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The Dynamics of Students' Heuristic Failure and Mathematical Perseverance in Solving Contextual Problems Across Cognitive Demand Levels

Asta Denata Aulya[1*], Sugi Hartono[2]
[1,2] Universitas Negeri Surabaya, Kampus Ketintang Surabaya 60231, Indonesia
Email: asta.23074@mhs.unesa.ac.id

Abstract

This study aimed to explore the dynamics of heuristic failure and mathematical perseverance among students in solving contextual problems across different cognitive demand levels. A descriptive qualitative approach was employed involving three purposively selected tenth-grade students representing high, medium, and low mathematical ability. The research instruments consisted of a mathematics ability test, a contextual problem-solving test on Systems of Linear Equations in Three Variables comprising Procedures with Connections (PWC) and Doing Mathematics (DM) tasks, and retrospective interviews. Data were analyzed using the heuristic failure framework adapted from Schoenfeld and the Three-Phase Perseverance Process (3PP) proposed by DiNapoli and Miller. The findings revealed that the high-ability student experienced heuristic failure at the execution and reflection stages and discontinued effort after reaching an impasse on the DM task. The medium-ability student revised errors and sustained productive effort until obtaining a correct solution, whereas the low-ability student experienced more dominant heuristic failures and did not continue solving the problem after encountering an impasse. The findings indicate that mathematical ability alone did not determine perseverance patterns, particularly when students encountered an impasse in the more cognitively demanding DM task.

Keywords: Heuristic Failure, Mathematical Perseverance, Cognitive Demand Level, Contextual Problems

Abstrak

Penelitian ini bertujuan untuk mengeksplorasi proses commognitive seorang calon guru matematika berkemampuan tinggi dalam mengajukan masalah matematika dengan mengkaji bagaimana empat komponen commognitive, yaitu word use (penggunaan kata), visual mediators (mediator visual), routines (rutinitas), dan narratives (narasi), muncul pada tahap orientasi, koneksi, generasi, dan refleksi. Penelitian ini menggunakan pendekatan deskriptif kualitatif. Partisipan dipilih secara purposive berdasarkan perolehan skor tertinggi pada tes pemecahan masalah matematika. Data dikumpulkan melalui tes pemecahan masalah, tugas pengajuan masalah matematika, dan wawancara semi-terstruktur. Data dianalisis menggunakan tahapan reduksi data, penyajian data, dan penarikan kesimpulan, sedangkan keabsahan data dijamin melalui triangulasi. Hasil penelitian menunjukkan bahwa keempat komponen commognitive muncul pada seluruh proses pengajuan masalah matematika. Pada tahap orientasi, partisipan merepresentasikan ide-ide matematika menggunakan simbol dan istilah matematika yang tepat. Pada tahap koneksi, partisipan membangun hubungan konseptual dengan mengaitkan turunan dan integral sebagai operasi yang saling berkebalikan. Pada tahap generasi, partisipan menyusun masalah matematika yang lebih kompleks dengan melibatkan fungsi eksponensial. Pada tahap refleksi, partisipan memverifikasi solusi melalui substitusi dan justifikasi logis. Temuan ini menunjukkan bahwa pengajuan masalah matematika bukan sekadar aktivitas prosedural, melainkan merupakan proses commognitive yang memperlihatkan interaksi dinamis antara pemahaman konseptual, komunikasi matematis, dan penalaran. Penelitian ini memperluas penerapan kerangka commognitive dalam konteks pengajuan masalah matematika serta memberikan implikasi pedagogis bagi pengembangan kompetensi pengajuan masalah pada calon guru matematika.

Kata kunci: Heuristic Failure, Mathematical Perseverance, Level Cognitive Demand, Masalah Kontekstual

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Introduction

Contextual problem solving is an essential component of mathematics learning because it requires students to connect mathematical concepts with real-life situations, construct appropriate mathematical models, select solution strategies, and interpret the obtained results within the context of the problem (Brinus et al., 2019; Kohen & Nitzan-Tamar, 2022). However, students do not always successfully complete these processes, particularly when they experience difficulties in integrating relevant information, selecting appropriate strategies, or executing the selected procedures (Herold-Blasius, 2025; Jaenudin et al., 2025). Such difficulties may lead to heuristic failure, in which the strategy employed by a student fails to produce meaningful progress and may eventually develop into an impasse during problem solving (DiNapoli & Miller, 2022; Schoenfeld, 1992). Students may respond differently to these obstacles: some discontinue their efforts, whereas others review, revise, or reinitiate their solution strategies, reflecting different patterns of mathematical perseverance (Barnes, 2019; DiNapoli & Miller, 2022). In the present study, these processes were examined among tenth-grade students solving contextual problems involving Systems of Linear Equations in Three Variables across different levels of cognitive demand.

To support students in solving mathematical problems, numerous studies have emphasized the importance of heuristic strategies in problem solving (Wakhata et al., 2023; Hartono et al, 2025). Heuristics are regarded as a set of strategies and techniques that assist students in understanding problems, exploring possible solution paths, and determining appropriate procedures to obtain valid solutions (Kohen & Nitzan-Tamar, 2022). Within mathematical problem solving, heuristic use is commonly associated with understanding the problem, planning a solution, implementing the plan, and reviewing the obtained solution (Agusfianuddin et al., 2025). Through the application of these strategies, students are expected to organize available information systematically and develop more structured mathematical reasoning when solving contextual problems. Nevertheless, employing heuristic strategies does not always lead to successful solutions because students may encounter obstacles that prevent their chosen strategies from being carried through or from producing the expected outcomes.

Failure in applying problem-solving strategies should not be viewed as a phenomenon separate from the problem-solving process but rather as an inherent part of students' attempts to construct solutions. During mathematical thinking, students may experience situations in which they become stuck or reach an impasse, requiring them to redirect their thinking in order to continue solving the problem (Barnes, 2019). This phenomenon is consistent with the findings of DiNapoli & Miller (2022), who described obstacles in problem solving as setbacks or perceived impasses, situations in which students feel unable to proceed and are uncertain about how to continue. Under such circumstances, strategies that initially appeared appropriate may become unsuccessful, fail to produce meaningful progress, or even lead students to dead ends that hinder further problem solving. Despite encountering similar obstacles, however, students do not necessarily respond in the same way when experiencing strategy failure.

The diversity of students' responses to strategy failure indicates that obstacles encountered during problem solving do not always result in the termination of the thinking process. Some students persist with their efforts, whereas others shift to alternative approaches when their initial strategies prove ineffective. In mathematics education, such behaviors are closely related to the concept of *mathematical perseverance*, defined as students' capacity to initiate, sustain, and re-engage in productive struggle when confronted with obstacles, setbacks, and discouragement during problem solving (DiNapoli & Miller, 2022). Furthermore, perseverance is reflected in students' ability to modify or adapt their solution plans when difficulties arise, allowing the search for a solution to continue. Nevertheless, not all students are able to sustain these efforts, as some discontinue the problem-solving process when they perceive the encountered obstacles to be too difficult to overcome (Rasyid, 2025). These variations suggest that the characteristics of mathematical tasks also influence how students employ problem-solving strategies and demonstrate perseverance throughout the problem-solving process.

