\ o$ so that $0 \leq c - g\ (n\ ) \leq f\ (\ n\ )$ for all $n \geq n\ o$ [2]. Big-Theta Notation (Θ) represents a strict constraint, defined as $\Theta\ (g\ (\ n\ )\ ) = \{\ f\ (\ n)\}$ there is a constant positive $C\ 1$, $C\ 2$, and $n\ o$ so that $0 \leq c\ 1 - g\ (\ n\ ) \leq f\ (\ n\ ) \leq c\ 2 - g\ (\ n\ )$ For all $n \geq n\ o$ [1]. This tripartite notation system allows the characterization of complexity. Which appropriate in various perspective analytic different [3]. Knuth emphasizes that these notations serve not only as computational tools, but also as fundamental mathematical constructs that enable rigorous algorithmic analysis [4]. Formal definitions establish the theoretical framework necessary to perform systematic complexity evaluations, providing the mathematical rigor necessary for academic research [5].

B. Foundation Mathematics

The mathematical foundation underlying asymptotic analysis comes from advanced mathematical concepts including limits, calculus, and discrete mathematics [6]. Sedgewick and Wayne showed that asymptotic analysis requires an understanding of growth rates, where functions are classified by their dominant terms as the input size approaches infinity [7]. Their mathematical framework uses the theory limits, specifically lim lim

$f(n)$ / g (n), for build $n \to \infty$connection asymptotic between function [1]. Graham et al. provide a comprehensive mathematical tool for asymptotic analysis, including summation and generating function techniques [8]. Its mathematical foundation includes several key principles: first, the dominance of the highest-order terms in polynomial expressions; second, the insignificance of constant factors in asymptotic growth; and third, the application of mathematical induction. For construction proof formal [8]. Principle- This principle forms the theoretical basis for evaluating algorithm complexity.

The mathematical rigor required for asymptotic analysis demands precise formulation of assumptions, clear statements of theorems, and systematic construction of proofs [9]. Sipser emphasizes that the mathematical foundation must include an understanding of discrete probability, combinatorics, and algebraic manipulation techniques that are essential for complexity analysis [9]. Exchange element the smallest that have been selected with elements in the unsorted part of the array. This process will continue until all elements in the array are completely sorted[12]. The Selection Sort algorithm is the simplest algorithm, but this algorithm is not efficient in large data sets [13][14].

C. Pseudocode

Algorithm Selection Sort started with look for smallest element from subarray Which Not yet sorted. Element then exchanged with the first element of the subarray. This step is repeated for all elements until the array is sorted. Here is an example of a pseudocode for the Selection Sort algorithm:

```
Void selectionsort(int arr[], int n)
{
int i, min, time;
for(int i=0;i<n- 1;i++)
{
min=i;
for(int j=i+1;j<n;j++)
{
if(arr[j]<arr[min]) min=j;
}
temp=arr[min]; arr[min]=arr[j]; arr[j]=temp;
}
```

The principles of asymptotic analysis guide the systematic evaluation of algorithmic efficiency through a methodological framework that has been established [10]. Roughgarden identified three basic principles: worst-case analysis for determining upper bounds, average-case analysis for evaluating practical performance, and best-case analysis for determining lower bounds [10]. These principles collectively provide a comprehensive algorithmic characterization. The asymptotic dominance principle states that higher-order terms determine computational complexity, making lower- order terms and constants asymptotically insignificant [1]. This principle allows comparison of algorithms based on fundamental growth characteristics rather than specific details. implementation [7]. Methodology This emphasize in scalability analysis, examining the behavior of the algorithm as the input size increases towards infinity [3].

Aho et al. showed that the principles of asymptotic analysis should combine theoretical rigor and practical application [8]. These principles include: methodology. calculation operation Which systematic, verification of mathematical proofs, and a comparative analysis framework for evaluation algorithm [8]. Principles This building the foundation For do analysis theoretical Which strict to algorithmic complexity.

