



Analysis of Artificial Intelligence Assisted Proof Process Through Principle of Mathematical Induction in Real Analysis Course

Isnawati Lujeng Lestari¹, Mayang Sari¹, Gusti Uripno², Siti Suprihatiningsih³, Firda Hariyanti¹, Ebenezer Bonyah⁴

¹Universitas Nahdlatul Ulama Pasuruan, Pasuruan, Indonesia

²Universitas PGRI Ronggolawe, Tuban, Indonesia

³Universitas Katolik Santo Agustinus Hippo, Landak, Indonesia

⁴Akenten Appiah Menka University of Skills Training and Entrepreneurial Development, Kumasi, Ghana
Email: s.suprihatiningsih@sanagustin.ac.id

Abstract

The low proficiency of Mathematics Education students in constructing mathematical proofs, especially using the principle of mathematical induction, highlights the need for enhanced learning approaches. One promising method is the integration of Artificial Intelligence (AI) into the proof process within Real Analysis courses. This study aims to describe how students carry out mathematical induction proofs with the assistance of AI. Ten voluntary students enrolled in Real Analysis participated in an initial test involving divisibility problem. From this group, two students were selected through maximum variation sampling based on their answer diversity and communication skills. One student employed a modulo-based approach, while the other used the divisibility-definition concept. Overall, the results demonstrate that AI significantly supports students in understanding problems, planning proofs, implementing strategies, and revising their reasoning. AI played a critical role in concept generation, solution evaluation, and embedded reflection across each stage of Polya's problem-solving framework, combined with the three aspects of AI-assisted proof: construction, evaluation, and revision.

Keywords: AI Assisted, Proof, Principle of Mathematical Induction

Abstrak

Rendahnya kemampuan mahasiswa Pendidikan Matematika dalam menyusun pembuktian matematis, khususnya melalui prinsip induksi matematika, menunjukkan perlunya pendekatan pembelajaran yang lebih efektif. Salah satu metode yang menjanjikan adalah integrasi Artificial Intelligence (AI) dalam proses pembuktian pada mata kuliah Analisis Riil. Penelitian ini bertujuan untuk mendeskripsikan bagaimana mahasiswa melakukan pembuktian dengan metode induksi matematika melalui bantuan AI. Sepuluh mahasiswa yang mengikuti mata kuliah Analisis Riil secara sukarela mengikuti tes awal yang memuat masalah keterbagian. Dari kelompok ini, dua mahasiswa terpilih melalui teknik sampling variasi maksimum berdasarkan keragaman jawaban dan keterampilan komunikasinya. Satu mahasiswa menggunakan pendekatan berbasis modulo, sedangkan satu lainnya menggunakan konsep definisi keterbagian. Secara keseluruhan, hasil penelitian menunjukkan bahwa AI secara signifikan membantu mahasiswa dalam memahami masalah, merancang pembuktian, melaksanakan strategi, dan merevisi pemikirannya. AI berperan penting dalam membentuk konsep, mengevaluasi solusi, serta memberikan refleksi yang terintegrasi pada setiap tahap dalam kerangka pemecahan masalah Polya, yang dikombinasikan dengan tiga aspek pembuktian berbantuan AI: konstruksi, evaluasi, dan revisi.

Kata kunci: Berbantuan AI, Bukti, Prinsip Induksi Matematika

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Introduction

The low proof ability of students of the Mathematics Education urgently need for analysis of students' errors or thought processes through the proof process that carried out by students (Hartono, 2025). The main activity in learning mathematics is proof (Sin' et al., 2024). Thus, proof skills are very important for every mathematics student and mathematics education (Zwaneveld et al., 2024). One of the proof methods in mathematics that involves natural numbers is the principle of mathematical induction (Rosen, 2011). Proof using the mathematical induction method is axiomatic deductive even

though it uses the term induction (Rips & Asmuth, 2007). In addition to being taught in higher education, this method is also taught to high school students. Thus, mathematics education students must master this proof method.