The characteristics of mathematical tasks constitute one of the factors influencing how students think and act while solving problems (Siswono et al, 2026; Hartono et al, 2024). Wakhata et al. (2023) explained that *cognitive demand* is closely associated with students' opportunities to engage in deep mathematical thinking and reasoning while working on mathematical tasks. In addition, task characteristics are related to the development of students' conceptual understanding during learning. Tasks with different levels of cognitive demand present different challenges, thereby influencing how students apply problem-solving strategies and sustain their efforts when encountering obstacles. Thus, cognitive demand is not merely associated with task difficulty but also with the opportunities it provides for various cognitive processes to emerge during mathematical problem solving. Differences in these cognitive processes may lead to variations in students' responses to obstacles, particularly when viewed in relation to their mathematical ability.

Differences in students' mathematical ability may influence how they understand problems, select solution strategies, and respond to obstacles encountered throughout the problem-solving process. This is consistent with the findings of Jaenudin et al. (2025) who reported that differences in mathematical ability are reflected in distinct problem-solving characteristics, with high-achieving students tending to complete each stage of problem solving more effectively than low-achieving students. Other studies have likewise shown that students experiencing difficulties in mathematics often fail to sustain their efforts when confronted with obstacles during problem solving (Zibar Parisu, 2025). These findings indicate that mathematical ability is associated not only with successful problem resolution but also with how students cope with strategy failure and maintain their efforts throughout the search for a solution. Nevertheless, the relationship among heuristic failure, mathematical perseverance, and students' mathematical ability remains insufficiently explored.

Research on heuristics in mathematical problem solving has expanded considerably, demonstrating that appropriate strategy use plays a crucial role in supporting successful problem solving (Herold-Blasius, 2025; Saygılı, 2017; Schoenfeld, 1992) Meanwhile, studies on *mathematical perseverance* have shown that sustaining and renewing effort when facing obstacles constitutes an essential component of successful mathematical problem solving (Barnes, 2019; Dinapoli & Miller, n.d.; Wilburne & Dause, 2017). Other studies have also demonstrated that the cognitive demand of mathematical tasks influences students' thinking processes and strategy use during problem solving (Wakhata et al., 2023). However, previous research has primarily focused on the effectiveness of heuristic strategies, the development of mathematical perseverance, or the influence of cognitive demand on students' performance. Few studies have specifically examined how heuristic failure emerges during problem solving, how students respond to such failures through mathematical perseverance, and how these two constructs interact across students with different levels of mathematical ability. Furthermore, the relationship among heuristic failure, mathematical perseverance, and cognitive demand has rarely been investigated within a single integrated framework. This gap provides an opportunity to gain a deeper understanding of students' thinking processes when encountering obstacles while solving contextual mathematical problems.

A deeper understanding of students' thinking processes is essential for revealing how they respond to obstacles and sustain their efforts during mathematical problem solving. Accordingly, this study aims to explore the dynamics of heuristic failure and mathematical perseverance among students with high, medium, and low mathematical ability when solving contextual problems at the Procedures with Connections and Doing Mathematics cognitive demand levels. Theoretically, this study contributes to mathematical problem-solving research by integrating heuristic failure, mathematical perseverance, mathematical ability, and cognitive demand within a single analytical framework. Practically, the findings may assist teachers in identifying the stages at which students experience difficulties, designing mathematical tasks with graduated levels of cognitive demand, and providing appropriate scaffolding that encourages students to review their representations, evaluate the effectiveness of their strategies, and reinitiate their efforts after reaching an impasse.

Method

This study employed a qualitative multiple-case study design with a descriptive orientation to explore the dynamics of students' heuristic failure and mathematical perseverance in solving contextual problems across different cognitive demand levels. Each of the three selected participants was treated as an individual case representing high, medium, or low mathematical ability. The multiple-case study design enabled an in-depth examination of each participant's problem-solving process, followed by a cross-case comparison to identify similarities and differences across mathematical ability categories and cognitive demand levels. Data were collected from the participants' written solutions and retrospective interviews conducted after they had completed the problem-solving tasks.

The study was conducted with 36 tenth-grade students at 17 Agustus 1945 Senior High School, Surabaya. All participants completed a mathematics ability test covering the prerequisite concepts required for solving Systems of Linear Equations in Three Variables (SLETV). The test results were used to classify the students into high, medium, and low mathematical ability groups based on predetermined score ranges.

Table 1. Mathematical Ability Classification Criteria

Category	Score Range	Ability Indicators
High	80-100	Demonstrated mastery of all prerequisite concepts for solving Systems of Linear Equations in Two Variables (SLETV), performed accurate computations, and correctly constructed mathematical models for routine problems.
Medium	60-79	Demonstrated mastery of most prerequisite concepts but occasionally made computational errors or experienced minor difficulties in constructing mathematical models.
Low	< 60	Demonstrated limited understanding of fundamental algebraic concepts, frequently made errors in basic arithmetic operations, and was unable to construct appropriate mathematical models.

Subsequently, the research participants were selected using purposive sampling based on the results of the mathematics ability test and recommendations from the mathematics teacher. Three students were selected to represent the three mathematical ability groups: a high-ability student (HS), a medium-ability student (MS), and a low-ability student (LS).

The research instruments consisted of a mathematics ability test, a contextual problem-solving test on Systems of Linear Equations in Three Variables (SLETV), and a retrospective interview protocol. Each instrument was designed to collect different types of data in accordance with the objectives of the study. The mathematics ability test served as the preliminary instrument for classifying students according to their mathematical ability. The test assessed prerequisite knowledge required for solving SLETV problems, including algebraic operations, linear equations, systems of linear equations in two variables, elimination and substitution procedures, and mathematical modeling. The test results were used to classify students into high, medium, and low mathematical ability groups prior to participant selection.

The primary research instrument was a contextual problem-solving test on SLETV consisting of two problems representing different cognitive demand levels. The tasks were developed based on the cognitive demand framework proposed by Stein et al. The first problem was categorized as Procedures with Connections (PWC) because it required students to connect contextual information with appropriate mathematical representations while applying previously learned procedures. In contrast, the second problem was categorized as Doing Mathematics (DM) because it required students to explore solution strategies, engage in mathematical reasoning, and make decisions throughout the problem-solving process. The classification of the problems according to their cognitive demand levels is presented in Table 2.

Table 2. Classification of Contextual Mathematical Problems by Cognitive Demand Level

Problem	***Cognitive demand* Level**
Problem 1	Procedures with Connections
Problem 2	Doing Mathematics

A retrospective interview was conducted after the students had completed the contextual problem-solving test. The interview aimed to explore students' thinking processes that could not be fully identified from their written solutions. The interview questions focused on the problem-solving process, the occurrence of heuristic failure at each stage of problem solving, and students' responses when encountering obstacles during problem solving. The interview data were used to confirm and enrich the findings derived from the analysis of the students' written solutions.

