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Exploring the Commognitive Processes of a Prospective Mathematics Teacher in Mathematical Problem Posing

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Abstract

This study aims to explore the commognitive processes of a high-performing prospective mathematics teacher during mathematical problem posing by examining how the four commognitive components word use, visual mediators, routines, and narratives emerge across the orientation, connection, generation, and reflection stages. This study employed a qualitative descriptive approach. The participant was purposively selected based on obtaining the highest score on a mathematical problem-solving test. Data were collected through a problem-solving test, a problem-posing task, and semi-structured interviews. The data were analyzed using data reduction, data display, and conclusion drawing, while credibility was established through data triangulation. The findings revealed that all four commognitive components emerged throughout the mathematical problem-posing process. During the orientation stage, the participant represented mathematical ideas using appropriate mathematical symbols and terminology. During the connection stage, the participant established conceptual relationships by linking derivatives and integrals as inverse operations. During the generation stage, the participant formulated a more sophisticated mathematical problem involving exponential functions. During the reflection stage, the participant verified the solution through substitution and logical justification. These findings indicate that mathematical problem posing is not merely a procedural activity but a commognitive process in which conceptual understanding, mathematical communication, and reasoning interact dynamically. This study extends the application of the commognitive framework to mathematical problem posing and provides pedagogical insights for developing prospective mathematics teachers' problem-posing competencies.

Keywords: Commognitive Theory, Mathematical Problem Posing, Prospective Mathematics Teachers, Qualitative Research

Abstrak

Penelitian ini bertujuan untuk mengeksplorasi proses commognitive seorang calon guru matematika berkemampuan tinggi dalam mengajukan masalah matematika dengan mengkaji bagaimana empat komponen commognitive, yaitu word use (penggunaan kata), visual mediators (mediator visual), routines (rutinitas), dan narratives (narasi), muncul pada tahap orientasi, koneksi, generasi, dan refleksi. Penelitian ini menggunakan pendekatan deskriptif kualitatif. Partisipan dipilih secara purposive berdasarkan perolehan skor tertinggi pada tes pemecahan masalah matematika. Data dikumpulkan melalui tes pemecahan masalah, tugas pengajuan masalah matematika, dan wawancara semi-terstruktur. Data dianalisis menggunakan tahapan reduksi data, penyajian data, dan penarikan kesimpulan, sedangkan keabsahan data dijamin melalui triangulasi. Hasil penelitian menunjukkan bahwa keempat komponen commognitive muncul pada seluruh proses pengajuan masalah matematika. Pada tahap orientasi, partisipan merepresentasikan ide-ide matematika menggunakan simbol dan istilah matematika yang tepat. Pada tahap koneksi, partisipan membangun hubungan konseptual dengan mengaitkan turunan dan integral sebagai operasi yang saling berkebalikan. Pada tahap generasi, partisipan menyusun masalah matematika yang lebih kompleks dengan melibatkan fungsi eksponensial. Pada tahap refleksi, partisipan memverifikasi solusi melalui substitusi dan justifikasi logis. Temuan ini menunjukkan bahwa pengajuan masalah matematika bukan sekadar aktivitas prosedural, melainkan merupakan proses commognitive yang memperlihatkan interaksi dinamis antara pemahaman konseptual, komunikasi matematis, dan penalaran. Penelitian ini memperluas penerapan kerangka commognitive dalam konteks pengajuan masalah matematika serta memberikan implikasi pedagogis bagi pengembangan kompetensi pengajuan masalah pada calon guru matematika.

Kata kunci: Teori Commognitive, Pengajuan Masalah Matematika, Calon Guru Matematika, Penelitian Kualitatif.

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Introduction

Such communication can be verbal or nonverbal and can take place with oneself or with others. This is consistent with the views of Sfard (2012), who emphasizes thinking through the terms cognitive and communication. The combination of cognition and communication is known as commognition. Commognition emphasizes interpersonal communication and individual thought that involves cognition (thinking) and is individual in nature, and requires external support but does not necessarily have to be interpersonal. These can be classified into four components known as the commognitive framework: word use, visual mediators, routines, and narratives. Word use involves the use of words for numbers, geometric concepts, equations, and others. Visual mediators include the use of objects such as diagrams, symbols, graphs, and others; narratives include axioms, definitions, and theorems; and routines include defining, estimating, proving, and abstraction (Sfard, 2021) .