E. Selection Sort Algorithm Definition Algorithm Selection Sort

The Selection Sort algorithm is an algorithm that sorts data based on comparison. This algorithm will examine an array of elements and will try to find the smallest element in the array. Then this algorithm swaps the smallest element with the element in the first position. Then after it is done, it tries to select
the smallest element from the unsorted part of the array after performing each iteration. Then he Characteristics Base Algorithm Selection Sort. Selection Sort is a comparison-based sorting algorithm that has the basic characteristic of working in-place and has a lower number of data exchanges compared to other simple algorithms. However, this algorithm has a time complexity. $O(n2)$ in every condition input, so it doesn't efficient for large data [15].

F. Analysis Complexity Previous theoretical work exists

In computer science, algorithms are used to solve various problems in an organized manner. The effectiveness of an algorithm is assessed through complexity analysis, which assesses the amount of resources (time and memory) required depending on the size of the input. This assessment is very important because it directly affects the performance of the software, especially when handling large amounts of data [28][27].

1. Donation Donald E. Knuth

In his book entitled The Art of Computer Programming, Knuth serve runway mathematics solid to understand algorithms, especially algorithms for sorting and searching. He introduced the methods of average analysis, worst-case analysis, and probabilistic approaches to evaluating the performance of algorithms [29].

2. Cormen and his colleagues. and Compilation of Book Analysis

"Introduction Algorithm" Which written by Cormen and his colleagues. Serve method Whichregular to analyze algorithms using asymptotic notation ,Which covers notation Big-O, Omega, And Theta. They Also

introduce approach in construct an algorithm like for And conquer, greedy, and dynamic programming [31][30].

3. *Visual and Experimental Methods by Sedgewick and Wayne*

Sedgewick and Wayne contributed through experimental methods in analyzing algorithms. They combined theoretical methods and data visualization to provide a practical understanding of algorithm performance, especially in the area of sorting algorithms.

4. A Study of Data Structures and Complexity by Weiss. Mark Allen

Weiss emphasizes the importance of analyzing algorithms in relation to data structures, as well as presenting the efficiency of various operations (such as searching, inserting, And deletion) on diverse structure such as trees and hash tables [32]. The theoretical work that has been done has created a very solid framework for analyzing algorithms. However, there is still scope for integrating theoretical and empirical approaches in research on algorithm performance. The mastery of Good to theory very crucial as foundation to create more flexible and effective algorithms in the context of use in everyday life.

5. The Gap in literature At the moment

Despite significant advances in algorithmic theory—particularly in the development of formal models such as Big-O notation—there remains a critical gap between theoretical complexity and practical implementation. This study aims to investigate these overlooked aspects, particularly regarding simple algorithms like Selection Sort, which are often excluded from contemporary algorithmic research.

- *Lack of Evaluation of Simple Algorithms*

  Most algorithm textbooks and research papers tend to focus on advanced algorithms with lower time complexities, often disregarding simpler ones such as Selection Sort. For instance, Baase and Van Gelder [33] provide minimal theoretical treatment of Selection Sort, describing it only briefly in the context of introductory examples. Similarly, Cormen et al. [34] and Sedgewick & Wayne [35] emphasize efficient algorithms like Merge Sort, Quick Sort, and Heap Sort, while relegating Selection Sort to a marginal position without detailed exploration. This omission is problematic because Selection Sort remains relevant in educational settings, where its deterministic behavior and conceptual simplicity make it ideal for teaching core algorithmic principles. Moreover, it is often still used in embedded systems, where memory and resource constraints make simpler, predictable algorithms preferable. The lack of in-depth theoretical and applied analysis of such algorithms represents a blind spot in the current literature.

- *Discrepancy Between Theory and Practice*

  As noted by Cormen et al. [34], algorithm analysis often centers on asymptotic behavior (e.g., Big-O) without considering empirical performance under real-world hardware conditions. Factors like cache utilization, branch prediction, and memory hierarcy are rarely addressed. This hierarchy are rarely addressed. This results in a disconnect between theoretical efficiency and practical execution times.

- *Over-Reliance on Idealized Models*

  Many analyses use the RAM (Random Access Machine) model, which assumes constant-time access for all operations and ignores delays from system-level behaviors such as caching, latency, and instruction pipelining. Sedgewick and Wayne [35] acknowledge these limitations but do not integrate them into their complexity evaluations. Consequently, algorithms may perform quite differently on real hardware than theoretical models suggest.