Data were collected in several stages. The first stage involved administering a mathematics ability test to all students in one tenth-grade class to obtain information about their levels of mathematical ability. Based on the test results, the students were classified into high, medium, and low mathematical ability groups. Subsequently, the research participants were selected using purposive sampling based on the mathematics ability test results and recommendations from the mathematics teacher. The second stage involved administering the contextual SLETV problem-solving test to the three selected participants representing the three mathematical ability groups. During the test, the participants were instructed to solve each problem independently and document their complete solution processes on the provided answer sheets. The test was initially planned to be administered in a quiet and conducive classroom environment. However, during the test administration, an unanticipated presentation by a private tutoring institution was conducted in the same classroom. Consequently, the classroom became relatively noisy and less conducive, which may have affected the participants' concentration during the problem-solving process. The high-ability participant also reported difficulty concentrating under these conditions during the retrospective interview. Therefore, this situational context was considered when interpreting the findings, particularly the participant's response after encountering an impasse.

After completing the test, each participant took part in a retrospective interview. The interview was conducted to explore the students' thinking processes while solving the problems, including how they understood the problems, selected solution strategies, encountered obstacles, and responded to difficulties throughout the problem-solving process. The overall data collection procedure employed in this study is illustrated in Figure 1.

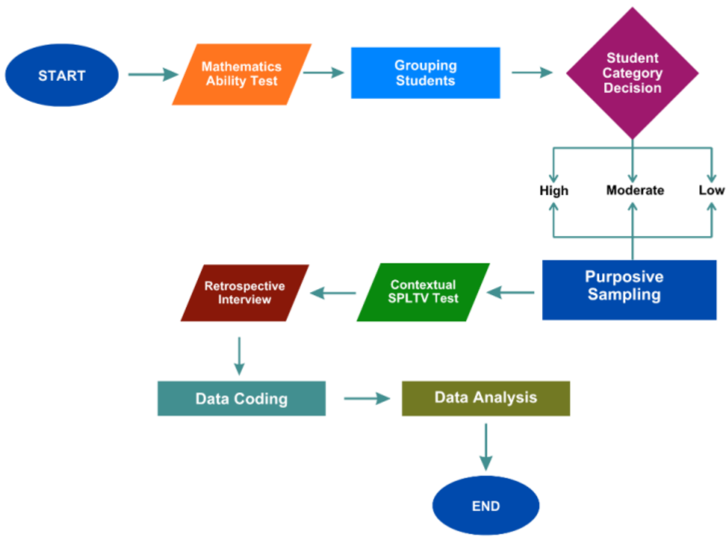

Figure 1. Research Procedure

The data, consisting of students' written solutions and retrospective interview transcripts, were analyzed qualitatively through several stages. The first stage involved data reduction, in which information relevant to the research objectives was selected. The second stage involved coding based on the indicators of heuristic failure and mathematical perseverance. The coding of heuristic failure was guided by the six stages of problem solving adapted from Schoenfeld (1992), *understanding problem*, *representation*, *strategy selection*, *execution*, *monitoring*, and *reflection*. Meanwhile, the coding of mathematical perseverance was based on the *Three-Phase Perseverance Process* (3PP) framework proposed by DiNapoli & Miller (2022), which consists of the entrance, initial attempt, impasse, and additional attempt phases.

Each piece of evidence identified from the students' written solutions and interview transcripts was coded according to the corresponding indicators. Subsequently, source triangulation was conducted by comparing the written and interview data to ensure the accuracy of the coding and the credibility of the interpretations. The coded data were then analyzed to identify the dynamics of heuristic failure and mathematical perseverance demonstrated by each participant while solving contextual problems across different cognitive demand levels. Finally, the patterns identified among the high-, medium-, and low-ability participants were compared to characterize the dynamics associated with each mathematical ability group.

Table 3. Coding Framework for Heuristic Failure

Stage	Code
Understanding Problem	UP
Representation	REP
Strategy Selection	STR
Execution	EXE
Monitoring	MON
Reflection	REF

Each coding category comprised several subcategories representing successful performance, heuristic failure, or incomplete performance at each stage of the problem-solving process..

Table 4. Coding Framework for Mathematical Perseverance

Phase	Code	Indicator
Entrance Phase	CLA	Clarity
	INO	Initial Obstacle
Initial Attempt Phase	INE	Initial Effort
	SUE	Sustained Effort
	OUE	Outcome of Effort
Impasse	IMD	Impasse Detection
	RES	Response to Impasse
Additional Attempt Phase	REI	Re-Initiated Effort
	RES 2	Re-sustained effort
	OUN	Outcome of New Effort

These codes were used to identify and compare the patterns of heuristic failure and mathematical perseverance demonstrated by each participant while solving contextual problems across different cognitive demand levels. To ensure the credibility of the data, source triangulation was conducted by comparing the students' written solutions with the retrospective interview data. Both data sources were analyzed in an integrated manner to ensure the consistency of the findings and enhance the accuracy of the interpretations of students' thinking processes during contextual problem solving. In addition to source triangulation, member checking was conducted by asking each participant to confirm the researchers' interpretations of their written solutions and interview responses. The participants were given opportunities to clarify or correct interpretations that did not accurately represent their problem-solving experiences. Any clarifications provided by the participants were incorporated into the final analysis.

Result and Discussion

To provide an overview of the occurrence of heuristic failure and mathematical perseverance indicators, the analysis results of the three participants across the different cognitive demand levels are summarized in Tables 5 and 6.

Table 5. Occurrence of Heuristic Failure and Mathematical Perseverance Indicators in Problem 1

Student	*Heuristic*							*Mathematical perseverance*								
	UP	REP	STR	EXE	MON	REF	CLA	INO	INE	SUE	OUE	IMD	RES	REI	RES2	OUN
HS	✓	✓	✓	△	✓	△	✓	✓	✓	✓	✓	-	-	-	-	-
MS	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
LS	✓	✗	✗	✗	✗	△	△	✓	✓	✓	✓	✓	✗	✗	✗	✗

Table 6. Occurrence of Heuristic Failure and Mathematical Perseverance Indicators in Problem 2

Student	***Heuristic***							***Mathematical perseverance***								
	UP	REP	STR	EXE	MON	REF	CLA	INO	INE	SUE	OUE	IMD	RES	REI	RES2	OUN
HS	✓	✓	✓	△	✓	✗	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗
MS	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	-	-	-	-	-
LS	✓	✓	△	✗	✗	△	✓	✓	✓	✓	✓	✓	✗	✗	✗	✗

In Tables 5 and 6, the symbol ✓ indicates that an indicator was fulfilled, △ indicates partial fulfillment, ✗ indicates that an indicator was not fulfilled, and indicates that the indicator was not applicable because the corresponding phase did not occur during the problem-solving process. As shown in Tables 5 and 6, the three participants demonstrated distinct patterns of heuristic failure and mathematical perseverance across the two cognitive demand levels. In the Procedures with Connections (PWC) problem, HS completed nearly all problem-solving stages but omitted the final contextual calculation, MS recognized and corrected a computational error, whereas LS experienced difficulties beginning at the representation stage. In the Doing Mathematics (DM) problem, HS reached an impasse during execution and did not demonstrate an observable additional attempt, while MS completed the task without encountering an impasse. LS showed improvement by constructing an appropriate mathematical model in the DM problem, but difficulties in strategy selection and execution still developed into an impasse without an additional attempt. Overall, the participants differed not only in the stages at which heuristic failure emerged but also in how they recognized and responded to obstacles during problem solving. The findings for each participant are presented in the following sections.