The commognitive framework is designed not only for issues related to mathematics teaching and learning but can also be used to investigate the mindsets of prospective teachers. The commognitive framework has an impact on the professional development of prospective teachers and mathematical discourse in order to enhance students' exploratory abilities at the elementary and middle school levels (Metzuyanim & Tabach, 2017; Shabtay & Metzuyanim, 2017; Wittgenstein et al., 2009). This is the basis for the importance of conducting a commognitive investigation of prospective mathematics teachers, as it relates to their professional development. This is supported by the views of Presmeg (2016) and Kim, Choi, and Lim (2017) who argue that this framework can be used to investigate the mathematical thinking of prospective teachers within the context of school mathematics. One of the skills that prospective math teachers must master is problem posing.

In November 2020, researchers conducted a preliminary study on the commognitive abilities of fifth- and seventh-semester students in the Mathematics Education Program at PGRI Adi Buana University in Surabaya regarding problem-posing. The aim was to gain an understanding of problem-posing patterns based on commognitive abilities. A total of 15 students agreed to participate in the study, and each participant presented two different types of problem-posing: formulating problems based on a given situation and solving a problem first before formulating a problem. The results of the preliminary study provide an initial picture of the commognitive components of prospective teachers in formulating mathematical problems. The findings from the study indicate that out of the 15 participants, 6 students were unable to formulate a problem, 5 were able to formulate a problem but could not explain the process, and the remainder were able to formulate a problem.

This is consistent with the research by Isik and Kar (2012), which states that prospective teachers still face many difficulties in formulating problems.Problem formulation is considered the most essential aspect of mathematics and mathematical thinking (Silver et al., 1996). Problem formulation refers to the creation of new problems whose solutions are, at the very least, unknown to the formulator (Christou et al., 2005; Leung, 1997; Silver et al., 1996). Problem formulation also refers to the activity of transforming a specific question into another form of question. Mestre (2002) states that problem formulation is used as a tool to study cognitive processes and to identify students' knowledge, reasoning, and conceptual development. Cai and Rott (2024) Problem posing is a mental activity in which individuals formulate new problems or modify existing ones based on specific situations, concepts, or solutions. The following are the stages of problem posing according to Cai and Rott's framework: Orientation, in which the individual analyzes and understands the initial situation or problem presented; Connection, in which the individual links various pieces of information, data, or prior mathematical experiences to identify patterns or relationships among the available information to solve the problem; Generation: the phase of formulating or creating a new mathematical problem based on the results of the analysis and the connections that have been established; and Reflection: evaluating whether the newly posed problem can be solved.

The commognitive research that has been conducted is as follows Viirman (2015) investigated the teaching practices of seven mathematics teachers from a commognitive perspective. The results were presented in three categories: explanation, motivation, and question posing. Explanation refers to known mathematical facts, summaries and repetition, different representations, and everyday language. Ioannou (2016) used commognitive theory to observe the activities of pre-service mathematics teachers as they proved the theory of Abelian groups. The results showed that all commognitive components were present during the proof of Abelian groups, but two categories of errors were identified among the students: conceptual errors and errors in the proof process. This research was continued by Ioannou (2018), who used commognitive theory to observe the difficulties experienced by prospective mathematics teachers in group theory. The results showed that there were difficulties related to the object and meta levels in group theory, the relationship between the object and meta levels, and that commognitive conflict occurred when prospective teachers arrived at a solution. Tuset (2018) conducted research on the teaching of prospective teachers in achieving mathematics learning objectives using the commognitive framework. The results showed that the commognitive framework helped analyze the efforts of prospective teachers at two levels. Zayyadi et al. (2020) investigated the pedagogical and content knowledge of prospective mathematics teachers from a commognitive perspective. The results of this study identified the commognitive components of prospective teachers in mathematics learning based on content knowledge, namely word use, visual mediators, routines, and narratives.