- *Limited Integration with Modern Computing Architectures*

  The algorithm literature often fails to reflect the demands of parallel systems, GPU-based computation, or big data environments. Existing complexity analyses rarely incorporate modern workloads or architectural features such as concurrency or data locality. As a result, there is a lack of algorithmic strategies optimized for these emerging domains.

- *Lack of Interdisciplinary Applicability*

  Although algorithms are widely applied in domains like bioinformatics, finance, and computational linguistics, most literature remains highly technical and insular to computer science. There is little effort to simplify or adapt algorithms—especially simple ones—for cross-disciplinary usage. Bridging this gap requires making algorithms more accessible and practically grounded.

III. Methodology

A. Framework Work Theoretical

Framework theoretical For analysis complexity Selection Sort uses a systematic approach based on notation theory. asymptotic And methodology proof mathematical [1]. The analysis framework follows the established paradigm of evaluating algorithms through the enumeration of operations, modeling mathematics, And construction proof formal [3]. This approach ensures a rigorous theoretical evaluation that is consistent with academic standards for algorithm analysis research.

Approach analysis complexity use a layered methodology that includes three analytical perspectives: worst-case scenario analysis using Big-O notation, average-case evaluation through probabilistic analysis, and best-case checking for completeness [10].

Each analytical layer uses mathematical tools and proof techniques that conform to a defined complexity limit [2].Simplification of complex expressions into standard asymptotic forms [8].
This framework combines formal mathematical verification through constructive proof methodology, ensuring that complexity bounds are mathematically sound and academically rigorous [4].

B. Devices Mathematics used

This mathematical tool for theoretical analysis
This mathematical tool for theoretical analysis combines a number of draft And technique mathematics protocol analysis algorithm Which already exists, starting with the decomposition of the algorithm, continuing with the identification and enumeration of operations, and ending with a mathematical proof of the complexity bounds [1]. An important basis for rigorous complexity evaluation [8].

The main tools include summation analysis for evaluating repetition nesting, in where… represents model mathematics For operation comparison Selection Sort [1]. Induction mathematics serves as a formal proof technique to establish limit complexity in various size input [3]. Additional mathematical tools include recurrence relation analysis, probability theory for average case evaluation, and discrete mathematics for combinatorial analysis. [9]. Framework Work mathematical using limit theory to characterize asymptotic behavior, specifically examining the quadratic complexity bounds [7]. Algebraic manipulation techniques allow this toolkit includes a formal proof construction methodology, using direct proof, proof by contradiction, and constructive proof techniques appropriate for different aspects of complexity analysis [6]. Graph theory concepts support the visualization of algorithms and analysis structures, while probability theory discrete allow analysis statistics algorithm behavior on different input distributions [9].

C. Analysis Method

Evaluation algorithm Selection Sort done systematically through the following stages:

- Initialization Process Sequencing

  The algorithm starts by iterating from the first index to the last. index final in array. Every iteration aims to move the smallest element of an unsorted subarray to its proper position.
- Search Element Minimum

  Value of the unsorted elements. Process This involving comparison between element in a nested loop structure.
- ExchangeElements

  After element the smallest found, algorithm will perform one exchange to place the element in the correct position.
- Iteration/Repetition

  Step This will repeated until all over element in the array are in the right order. The algorithm will not stop until all position has processed And ensure the array is completely sorted. This Process produce amount comparison as much as $n\ n-1$. Model This state that the amount operation 2 grow in a way quadratic to size input n, Which show that complexity time algorithm This is $O\ (\ n\ 2\ )$ for all cases [1].

D. Proof Techniques Used

Analysis and validation of the efficiency of the Selection Sort algorithm is carried out by applying the following mathematical proof techniques:

- Direct Proof

  This technique is used to show the exact number combines a number of draft And technique mathematics of comparisons and exchanges performed by the algorithm. This proof is done by calculating the number of iterations of nested loops and reducing them to a quadratic mathematical formula.
- Loop Invariant

  This technique is used to show the exact number of comparisons and exchanges performed by the algorithm. This proof is done by calculating the number of iterations of nested loops and reducing them to a quadratic mathematical formula.
- Mathematical Induction

  This technique is used to prove that Selection Sort will always sort the array correctly, regardless of the number of elements.
  - Base Case: For n=1, the array is already sorted.
  - Induction Step: Assuming the algorithm works for n= k, it is also proven to apply for n= k + 1.
- Asymtotic Analysis

  Using the limit theory:

$$\lim_{n\to\infty} T\ n(n-1) : 2 = 1/2$$

  It can therefore be concluded that the execution time of the Selection Sort algorithm is at the upper limit of quadratic, namely $O(n^2)$ [1].