High-Ability Student (HS)

As summarized in Tables 5 and 6, the high-ability student (HS) exhibited distinct patterns of heuristic failure and mathematical perseverance across the Procedures with Connections (PWC) and Doing Mathematics (DM) problems. In the PWC problem, nearly all heuristic failure and mathematical perseverance indicators were fulfilled, although partial fulfillment was observed at the execution and reflection stages. In contrast, the DM problem involved an impasse that prevented the solution process from progressing to the final solution.

Problem 1: Procedures with Connections (PWC)

In solving the Procedures with Connections (PWC) problem, HS demonstrated a systematic problem-solving process. The participant successfully understood the information presented in the problem, represented it using an appropriate mathematical model, and selected the substitution method as the solution strategy. HS then applied the selected strategy consistently until the values of all required variables were successfully determined.

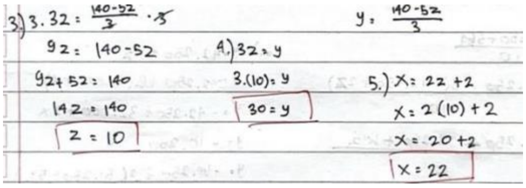

Figure 2. HS's Written Solution to the PWC Problem

However, the solution process was not fully completed. Although HS successfully determined the values of all variables, the final step of calculating the price of the *Art Package* was omitted, resulting in an incomplete solution. This finding indicates that heuristic failure occurred during the execution stage and subsequently affected the reflection stage. At the execution stage, the solution process was not completed to obtain the final answer, whereas at the reflection stage, the obtained results were not interpreted within the context of the problem. This finding was supported by the following interview excerpt.

> **HS:** *"Yes, I had reached that point, but I forgot to add them together. I forgot to calculate the final answer."*

The interview indicates that HS recognized that the solution had not reached the intended goal despite successfully obtaining all variable values. The incomplete solution was not caused by a misunderstanding of the problem or an inappropriate strategy selection but rather by overlooking the final calculation after determining all required variables. Despite experiencing heuristic failure during the execution stage, HS demonstrated mathematical perseverance throughout the problem-solving process. This was reflected in the participant's ability to maintain the substitution strategy from the beginning until all variable values had been obtained. Consequently, the entrance and initial attempt phases were completed successfully because no evidence of impasse emerged during the solution process.

> **HS:** *"For the first problem, I used substitution because the numbers were whole numbers, so the calculations were straightforward and the final answer was easy to obtain."*

This statement indicates that HS encountered no substantial obstacles while implementing the selected strategy. Therefore, the heuristic failure did not develop into an impasse but remained an incomplete execution of the final step. Consequently, the additional attempt phase did not occur because no obstacle required HS to restart or modify the solution process.

Problem 2: Doing Mathematics (DM)

When solving the Doing Mathematics (DM) problem, HS initially demonstrated a systematic problem-solving process. The participant successfully understood the given information, represented it using an appropriate mathematical model, and selected substitution as the solution strategy. HS consistently implemented this strategy through elimination and substitution, successfully deriving intermediate equations required for solving the problem.

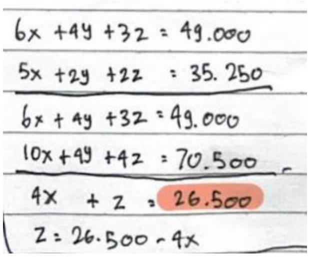

Figure 3. HS's Written Solution to the DM Problem

However, the solution process did not reach the expected final answer. HS encountered an error during the execution stage, causing the solution process to stop before obtaining the required solution. Consequently, heuristic failure occurred during the execution stage and subsequently affected the reflection stage. As a result, HS failed to produce a final solution that addressed the contextual problem, and the reflection stage was therefore considered unsuccessful. The interview data further supported this finding.

HS: *"Because it's an SLETV problem, there wasn't any requirement to use a particular method."*

HS: *"I used substitution, but it never seemed to end. The more I continued, the more confused I became because I kept going in circles."*

These excerpts indicate that HS independently selected the solution strategy based on personal understanding of the problem. However, the chosen strategy failed to lead to a complete solution, causing the problem-solving process to terminate before the intended goal was achieved. Unlike the PWC problem, the DM problem presented a higher level of complexity. Nevertheless, HS demonstrated mathematical perseverance during both the entrance and initial attempt phases. The participant identified the goal of the problem, recognized its level of difficulty, and immediately initiated the solution process by constructing a mathematical model.

HS: *"The goal was to determine the total amount that Rendi had to pay."*

HS: *"The second problem was more complicated because the numbers were in thousands and not whole numbers."*

HS: *"I turned them into variables to make the calculations easier."*

Furthermore, HS consistently maintained the selected strategy and made substantial progress through elimination and substitution.

HS: *"...then I used elimination, and after obtaining one variable..."*

HS: *"...once I found the value of z, substituting it into the other equations became easier."*

These statements indicate that the entrance and initial attempt phases were successfully completed because HS consistently maintained the selected strategy throughout most of the solution process. However, unlike the PWC problem, the solution process eventually developed into an impasse. After repeatedly applying substitution, HS was unable to determine the next step, causing the thinking process to come to a halt.

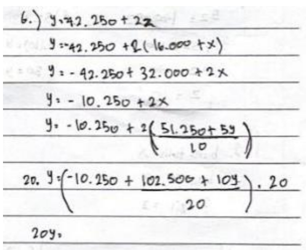

Figure 4. Excerpt of HS's Written Solution Showing the Impasse During the DM Problem

The interview further confirmed this condition.

HS: *"I used substitution, but it never seemed to end. The more I continued, the more confused I became because I kept going in circles."*

This statement indicates that HS recognized having reached an impasse, as the selected strategy no longer produced meaningful progress toward solving the problem. Despite recognizing the impasse, HS did not attempt to revise the solution or explore an alternative strategy.

HS: *"No, because the situation wasn't conducive. Even if I started over, I thought the result would be the same since I couldn't concentrate."*

This excerpt indicates that HS decided not to reinitiate the solution process after reaching an impasse. However, this decision was influenced not only by difficulties in determining an effective next step but also by the less conducive classroom environment during the test administration. At that time, a tutoring-program presentation was being conducted in the same classroom, resulting in noise and distractions that interfered with the participant's concentration. Therefore, the absence of an additional attempt should not be interpreted solely as a lack of willingness to persevere, but also as a response shaped by situational constraints. Nevertheless, within the observable problem-solving process, HS did not review the previous calculations, restart the solution, or explore an alternative strategy after recognizing the impasse. Accordingly, the indicators of re-initiated effort, re-sustained effort, and outcome of new effort were not identified. Overall, HS maintained the selected strategy until reaching an impasse but did not demonstrate an observable subsequent attempt to overcome the obstacle under the situational conditions present during the test administration.