Taken together, these studies show a consistent pattern: commognitive components can be identified across various mathematical activities proof, teaching practice, and content knowledge and are typically accompanied by difficulties at the boundary between object-level and meta-level discourse, as well as by commognitive conflict when prospective teachers reach a solution. However, these studies focus mainly on proving, teaching, and solving existing problems; none has specifically examined how the four commognitive components operate when prospective teachers generate new problems (problem posing), a skill argued to be essential for developing mathematical creativity and deeper conceptual understanding. The above studies on commogition indicate that there is a gap in the assessment of students' commogition skills when solving mathematical problems. This research is considered important because mathematics students must possess strong problem-formulation skills. This is because problem formulation can stimulate creativity and plays a key role in improving problem-solving skills and developing creative thinking skills.

The gap identified above is further reinforced by the finding of Isik and Kar (2012) that prospective teachers still face considerable difficulty in formulating problems, which contrasts with the claim that problem posing is essential to mathematical creativity. If this skill is important yet difficult, then a detailed, stage by stage account of how a successful problem-poser actually uses commognitive components is needed to inform how less successful prospective teachers might be supported. Therefore, this study aims to explore the commognitive processes of a high performing prospective mathematics teacher during mathematical problem posing by examining how the four commognitive components word use, visual mediators, routines, and narratives emerge across the orientation, connection, generation, and reflection stages. The novelty of this study lies in its use of the commognitive framework word use, visual mediators, routines, and narratives not to analyze proving or teaching, as in prior studies, but to trace, stage by stage (orientation, connection, generation, and reflection), how a high performing prospective mathematics teacher constructs, solves, and reflects on a newly posed mathematical problem, an aspect that has not been explicitly addressed in previous commognitive research. The findings are expected to contribute to a deeper understanding of prospective mathematics teachers' mathematical thinking and to provide pedagogical insights for designing learning environments that foster effective mathematical problem-posing competencies.

Method

This study is a descriptive study using a qualitative approach. The study was conducted among fourth- and fifth-semester students in the Mathematics Education program at PGRI Adi Buana University in Surabaya. Subjects were selected based on their problem-solving abilities, with those scoring the highest chosen. Data collection was carried out using primary and supplementary instruments. The primary instruments used were a problem-solving test, a problem-posing test, and student interview guidelines. Before these instruments were used, the problem-solving test, problem-formulation test, and interview guidelines were first validated by two mathematics education lecturers holding PhDs in mathematics education. The purpose was to assess whether the content, structure, and language used in the problem-solving test, problem-formulation test, and interview guidelines met the validity criteria. Next, in the data collection phase of this study, 22 students were asked to complete the problem-solving test. From the test results, research subjects with the highest scores were selected and then administered the problem-posing test. The data from the problem-posing test were analyzed through three stages: data reduction, data presentation, and drawing conclusions. The validity of the research results was tested using data triangulation (Moleong, 2007).

Result and Discussion

The students' cognitive performance in posing problems is as follows:

Orientation Stage

During the orientation stage, S1 first represents the problem. Figure 4.1 shows S1 writing down what is known using the symbols f'(x) = 1 - 2x and f(x) as the unknown. The following is a transcript of the interview with S1

P : Please list and explain what information is provided in the problem.
S1 : From the problem, we know the value of the first derivative and one value of the function, and we are asked to find the value of f(x).
P : What information do you already know about this problem?
S1 : I know that the first derivative is given, namely f'(x) = 1 - 2x.
P : Okay. Aside from that derivative, is there any other information known from the problem?
S1 :Yes, there is a function value at a certain point, namely f(3) = 4. This means that when x = 3, the value of the function f(x) is equal to 4.

Diketahui : $f'(x) = 1-2x$ $f(3) = 4$ Ditanya : $f(x)$?	Given: $f'(x) = 1 - 2x$ $f(3) = 4.$ Find: $f(x)$

Figure 1. Commognitive in representing the problem use visual mediator

During the orientation stage, the visual mediators used by S1 were evident when writing mathematical symbols, both during the interview and when representing the problem. The word use observed during the interview included S1 using the terms derivative and function. This matters because, in the commognitive framework, accurately naming and representing what is known and what is asked at the outset is what makes the subsequent stages possible: without first fixing the correct word use and visual mediators for derivative and function, S1 would not have had a stable representation to connect to the integral concept in the next stage.