IV. . THEORITICAL ANALYSIS

A. Algorithm Decomposition

This refers to partitioning a complex algorithm into smaller, more tractable components so that each part can be understood, implemented, and analysed more efficiently [16]. This approach, widely adopted in software engineering and algorithm design, enhances modularity and problem-solving effectiveness. The present theoretical investigation examines the **selection-sort algorithm** by means of **three six-element arrays** [15]. The primary objective is to sort each array in **ascending order** through **five manual**

iterations. The analytical stages are summarised below.

- Initialization and Selection of Minimum- Value Selection.

  Beginning at index 0, the algorithm searches the remaining sub-array for the smallest element. Once identified, this minimum value is **swapped** with the element at the current index. The procedure is repeated from index 0 to index (n − 2), yielding **n − 1 iterations.**
- Manual Iteration

  During every iteration, the current element is compared with each subsequent element. In-line comments document whether a swap occurs and which indices are involved.Three Array Examples Analyzed.

- **ArraysUnder Examination**

  Example 1

  Input: 6, 45, 34, 20, 100, 38
  Sorted Output: 6, 20, 34, 38, 45, 100

  Input: 89, 40, 33, 56, 99, 39
  Sorted Output: 33, 39, 40, 56, 89, 99

  Input: 28, 30, 37, 2, 78, 23
  Sorted Output: 2, 23, 28, 30, 37, 78

- **Observation on the Iterative Procces**

  Each example requires five iterations (n − 1). The number of swaps and the indices involved are recorded. No swap occurs in several iterations of Examples 1 and 2, whereas every iteration in Example 3 triggers a swap, indicating a more dynamic positional adjustment. T

  To clarify the iteration process of the selection sort algorithm, the following is a step-by-step visualization in table form. This visualization shows how the elements in the array are processed in each iteration, including the elements being compared, the minimum value found, the indices involved, and whether a data swap occurs.

Example Array: 6, 45, 34, 20, 100, 38
The following table shows the steps of the iteration in the selection sort algorithm.

Iteration	Current Array	Minimum Found	Minimum Indeks	Swap
1	6, 45, 34, 20, 100, 38	6	0	No
2	6, 45, 34, 20, 100, 38	20	3	45 ⇄ 20
3	6, 20, 34, 45, 100, 38	34	2	No
4	6, 20, 34, 45, 100, 38	38	5	45 ⇄ 38
5	6, 20, 34, 38, 100, 45	45	5	100 ⇄ 45

B. *Elementary Operations and Complexity*

- Identification of Elementary Operations

  Two fundamental operations are counted explicitly:
  - Comparison – Evaluating two array elements
  - Swap – Exchanging two elements when necessary For array size $n = 6$, done $n - 1 = 5$ iteration. In each iteration, the number of comparisons is $n - i - 1$. Hence, the total comparisons are $(n - 1) + (n - 2) + \cdots + 1 = n(n - 1)2$.
- Manual Trace and Visualization

  A complete operation trace is recorded, detailing every comparison, minimum update, and swap. This explicit trace provides a transparent account of algorithmic behaviour.
- Time Complexity

  Given the quadratic growth in the number of comparisons, selection sort exhibits $O(n^2)$ time complexity. Consequently, it is unsuitable for large-scale data sets because of the high comparison overhead [15].

C. *Mathematical Proof of Complexity*

Mathematical complexity is the analysis of the number of operations required by an algorithm to complete its task that can be completed in some situations. It is usually interpreted in terms of Big-O notation (such as $O(n^2)$, which indicates the execution time in terms of the number of elements n sorted).