Medium-Ability Student (MS)

As summarized in Tables 5 and 6, the medium-ability student (MS) exhibited a different pattern of heuristic failure and mathematical perseverance from that of the high-ability student (HS). Across both the Procedures with Connections (PWC) and Doing Mathematics (DM) problems, all heuristic failure and mathematical perseverance indicators were fulfilled. Nevertheless, the participant encountered several computational errors and obstacles during the problem-solving process. Unlike HS, who discontinued the solution process after reaching an impasse in the DM problem, MS recognized the errors, revised the solution, and continued the problem-solving process until obtaining the correct answer. These findings indicate that the impasse experienced by MS served as the starting point for an additional attempt, enabling the participant to successfully complete the problem.

Problem 1: Procedures with Connections (PWC)

In solving the Procedures with Connections (PWC) problem, MS demonstrated a systematic problem-solving process. The participant successfully understood the information presented in the problem, represented it using an appropriate mathematical model, selected substitution as the solution strategy, and implemented the procedure until the correct final answer was obtained.

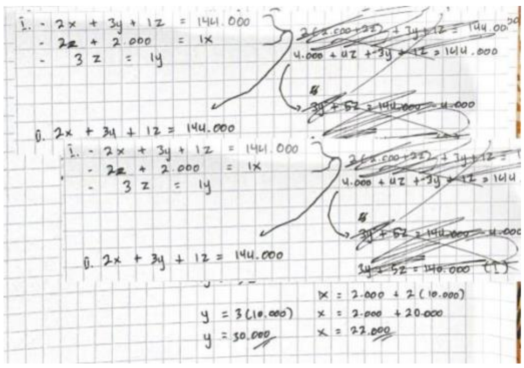

Figure 5. MS's Written Solution to the PWC Problem

This finding was supported by the following interview excerpt.

MS: *"I used substitution. Some people prefer elimination because it's easier, but I like combining both methods."*

During the solution process, MS made a computational error, as indicated by revisions in the written solution. However, the participant recognized the error before it affected the final answer.

MS: *"I made a calculation mistake because I wasn't thinking carefully enough. I missed something."*

MS: *"I checked my work again."*

These excerpts indicate that heuristic failure temporarily occurred during the execution stage due to a computational error. However, the error did not result in an unsuccessful solution because MS recognized the mistake, reviewed the solution process, and corrected the calculations before arriving at the correct answer. Consequently, the reflection stage was completed successfully. Although heuristic failure occurred during the execution stage, MS continued to demonstrate mathematical perseverance. The participant maintained the initially selected strategy throughout the solution process and evaluated the solution after recognizing the computational error.

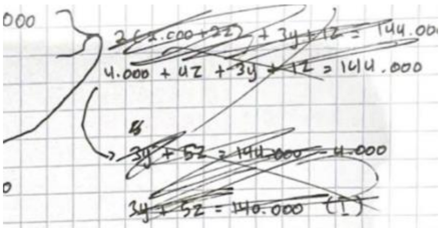

Figure 6. MS's Written Solution to the PWC Problem

This finding was further supported by the following interview excerpt.

MS: *"I'm confident that my final answer is correct."*

This statement indicates that after recognizing the error, MS did not discontinue the problem-solving process. Instead, the participant reviewed the solution, corrected the identified error, and continued working until obtaining the correct answer. Thus, the heuristic failure developed into an impasse, which was followed by an additional attempt involving review and revision of the solution. As a result, all phases of mathematical perseverance were successfully completed, leading to the correct final solution.

Problem 2: Doing Mathematics (DM)

In solving the Doing Mathematics (DM) problem, MS demonstrated a systematic problem-solving process from the beginning to the end of the task. The participant successfully understood the information presented in the problem, represented it using an appropriate mathematical model, selected an elimination–substitution strategy, and implemented the solution procedure until obtaining a correct answer that addressed the contextual problem.

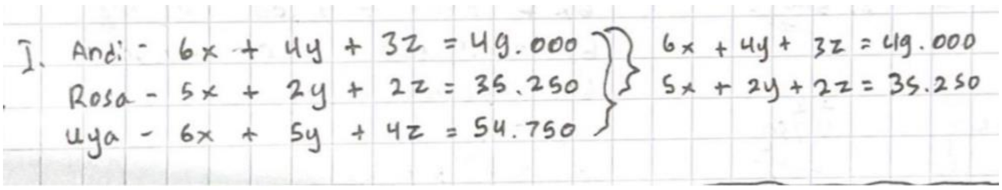

Figure 7. MS's Written Solution to the DM Problem

This finding was supported by the following interview excerpt.

MS: *"Here, Rendi buys the same number of notebooks as Rosa, so he buys five notebooks. He also buys the same number of pens as Andi, which is four, and finally three pencils. Therefore, I represented the mathematical model as 5x+4y+3z."*

After constructing the mathematical model, MS deliberately selected the elimination–substitution strategy based on the characteristics of the problem.

MS: *"Substitution alone is not recommended because, in the first problem, some variables could still be isolated, for example, 3z=y. However, in the second problem there are three variables, so substitution alone is less appropriate."*

These excerpts indicate that the participant consciously selected the solution strategy by considering the structure of the problem. After establishing the mathematical model, MS systematically applied elimination and substitution to determine the values of all variables correctly.

MS: *"After obtaining the new equations through elimination, I immediately continued with substitution."*

Furthermore, MS reviewed the completed solution, and no errors affecting the final answer were identified. These findings indicate that all stages of the heuristic framework were successfully completed, with no evidence of heuristic failure throughout the problem-solving process. The participant successfully understood the problem, selected an appropriate strategy, implemented the solution procedure systematically, and evaluated the final solution to obtain the correct answer. From the perspective of mathematical perseverance, MS also demonstrated a consistent problem-solving process throughout the task. The participant identified the problem goal, recognized that the problem was more complex than the previous one, and maintained the selected strategy until the final solution was obtained.

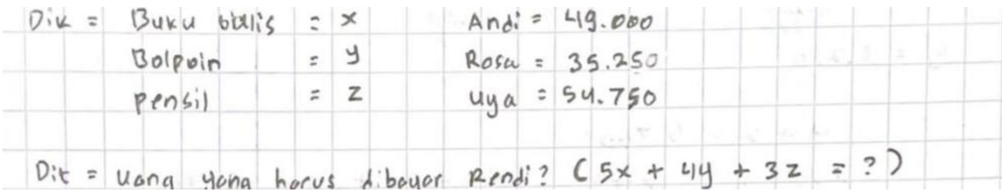

Figure 8. MS's Written Solution to the DM Problem

This finding was supported by the following interview excerpts.

MS: *"For the problem, I represented notebooks as x, pens as y, and pencils as z."*

In addition, MS recognized that the problem was more complex than the previous task.

MS: *"Yes, this problem is more complex and longer, but I'm already quite familiar with this kind of problem."*

These statements indicate that the entrance and initial attempt phases were successfully completed. Although the problem involved a higher level of complexity, MS maintained the selected strategy without encountering obstacles that disrupted the problem-solving process. This finding was further supported by the following interview excerpt.