Connection Stage

S1 connected various pieces of information to solve the problem. To determine the extent of S1's understanding of the problem, an interview with S1 was conducted. The following is an excerpt from that interview.

P : Take a look at the problem provided. What information can you gather from it?
S1 : We know the derivative of the function $f'(x) = 1 - 2x$ and the value of the function when $x = 3$, which is $f(3) = 4$. We are asked to find the function $f(x)$.
P. : When you saw that information, what was your first thought?
S1. : I thought that what was given was the derivative of the function, while what was asked for was the original function.
T: Why did you think that?
S1: Because the notation $f'(x)$ indicates the derivative of a function. If we're looking for the original function, then I have to use an integral.
P: So, what concept did you connect at that moment?
S1: I connected the concept of the derivative to the integral. I remembered that the integral is the inverse of the derivative.
P: So before you started calculating, what connections did you make?
S1: I connected the derivative to the integral, the result of the integral to the constant of integration, and then the constant of integration to the known value of the function. After all that information was connected, I then formulated the steps to solve the problem.

Based on the interview results, S1 connected the information in the problem to mathematical concepts previously learned. When given the information that $f'(x) = 1 - 2x$ and $f(3) = 4$, S1 reasoning involved connecting the concept of differentiation to the concept of integration as the inverse operation of differentiation. This connection is significant, not merely descriptive, because it shows S1 treating the derivative-integral relationship as a narrative resource a stored mathematical fact used to justify a course of action rather than applying integration mechanically; this is what allows the routine that follows (integrating term by term, then solving for the constant) to be executed with justification rather than by rote.

P : Why did you choose that method?
S1 : Because that's the method I usually use when I encounter a problem like this, ma'am.
P : Have you ever solved a similar problem before?
S1 : Yes, ma'am, back in my Calculus 1 class.
P : Okay. Are there any other methods you could have used besides the one you chose?
S1 : As far as I remember, that's the only method—it's the easiest one, ma'am.
P : Did you have any trouble solving that problem?
S1 : No, ma'am.
P : How did you make sure each step was done correctly?
S1 : I double-checked by finding the derivative of $f(x)$. If it's correct, the derivative should match what's given in the problem, which is $f'(x) = 1 - 2x$.
P : Is there anything else you did to make sure your work was correct?
S1 : I also checked that $f(3) = 4$, ma'am. I substituted $x = 3$ into the function $f(x) = -x^2 + x + 10$. The result was $f(3) = -9 + 3 + 10 = 4$, so it matches what's given in the problem.

From the work and the interview, it is evident that S1 solved the problem using the concept of integration, namely $f(x) = \int(1 - 2x)\,dx = x - x^2 + C$. The way the integration concept was written to find the value of f(x) demonstrates a narrative approach. After finding the value of f(x), S1 used the information already given in the problem—namely, $f(3) = 4$—to determine the value of the constant C. The routine followed by Student 1 is shown in Figure 2c. Based on this figure, Student 1 substituted the value $x = 3$ into the equation $f(x) = x - x^2 + C$, resulting in $C = 10$, as shown in the figure. The narrative used by S1 is shown in Figure 2b, where S1 substitutes the value of C into the equation $f(x) = x - x^2 + C$, resulting in $f(x) = x - x^2 + 10$.