- Best case analysis

  In selection sort, even though the initial data is already in an ascending order, the algorithm will still make comparisons between as many elements as possible. n (n - 1) 2* (n (n - 1) : 2n (n -1) time. This is because selection sort does not have a mechanism for stop early if the data is already sorted. For example, If data beginning is: 6, 20, 34, 38, 45, 100 Fixed Algorithms will look for element the smallest in unordered parts and compare each pair.
- Average Case Analysis

  In the average case, the elements are arranged randomly. Selection sort will still perform the same number of comparisons as in the best and worst cases, namely:
  $T(n) = n(n - 1)2T(n) =$ n(n -1):2 $T(n) = 2n(n - 1)$. The number of exchanges also remains at a maximum of one per iteration (a total of n - 1 swaps). Therefore, even though the data is randomized, the number of main steps remains the same, so the time complexity in the average case is also $O(n^2)$ generating data exchanges.
- Worst Case Analysis

  In the worst case — for example, when the data is

arranged in descending order, such as: 100, 45, 38, 34, 20, 6 — the algorithm must still perform data swapping:
$n(n-1)2 *(n(n-1):2)*2n(n-1)$
• comparisons, and
• n - 1 swaps (because each iteration will inevitably find an element smaller than the current element and must be swapped). However, the number of comparisons remains the same, so the time complexity remains: $O(n2)$. Based on the above analysis, Selection Sort has a time complexity of $O(n^2)$ in all cases (best, average, worst), because the number of comparisons performed is constant and follows the formula for the sum of an arithmetic series.This indicates that this algorithm is not efficient for large data sets, despite being simple and using constant memory O(1).

D. *Analysis Comparative*

An element smaller than the current element and must be swapped). However, the number of comparisons remains the same, so the time complexity remains: $O(n2)$ Comparison with other sorting algorithms Choosing the appropriate sorting algorithm can have a significant impact on program efficiency, especially when managing large amounts of data. AlthoughSelection Sort is considered less efficient in analysis. Asymptotically, this method is still used in simple systems or for teaching basic algorithms. This article compares this algorithm with other algorithms in terms of complexity and actual performance [37].

Table 3. Comparison of Sorting Algorithms Based on Time Complexity, Space, and Stability

Algorithm	Best Time	Average Time	Worst Time	Space	Stable	Suitable for
Selection Sort	O(n²)	O(n²)	O(n²)	O(1)	No	Data small, educatior
Insertion Sort	O(n)	O(n²)	O(n²)	O(1)	Yes	Data small, almost sorted
Merge Sort	O(n log n)	O(n log n)	O(n log n)	O(n)	Yes	Data big, stable needed
Quick Sort	O(n log n)	O(n log n)	O(n²)	O(log n)	No	Performance tall, No need stability

Selection Sort still more superior in matter simplicity and predictability—suitable for systems with limited resources or for learning purposes.

- Merge Sort is preferred in applications that require stability and can be scaled through parallel processing.
- Quick Sort still become choice main in Lots library standard Because speed its performance in condition average
- Insertion Sort effective used on list Which small sized and when data is almost regular [38][39].

Theoretical evaluation of the algorithm is a crucial element in process development device soft. The theoretical position explains how an algorithm can be categorized according to its time and space complexity, stability, and relationship to data structures. In the research academic, method This useful For distinguish between basic algorithms and more complex algorithms. This analysis is very important as a basis for selecting algorithms in real applications [40]. Computational Model: Many studies use the RAM (Random Access Machine) model as a basis for assumptions [41][42].

Table 4. Comparison of Sorting Algorithm Based on Paradigm, Efficiency, and Stability

Algorithm	Paradigm	Average Complexity	Stablility	Theory Related
Selection Sort	Brute Force	O(n²)	No	Iterative, deterministic, simple
Insertion Sort	Incremental	O(n²)	Yes	Suitable for almost sorted lists
Merge Sort	Divide-Conquer	O(n log n)	Yes	Recursion stability, optimal, supports theoretical proof
Quick Sort	Divide-Conquer	O(n log n)	No	Probabilistic proof, flat average analysis

- Asymptotic Complexity: Assessing the effectiveness of algorithms on a large scale by utilizing Big-O, Omega, and Theta notations.
- Algorithm Stability: Whether the algorithm can

maintain the order of elements that have the same key.
- Principles of Algorithm Design: Such as divide and conquer, greed, coercion, and dynamic programming.
- Sequencing Selection considered as algorithm education and appropriate used in system Which own memory limitation due to the need for O(1) additional space. Insertion Sort very effective in situation in where data almost sorted And become base for development more optimal hybrid algorithm.