MS: *"No, I didn't."*

This response indicates that the solution process proceeded smoothly without reaching an impasse. Consequently, the additional attempt phase did not occur because MS successfully completed the problem using the initial strategy without requiring revisions or alternative solution strategies. Overall, MS demonstrated the ability to overcome minor computational errors through an additional attempt in the PWC problem while successfully solving the more cognitively demanding DM problem without experiencing an impasse.

C. Low-Ability Student (LS)

As summarized in Tables 5 and 6, the low-ability student (LS) exhibited more instances of heuristic failure than both the high-ability student (HS) and the medium-ability student (MS). In the Procedures with Connections (PWC) problem, heuristic failure emerged at the representation stage and continued through the execution stage. In contrast, although LS successfully constructed an appropriate mathematical model in the Doing Mathematics (DM) problem, difficulties arose when selecting and implementing an appropriate solution strategy, causing the problem-solving process to terminate before reaching the final solution. From the perspective of mathematical perseverance, LS demonstrated an initial effort to solve the problems but did not engage in any additional attempt after reaching an impasse.

Problem 1: Procedures with Connections (PWC)

When solving the Procedures with Connections (PWC) problem, LS successfully identified the essential information and understood the objective of the problem. However, difficulties emerged when the participant attempted to represent the contextual information as a mathematical model. Although LS assigned variables, the resulting mathematical model did not accurately represent the information provided in the problem, preventing the solution process from proceeding appropriately.

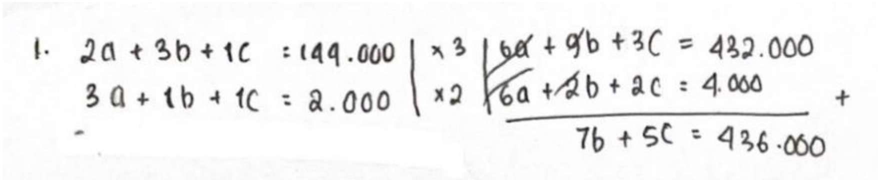

Figure 9. LS's Written Solution to the PWC Problem

This finding was supported by the following interview excerpts.

LS: *"No."*

LS: *"Yes, I still don't understand."*

These excerpts indicate that heuristic failure first occurred during the representation stage. The participant's inability to construct an appropriate mathematical model prevented the contextual information from being transformed into a solvable mathematical representation. The error at the representation stage subsequently affected the strategy selection stage. LS selected the elimination method not because of a sound procedural understanding but simply because the method had been used previously.

LS: *"Elimination, because it's easier."*

LS: *"I don't know."*

LS: *"I think they should be added together."*

These statements indicate that the selected strategy was not supported by sufficient procedural understanding. Consequently, LS was unable to carry out the elimination process systematically, and the heuristic failure continued into the execution stage.

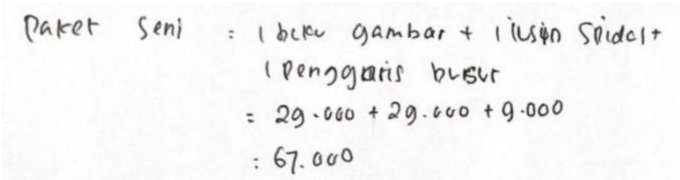

Figure 10. LS's Written Solution to the PWC Problem

The interview further supported this finding.

LS: *"I calculated it manually... I just made it up."*

This statement indicates that LS continued writing an answer despite not following an appropriate solution procedure. Furthermore, the participant did not review the completed work, resulting in a final answer that failed to address the problem correctly. Despite experiencing multiple instances of heuristic failure, LS demonstrated mathematical perseverance during the initial phases of problem solving. The participant identified the goal of the problem and attempted to initiate the solution using a previously remembered strategy.

LS: *"Um... calculating... the cost price, determining the original price."*

In addition, LS attempted to solve the problem using the remembered strategy. However, when applying this strategy, the participant was unable to determine the next step, causing the solution process to develop into an impasse.

LS: *"The difficult part was the one about the higher price."*

This statement indicates that LS recognized the presence of an obstacle but did not know how to proceed. Consequently, the heuristic failures that occurred during the representation and strategy selection stages developed into an impasse. After reaching the impasse, LS made no further attempt to revise the solution or explore an alternative strategy.

LS: *"As long as I wrote an answer."*

LS: *"No, I just wanted to finish quickly. The important thing was to write something."*

These excerpts indicate that LS chose to discontinue the problem-solving process after reaching an impasse. Unlike MS, who reviewed and corrected the solution, LS provided an answer without revising the solution process. Consequently, the response to impasse, re-initiated effort, re-sustained effort, and outcome of new effort phases were absent from the solution process.

Problem 2: Doing Mathematics (DM)

When solving the Doing Mathematics (DM) problem, LS demonstrated some improvement compared with the performance on the Procedures with Connections (PWC) problem. The participant successfully understood the information presented in the problem and represented it using an appropriate mathematical model. LS correctly assigned variables and formulated the system of linear equations based on the purchasing information provided in the problem.

$$
\begin{aligned}
6t + 4b + 3p &= 49.000 \quad * \text{ Andin} \\
5t + 2b + 2p &= 35.250.00 \quad \text{Rosa} \\
6t + 5b + 4p &= 54.750.00 \quad \text{Uya}
\end{aligned}
$$

Figure 11. LS's Written Solution to the DM Problem

However, difficulties emerged once the mathematical model had been constructed. LS selected the elimination method but lacked a complete understanding of the procedure, making it impossible to continue the elimination process until the values of the variables were obtained. This finding was supported by the following interview excerpts.

LS: *"Elimination."*

LS: *"I don't know, Ma'am. As far as I remember, elimination means adding them together. Since both signs are positive, I thought they should be added."*

These excerpts indicate that although LS recognized the appropriate strategy, the participant did not fully understand the elimination procedure. Consequently, heuristic failure occurred during the strategy selection stage and continued into the execution stage because the participant was unable to complete the solution process correctly. As a result, LS did not review either the solution process or the final answer. The written solution showed that the participant directly wrote a final answer without following a systematic solution procedure.

LS: *"No, I just wanted to finish quickly. The important thing was to write something."*

This statement indicates that LS did not engage in monitoring during the solution process. Although the participant understood the objective of the problem, the final answer was not obtained through an appropriate mathematical procedure. Therefore, the reflection stage was considered only partially fulfilled. From the perspective of mathematical perseverance, LS nevertheless demonstrated an initial effort to solve the problem. Unlike in the PWC problem, LS successfully constructed an appropriate mathematical model and identified the problem objective before attempting to apply the elimination method. This finding was supported by the following interview excerpt.