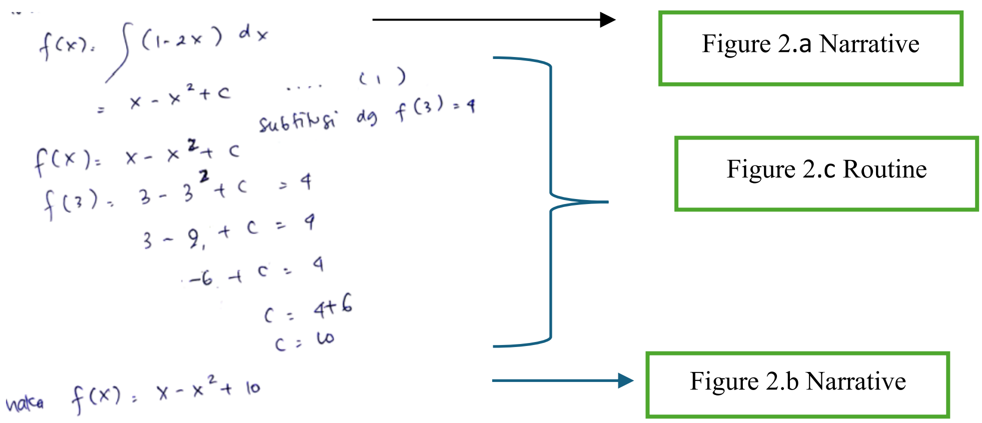

Figure 2. Commognitive in problem-solving

Generation stage

After resolving the initial issues, S1 then raised new issues. The results of the work and the interviews are as follows.

$f'(x) = 1 - 2x + 3e^x$ dan $f(3) = 4$. Tentukan $f(x)$	f'(x) = 1 - 2x + 3e^x and f(3) = 4. Determine the function f(x).

Figure 3. Commognitive results of proposing a new problem use visual mediator and word use

P : Can you create a new problem or question that is more complex than the previous one?
S1 : Yes, I can.
P : All right. Based on what you've already worked on, why did you create this new problem: Given that f'(x) = 1 - 2x + 3e^x and f(3) = 4. Determine the function f(x).
S1 : Because I'm confident I can solve this new problem.
P : All right. What concepts did you use to solve this problem?
S1 : I immediately recognized that this problem is similar to the previous one, but more complex because of the additional term 3e^x. So I knew it was still about finding the original function from its derivative, but with a more complicated derivative form.
P : Okay, were there any other concepts?
S1 : I remembered that to find the original function from its derivative, we need to perform an integral. So I'll integrate 1 - 2x + 3e^x. The addition of e^x reminded me of the integration rule for exponents.

From Figure 3 and the interview results, S1 created a new problem based on what had been solved previously. S1 used the concepts of derivatives and integrals, then added an exponential function. S1's use of visual mediation occurred when writing down the mathematical symbols, and narrative mediation was evident when S1 explained the reasoning behind creating the problem. Next, S1 solved the problem they had created using the concepts explained in the interview above. S1's solution is as follows. This is evidence of narrative and routine working together, not merely in parallel: S1's explanation of why the exponential term was added shows that the modification was guided by a stored narrative about how integration rules generalize, and it is this narrative that licenses the routine of integrating a more complex expression rather than S1 simply guessing the answer would still work.

P : List and explain what information is provided in the problem.
S1 : The problem provides the first derivative of a function, and we are asked to determine the function f(x) or the original function.
P : What information do you already know from this problem?
S1 : I know that the first derivative is given, namely $f'(x) = 1 - 2x + 3e^x$.
P : Good. Aside from that derivative, is there any other information known from the problem?
S1 Yes, there is a function value at a certain point, namely $f(3) = 4$. This means that when $x = 3$, the value of the function f(x) is equal to 4.

During the interview regarding problem representation, the vocabulary used by S1 consisted of mathematical terms such as function, integral, and derivative. The visual mediator used by S1 was writing down mathematical symbols. Subsequently, S1 solved the problem that had been presented. The excerpt from the interview and S1's solution are as follows.

Figure 5. Commognitive in problem-solving

Figure 5.a Narrative

P : What steps did you take to solve this problem?
S1 : First, I integrated each term of f' (x):
$f(x) = \int(1 - 2x + 3e^x)dx = x - x^2 + 3e^x + C$.
Then I used the value $f(3) = 4$ to find C.
P : What was the next step?
S1 : Finding the value of C. Once C was found, I substituted it, and I got $C = 10 - 3e^3$. So $f(x) = x - x^2 + 3e^x + 10 - 3e^3$.