Merge Sort in a way theoretical is method Which most efficient for all situations due to its stable nature and time complexity $O(n \log n)$ which always consistent.

- Quick Sort is very effective on average, but its theoretical position prone to to condition worst $O(n^2)$, which depends on the pivot selection.

The theoretical position of the sorting algorithm provides a deep understanding of the proper way and time For use the algorithm. Selection Sort has mark education Which tall, temporary Merge Sort and Quick Sort excel in terms of performance. In the future, theoretical understanding will remain the basis for flexible and effective software engineering.

V. Result and Discussion

A. Theoritical Findings

The results of a theoretical analysis of the selection sort algorithm show that this algorithm has a time complexity of $O(n^2)$ [15]. This complexity was confirmed through a
detailed manual approach, by analyzing the number of basic operations in the form of comparisons and data exchanges in each iteration. In the three case studies analyzed, each consisting of six elements, the algorithm required five iterations to complete the sorting process. Each iteration involved searching for the minimum element in the unsorted portion of the array, followed by an exchange operation if necessary. This pattern reflects the number of comparisons performed, which is n(n - 1):2 , which is a characteristic of algorithms with quadratic complexity. Furthermore, eventhough the number of comparisons reaches 15 times for n=6, the number of exchanges performed is relatively small. This indicates the efficiency of the algorithm in terms of saving exchange (swap) operations, even though the number of comparisons remains high. Therefore, selection sort is more suitable for small datasets, where clarity of logic and simplicity of implementation are prioritized over time efficiency at a large scale [15].

B. Implication

Theoretical analysis of the Selection Sort algorithm confirms that it maintains $O(n^2)$ time complexity across all input types and makes it example ideal For introduce the concept of deterministic algorithms in time complexity analysis. This is due to the structure nested loop that produces the number of comparisons as many as n(n-1):2 [15]. which is a characteristic of algorithms with quadratic complexity. Its simplicity allows for a deep understanding of how control structures affect algorithm efficiency. Due to its stable execution pattern and ease of mathematical modeling, Selection Sort is relevant as a basic model for understanding the fundamental concepts of Big-O notation and the influence of loop structures on algorithm efficiency. Selection Sort has high educational value in algorithm and data structure learning. The simplicity of its steps, the search for the minimum element, and the exchange process facilitate understanding of nested loops, the comparison and exchange of elements, the introduction of asymptotic notation such as $O(n^2)$, and proof techniques like loop invariants and mathematical induction [15].

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VI. Conclusion

This comprehensive theoretical analysis of the Selection Sort algorithm reveals fundamental insights into its computational behavior and practical applications in computer science. The research demonstrates that Selection Sort maintains a uniform quadratic time complexity of $O(n^2)$ across all input conditions, performing exactly n(n-1)/2 comparisons

regardless of the initial data arrangement. This deterministic characteristic distinguishes it from other sorting algorithms whose performance varies based on input characteristics, making Selection Sort highly predictable in its execution pattern.The mathematical analysis confirms the algorithm's nested loop structure produces consistent behavior with minimal swap operations, validated through rigorous proof techniques including direct proof, loop invariants, and mathematical induction. While the quadratic complexity limits its scalability for large datasets, Selection Sort offers significant advantages in specific contexts, particularly its O(1) space complexity that makes it ideal for memory- constrained systems and scenarios where exchange operations are computationally expensive.

The study establishes Selection Sort's exceptional educational value as a pedagogical tool for teaching fundamental algorithmic concepts. Its simplicity facilitates understanding of Big-O notation, nested loops, and complexity analysis principles, making complex theoretical concepts accessible to students learning computer science fundamentals. The algorithm serves as an exemplary model for introducing asymptotic analysis and mathematical proof techniques in academic settings. Despite its limitations for large-scale data processing, this research concludes that Selection Sort's consistent performance predictability, educational significance, and implementation simplicity ensure its continued relevance in computer science education and specialized applications requiring deterministic behavior and minimal memory usage. The study provides a comprehensive theoretical framework that fills a significant gap in the literature by offering thorough analysis of fundamental algorithms often overlooked in favor of more sophisticated sorting methods, thereby contributing valuable insights to both algorithmic theory and educational methodology.

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