LS: *"To determine the total amount that Rendi had to pay."*

This statement indicates that LS recognized the problem goal and understood that the task required several solution steps. In addition, the participant demonstrated an initial effort by constructing the mathematical model before attempting the elimination procedure. However, during the elimination process, LS became unable to determine the next step, causing the solution process to develop into an impasse.

$$
\begin{aligned}
5t + 2b + 2p &= 35.250 \;\; |\times 4 \quad 20t + 8b + \cancel{8p} = 70.500 \\
6t + 5b + 4p &= 54.750 \;\; |\times 2 \quad 12t + 10b + \cancel{8p} = 105.500 \\
& \qquad\qquad\qquad\;\; \overline{32t + 18b \qquad = 176.000} \; +
\end{aligned}
$$

Figure 12. LS's Written Solution Showing the Impasse During the DM Problem

LS: *"I got confused at that part."*

This statement indicates that LS recognized the obstacle but did not know how to continue the solution process. Accordingly, the heuristic failure that emerged during the strategy selection stage developed into an impasse. Despite recognizing the impasse, LS made no attempt to revise the solution or explore an alternative strategy.

LS: *"No, I just wanted to finish quickly. The important thing was to write something."*

This excerpt indicates that LS chose to discontinue the solution process after reaching an impasse. Unlike MS, who reviewed and revised the solution, LS terminated the problem-solving process without making any corrections. Consequently, the response to impasse, re-initiated effort, re-sustained effort, and outcome of new effort phases were absent. Overall, LS experienced heuristic failure from the early and middle stages of problem solving, which developed into an impasse that was not followed by any additional effort.

The findings revealed that the high-ability student (HS) demonstrated different problem-solving dynamics across the Procedures with Connections (PWC) and Doing Mathematics (DM) problems. In the PWC problem, HS successfully understood the problem, represented the contextual information using an appropriate mathematical model, and maintained the selected solution strategy throughout nearly the entire problem-solving process. Heuristic failure occurred only at the final stage when the participant failed to complete the last step required to obtain the answer within the problem context, and therefore did not develop into an impasse. In contrast, in the DM problem, HS encountered difficulties during the execution stage, causing the solution process to develop into an impasse and terminate before the final solution was reached.

These findings suggest that successful performance during the understanding the problem, representation, and strategy selection stages does not necessarily guarantee successful completion of the entire problem-solving process. According to Schoenfeld (1992), successful mathematical problem solving depends not only on understanding the problem and selecting an appropriate strategy but also on monitoring and evaluating the solution process. In the present study, HS was able to identify appropriate solution strategies for both problems but experienced difficulty maintaining the effectiveness of the selected strategy when solving the problem with a higher cognitive demand. Consequently, the heuristic failure experienced by HS was not attributable to inadequate problem comprehension but rather to difficulties in executing and completing the selected strategy.

This pattern is also consistent with Wakhata et al. (2023), who emphasized that tasks with higher cognitive demand require students to coordinate several forms of reasoning and make strategic decisions throughout the solution process. Although HS possessed sufficient conceptual knowledge to understand and represent the DM problem, the greater strategic and procedural demands of the task contributed to difficulties in maintaining an effective solution pathway. In addition, Herold-Blasius (2025) highlighted the importance of self-regulation in evaluating whether a selected strategy continues to produce meaningful progress.

From the perspective of mathematical perseverance, HS demonstrated contrasting characteristics across the two problems. In the PWC problem, the solution process progressed without reaching an impasse, allowing the entrance and initial attempt phases to be completed successfully. In contrast, during the DM problem, HS recognized that the selected strategy was no longer producing meaningful progress, resulting in an impasse. No observable additional attempt, such as reviewing the calculations, exploring an alternative strategy, or restarting the solution process, was identified. However, the participant's decision to discontinue the process should be interpreted cautiously because the test was conducted in a less conducive classroom environment due to an unanticipated tutoring-program presentation. The participant explicitly reported difficulty concentrating under these conditions. Thus, the absence of an additional attempt reflected an interaction between the mathematical obstacle and the situational conditions experienced during the test.

Furthermore, These findings remain consistent with the Three-Phase Perseverance Process proposed by DiNapoli and Miller (2022), which emphasizes that mathematical perseverance is demonstrated not only through initial engagement but also through students' responses after encountering an impasse. In the present study, HS demonstrated sustained effort during the initial attempt but did not proceed to the additional attempt phase. Nevertheless, this pattern should not be interpreted as a stable personal characteristic because perseverance may be influenced by both cognitive and environmental factors. This finding suggests that high mathematical ability does not automatically ensure that students will demonstrate all phases of mathematical perseverance, particularly when they encounter cognitively demanding problems under less supportive situational conditions.

The findings revealed that the medium-ability student (MS) demonstrated a relatively complete problem-solving process across both the Procedures with Connections (PWC) and Doing Mathematics (DM) problems. In the PWC problem, MS experienced a computational error during the solution process. However, the error was recognized and corrected through reviewing the completed work, preventing it from affecting the final answer. In contrast, in the DM problem, MS successfully understood the problem, constructed an appropriate mathematical model, selected an elimination–substitution strategy, and completed the entire solution process without encountering an impasse. These findings indicate that MS was able to maintain a consistent problem-solving process despite the different levels of cognitive demand presented by the two problems.

From the perspective of heuristic failure, the computational error observed in the PWC problem did not develop into an unsuccessful solution because it was promptly recognized and corrected through self-review. Conversely, no heuristic failure was identified in the DM problem, as MS successfully understood the given information, selected an appropriate strategy, systematically implemented the solution procedure, and evaluated the completed solution to obtain the correct answer within the problem context. These findings suggest that successful problem solving depends not only on understanding the problem and selecting an appropriate strategy but also on the ability to monitor and evaluate the solution process. This finding is consistent with Schoenfeld (1992), who argued that monitoring and metacognitive control are essential components of successful mathematical problem solving. Through these processes, errors can be identified and corrected before developing into failures in subsequent stages of problem solving. The ability of MS to identify and revise the computational error also reflects self-regulatory behavior during mathematical problem solving. Herold-Blasius (2025) explained that successful problem solvers do not merely apply a strategy but also monitor its effectiveness and regulate their actions when errors or obstacles arise. In the present study, this behavior enabled MS to prevent a temporary computational error from affecting the final solution.

From the perspective of mathematical perseverance, MS exhibited a pattern distinct from those of both HS and LS. In the PWC problem, the computational error did not interrupt the solution process. Instead, MS used the error as feedback to review the solution, revise the incorrect procedure, and continue working until the correct answer was obtained. Consequently, the heuristic failure developed into an impasse, which was successfully overcome through an additional attempt. In contrast, no impasse occurred in the DM problem because the selected strategy effectively guided MS to the correct solution without requiring revision or an alternative strategy.

These findings are consistent with the Three-Phase Perseverance Process (3PP) proposed by DiNapoli & Miller (2022), which emphasizes that mathematical perseverance is demonstrated not only through students' initial efforts to solve a problem but also through their ability to recognize obstacles, engage in re-initiated effort, and sustain those efforts until the solution goal is achieved. In the present study, these characteristics were reflected in MS's ability to review the identified error, revise the solution process, and continue working until the correct solution was obtained. This finding suggests that errors encountered during mathematical problem solving do not necessarily lead to failure; instead, they may serve as a catalyst for productive additional effort.