Figure 5.c Routine

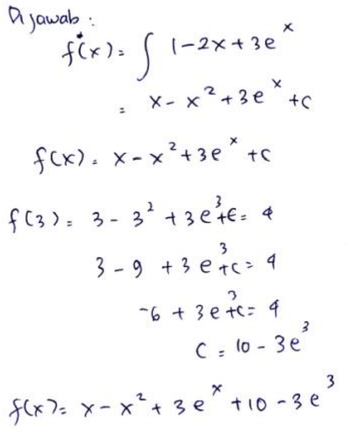

Figure 5.b Narrative

From the work and the interview, it is evident that S1 solved the problem using the concept of integration, namely $f(x) = \int(1 - 2x + 3e^x)\,dx = x - x^2 + 3e^x + C$. The narrative used by S1 involved writing out the integral concept used to find the value of f(x). After finding the value of f(x), S1 used the given information in the problem—namely, $f(3) = 4$—to determine the value of the constant C. The routines performed by S1 are shown in Figure 4.5c, which illustrates the steps used to solve the problem. Based on the figure, S1 substitutes the value $x = 3$ into the equation $f(x) = x - x^2 + 3e^x + C$, resulting in $C = 10 - 3e^3$ as shown in the figure. The narrative of S1's process is shown in Figure 4.5b, where S1 substitutes the value of C into the equation $f(x) = x - x^2 + 3e^x + C$, resulting in $f(x) = x - x^2 + 3e^x + 10 - 3e^3$.

Reflection Stage

After completing the problem, Student 1 reflected by evaluating their work. The results of their work and the interview are as follows:

P: Did you double-check your final answer?
S1: Yes, ma'am, because I was afraid I might not have been thorough enough while working on it.

P: How did you make sure that each step was done correctly?
S1: I tried checking again by deriving f(x). If it's correct, the derivative should match the value given in the problem, which is $f'(x) = 1 - 2x + 3e^x$.
P: Is there anything else you do to make sure your work is correct?
S1: I also checked that f(3) = 4. I substituted x = 3 into the function $f(x) = x - x^2 + 3e^x + 10 - 3e^3$. The result is $f(3) = 3 - 3^2 + 3e^3 + 10 - 3e^3 = 4$, so it matches what's in the problem.
P: Did you try solving the problem in another way to verify the result?
S1: No, ma'am, because I think this method is the easiest.

Based on the work and the interview, S1 reflected on what they had done by substituting the value of x into the equation f(x). The narrative used by S1 was substituting the value x = 3 into the equation $f(x) = x - x^2 + 3e^x + 10 - 3e^3$ to ensure that the resulting function was correct. The importance of this step lies in what it reveals about S1's routine as a whole: by re-deriving f(x) and re-checking f(3) = 4, S1 treated verification itself as a routine grounded in the same narrative used to solve the problem, rather than treating the first answer as automatically correct, this is what distinguishes reflection as metacognitive monitoring from simply finishing a calculation.

The findings demonstrate that the four commognitive components word use, visual mediators, routines, and narratives did not function independently but interacted dynamically throughout the orientation, connection, generation, and reflection stages of mathematical problem posing. Rather than appearing as isolated features of mathematical discourse, these components collectively supported S1 in understanding the problem, constructing mathematical relationships, generating a new problem, and evaluating the correctness of the solution. This finding reinforces Sfard's (2008) proposition that mathematical thinking is fundamentally a form of communication in which cognition is expressed through discourse. The present study further suggests that successful mathematical problem posing depends not only on possessing each commognitive component but also on coordinating them continuously across successive stages of problem posing. In this respect, problem posing should be viewed as a communicative process in which conceptual understanding and procedural reasoning develop simultaneously rather than sequentially.

The orientation and connection stages illustrate how mathematical discourse is initially established before new mathematical ideas can be generated. During orientation, S1 employed appropriate mathematical terminology and symbolic representations to identify the given information and clarify the mathematical objects involved. These findings are consistent with Viirman (2015), who argued that mathematical language and representations play a central role in constructing mathematical meaning, and with Rahmawati and Sulaiman (2021), who reported that accurate use of mathematical symbols facilitates conceptual understanding. However, the present findings extend previous studies by showing that word use and visual mediators are not merely representational tools; instead, they establish a stable mathematical discourse that enables subsequent conceptual connections. During the connection stage, S1 linked derivatives and integrals as inverse operations and integrated prior mathematical knowledge to justify the selected solution strategy. This finding supports Hechter and Combrinck (2022), who emphasized that conceptual understanding is reflected in the ability to relate mathematical ideas meaningfully. Within the commognitive perspective, this derivative–integral relationship functioned as a narrative that justified subsequent routines, demonstrating that procedural actions were guided by conceptual discourse rather than by memorized algorithms alone.

The most significant contribution of this study emerges from the generation stage, where S1 successfully formulated a more complex mathematical problem by incorporating an exponential function into the original task. Previous commognitive studies have primarily examined mathematical proving, classroom discourse, and problem solving (Ioannou, 2016, 2018; Tuset, 2018; Zayyadi et al., 2020), whereas little attention has been given to how commognitive components operate during mathematical problem posing. The present findings indicate that generating a new mathematical problem requires students not only to recall existing concepts but also to reorganize and transform them into a new mathematical structure. The explanations provided by S1 demonstrate that narratives, routines, word use, and visual mediators operated interactively to support justified problem modification rather than arbitrary changes. These findings are consistent with Silver (1997) and Cai and Hwang (2020), who argued that problem posing promotes mathematical creativity and conceptual flexibility. From a commognitive perspective, mathematical creativity emerges through the transformation of existing mathematical discourse into new mathematical discourse.

The reflection stage further demonstrates that successful problem posing extends beyond constructing new problems to critically evaluating their mathematical validity. S1 verified the generated solution by differentiating the obtained function and substituting the given values, indicating that verification itself formed an integral part of the commognitive process rather than merely serving as a final checking procedure. This finding agrees with Kusaka (2022) and Oudman et al. (2022), who emphasized the importance of reflection and metacognitive monitoring in mathematical problem solving. Within the commognitive framework, reflection represents the stage in which mathematical claims are validated through discourse, ensuring consistency between narratives, routines, and mathematical representations. Consequently, reflection strengthens the coherence of the entire problem-posing process and confirms the legitimacy of the newly generated mathematical problem.

Although the present findings are based on a single high-performing prospective mathematics teacher and therefore cannot be generalized to all prospective teachers, they provide a detailed account of how successful commognitive processes unfold during mathematical problem posing. The findings contrast with those of Isik and Kar (2012), who reported that many prospective teachers experience difficulties in formulating mathematical problems, suggesting that different levels of mathematical ability may produce different commognitive patterns, particularly during the generation stage. Theoretically, this study extends Sfard's commognitive framework by demonstrating how the interaction among word use, visual mediators, routines, and narratives supports the generation of new mathematical problems rather than merely explaining mathematical proofs or solving existing tasks. Practically, these findings suggest that mathematics teacher education programs should explicitly develop all four commognitive components through structured problem-posing activities that encourage prospective teachers to represent, justify, communicate, and reflect on their mathematical reasoning.

Conclusion

In conclusion, this study demonstrates that a high-performing prospective mathematics teacher employed all four commognitive components—word use, visual mediators, routines, and narratives—throughout the orientation, connection, generation, and reflection stages of mathematical problem posing. Rather than operating independently, these components interacted dynamically to support the representation, connection, construction, justification, and verification of mathematical ideas. These findings indicate that successful mathematical problem posing is not merely a procedural activity but a commognitive process in which conceptual understanding, mathematical communication, and procedural reasoning are continuously integrated. Theoretically, this study extends Sfard's commognitive framework by demonstrating how the four commognitive components collectively underpin the generation of new mathematical problems, thereby broadening previous commognitive research that has primarily focused on mathematical proving, teaching, and solving existing problems.

Practically, these findings suggest that mathematics teacher education programs should explicitly integrate commognitive-based problem-posing activities that encourage prospective teachers to use mathematical language precisely, represent mathematical ideas through appropriate symbols, justify their reasoning, establish meaningful conceptual connections, and critically evaluate the validity of the problems they generate. Such instructional practices have the potential to strengthen both conceptual understanding and mathematical creativity. Since this study involved only one high-performing prospective mathematics teacher, the findings should be interpreted as an in-depth illustration of successful commognitive processes rather than as representative of all prospective teachers. Therefore, future research should include participants with varying levels of mathematical ability and more diverse educational contexts to examine how commognitive processes develop across different learners and to identify potential commognitive conflicts that may emerge during mathematical problem posing.

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