Furthermore, these findings support Barnes (2019), who argued that perseverance in mathematical problem solving is influenced not only by cognitive ability but also by affective factors, particularly students' willingness to remain focused and sustain effort when facing difficulties. In the present study, MS maintained focus on the problem-solving goal despite initially making a computational error. Rather than terminating the solution process, the participant evaluated and corrected the error, ultimately arriving at the correct solution. Compared with both HS and LS, MS demonstrated the most comprehensive pattern of mathematical perseverance, as reflected in the ability to sustain effort, respond productively to obstacles, and successfully complete the problem-solving process.

The findings revealed that the low-ability student (LS) exhibited more frequent instances of heuristic failure than both the high-ability student (HS) and the medium-ability student (MS). In the Procedures with Connections (PWC) problem, heuristic failure emerged during the representation stage and subsequently continued through the strategy selection and execution stages. In contrast, although LS demonstrated improvement in the Doing Mathematics (DM) problem by constructing an appropriate mathematical model, difficulties remained in selecting and implementing an effective solution strategy, causing the problem-solving process to terminate before reaching the final solution. These findings indicate that although LS showed improvement in representing the problem under a higher cognitive demand, this improvement was not accompanied by the ability to execute an effective solution strategy.

From the perspective of heuristic failure, the difficulties experienced by LS were attributable not only to procedural errors but also to limited ability to connect contextual information with relevant mathematical concepts. In the PWC problem, the inability to construct an appropriate mathematical model prevented the selected strategy from being implemented correctly, causing the solution process to terminate at an early stage. In contrast, although LS successfully constructed an appropriate mathematical model in the DM problem, the participant lacked a complete understanding of the elimination procedure and therefore was unable to continue the solution process. These findings suggest that successful performance during the representation stage alone is insufficient to produce a correct solution unless it is accompanied by adequate procedural understanding during the strategy selection and execution stages. This finding is consistent with Schoenfeld (1992), who argued that successful mathematical problem solving depends on the interrelationship among problem understanding, strategy selection, strategy implementation, and monitoring throughout the solution process.

From the perspective of mathematical perseverance, LS demonstrated initial efforts to solve both problems through the entrance and initial attempt phases. Across both problems, the participant identified the problem goal and attempted to initiate the solution process using a previously remembered strategy. However, when the selected strategy no longer produced meaningful progress, LS reached an impasse and was unable to determine the subsequent solution steps. Unlike MS, who reviewed and revised the solution, LS chose to discontinue the problem-solving process and provided a response without following an appropriate mathematical procedure. These findings indicate that the heuristic failure experienced by LS developed into an impasse, but was not followed by an additional attempt. These findings are consistent with the Three-Phase Perseverance Process (3PP) proposed by DiNapoli & Miller (2022), which emphasizes that mathematical perseverance is demonstrated not only through students' initial efforts to solve a problem but also through their ability to respond to an impasse by engaging in re-initiated effort and re-sustained effort. In the present study, LS demonstrated mathematical perseverance only through the initial attempt phase. After recognizing that the selected strategy no longer produced progress, the participant neither reviewed the completed work, explored an alternative strategy, nor restarted the solution process. Consequently, the additional attempt phase was absent in both problems. The difficulties experienced by LS are also consistent with Jaenudin et al. (2025), who reported that students with lower mathematical ability tend to experience difficulties in coordinating problem representation, strategy selection, and procedural execution. Furthermore, Saygılı (2017) emphasized that successful engagement with non-routine problems requires students to select and adapt strategies rather than merely reproduce previously remembered procedures. In the present study, LS attempted to use elimination based on prior experience but lacked sufficient understanding to apply or modify the procedure when difficulties arose.

Furthermore, these findings support Wilburne (2017), who argued that students with inadequate problem-solving strategies tend to avoid evaluating their work and instead discontinue their efforts when encountering difficulties. The present findings further suggest that limited mathematical ability influences not only students' success in solving problems but also their ability to sustain effort when confronted with obstacles. Compared with HS and MS, LS exhibited the most extensive pattern of heuristic failure, as failures emerging during the early stages of problem solving developed into an impasse without any subsequent additional effort. These findings highlight the importance of strengthening students' representational skills, procedural understanding, and reflective practices to prevent them from abandoning the problem-solving process when encountering an impasse.

Taken together, the findings demonstrate that cognitive demand interacted with the participants' mathematical ability, strategic knowledge, and monitoring processes rather than determining problem-solving success in a linear manner. The PWC problem provided clearer procedural connections, enabling HS and MS to maintain their initial strategies, although HS omitted the final contextual calculation and MS temporarily made a computational error. In contrast, LS experienced difficulties beginning at the representation stage, indicating that even a task with clearer procedural connections may remain challenging when the required conceptual and representational knowledge is insufficient.

The DM problem required greater coordination among mathematical representation, strategy selection, execution, and decision-making. This higher cognitive demand contributed to an execution impasse for HS, was successfully managed by MS, and enabled LS to construct a more appropriate mathematical representation while still exposing limitations in procedural understanding. Thus, higher cognitive demand did not uniformly increase heuristic failure among all participants. Instead, it revealed different strengths and limitations in their problem-solving processes. This finding is consistent with Wakhata et al. (2023), who emphasized that tasks at different cognitive demand levels provide different opportunities for heuristic reasoning and strategic engagement. The relationship between cognitive demand and mathematical perseverance was also not linear because the participants' responses depended on how they recognized obstacles, evaluated their strategies, and decided whether to initiate an additional attempt.

Conclusion

In conclusion, this study demonstrates that heuristic failure and mathematical perseverance vary according to students' mathematical ability and the cognitive demand of the problems they solve. High-, medium-, and low-ability students exhibited distinct patterns of heuristic failure across the Procedures with Connections (PWC) and Doing Mathematics (DM) tasks. The high-ability student experienced only minor heuristic failure in the PWC task but encountered an impasse during the more demanding DM task without making an additional attempt to continue solving. The medium-ability student displayed the most effective problem-solving pattern by identifying errors, revising strategies, and sustaining effort until a correct solution was obtained across both tasks. In contrast, the low-ability student experienced heuristic failure beginning at the representation and strategy-selection stages, which ultimately developed into unresolved impasses due to the absence of further attempts after encountering obstacles. These findings indicate that higher mathematical ability is associated with fewer heuristic failures, but it does not necessarily guarantee stronger mathematical perseverance.

The findings further suggest that successful mathematical problem solving depends not only on conceptual and procedural knowledge but also on students' capacity to recognize, evaluate, and respond productively to obstacles encountered during the solution process. Therefore, mathematics instruction should incorporate contextual tasks with progressively higher cognitive demand while explicitly fostering students' perseverance through reflective questioning, error-analysis activities, and opportunities to reconsider and revise ineffective strategies. Such learning experiences can help students develop productive responses to impasses rather than abandoning their efforts. Future research is recommended to involve a larger and more diverse sample of participants and a broader range of contextual problems to further examine the relationship among heuristic failure, impasse, mathematical perseverance, and cognitive demand across different mathematical domains.

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