The principle of mathematical induction generally has three steps of proof (Rosen, 2011). These three steps are (a) Prove that $P(1)$ is true; (b) If $P(k)$ is true then $P(k+1)$ is true; (c) $P(n)$ is true for every n is a natural number (Bartle & Sherbert, 2011). Several obstacles are still found by students in applying proofs with this method (Hendriyanto et al., 2024). One obstacle is that students make assumptions as evidence based on what will be proven, which of course creates a paradox (Norton et al., 2023). Another obstacle is found when students want to direct $P(k+1)$ to be in accordance with the form in $P(n)$ (Ahmadi et al., 2019; Gonzales, 2020). In addition, students are unable to use previous theorems to help the proof process (Cipta et al., 2024). Based on these constraints, proof skills need to be optimized through learning in courses that include this topic, one of which is Real Analysis.

The low level of students' mathematical induction proof skills necessitates an in-depth analysis of the proof process using this method. The problem in this study focuses on thoroughly describing the proof process using the mathematical induction method carried out by students in real analysis courses. Students have many obstacles in solving problems involving mathematical induction (Relaford-Doyle & Núñez, 2021). These obstacles indicate that there needs to be a method approach to learning that includes mathematical induction (Belay et al., 2024). One of the courses that teaches mathematical induction is real analysis. Thus, students are expected to be able to improve their proof skills, in this case using the mathematical induction method, through real analysis courses.

One of the technologies that can help mathematics learning activities is Artificial Intelligence (Zhang, 2024). Artificial Intelligence can act as an assistant, media, and teaching material in mathematics education (Wardat et al., 2024). Teaching proof by integrating AI will be an opportunity and challenge in itself in improving students' abilities (Egara & Mosimege, 2024). The use of AI in solving mathematical problems can help in providing an evaluation of students' work results (Uripno et al., 2024). Thus, the application of AI as an aid in mathematics learning will affect students' cognitive processes in solving mathematical problems.

The principle of mathematical induction is a fundamental proof method in mathematics and can be viewed as a form of problem solving aligned with Polya's problem-solving framework (Polya, 2004). Proof, in this context, involves demonstrating the truth of a mathematical statement, which can be effectively expressed in the form of implications (Siswono et al., 2024; Hartono et al., 2025). Representing a mathematical statement as an implication facilitates a structured approach to problem solving in accordance with Polya's four stages: understanding the problem, devising a plan, carrying out the plan, and looking back (rechecking). To support this process, AI-integrated worksheets (LKPD) will be used to guide students through the proof process within this structured framework.

Previous studies have explored the integration of AI in proof-based learning. Mairing et al. (2024) conducted a quantitative study aimed at improving students' proof abilities using AI in Real Analysis courses; however, their study did not examine students' cognitive processes and focused only on the final results. Yoon et al. (2024) investigated how AI-generated prompts assist students in making decisions during proofs, but limited their study to problems involving divisibility. Meanwhile, Park and Manley (2024) conducted a qualitative study that emphasized three aspects of AI-assisted proof construction, evaluation, and revision but the research addressed a broad set of proof problems, limiting the depth of analysis for each type.

The novelty of the present study lies in its focus on three specific themes within mathematical induction proofs series formulas, inequalities, and divisibility (Bartle & Sherbert, 2011). Its primary contribution is offering a comprehensive description of students' cognitive processes while constructing mathematical induction proofs with AI assistance. Unlike previous research, this study combines Polya's problem-solving stages (Polya, 2004) with the three dimensions of AI-assisted proof proposed

by Park and Manley (2024): construction, evaluation, and revision. The study specifically targets the teaching of mathematical induction within the context of Real Analysis courses, aiming to describe how students solve induction problems through the support of AI tools.

Method

This qualitative descriptive study aims to explore the role of Artificial Intelligence (AI) in assisting students with mathematical proofs using the principle of mathematical induction in Real Analysis lectures. The problems provided focus on the topic of divisibility. The research framework is based on Polya's (2004) problem-solving model, which includes four stages: (1) Understanding the problem, students will formulate the mathematical statement in the form of an implication and identify the known and to-be-proven components; (2) Devising a plan, students will determine the appropriate proof strategy, such as direct or indirect proof; (3) Carrying out the plan, students will execute the proof steps previously identified; and (4) Looking back, this stage involves reviewing the results, identifying potential errors, and reflecting on the reasoning process. At each stage, the use of AI will be analyzed through three key aspects outlined by Park and Manley (2024): construction, evaluation, and revision. These aspects will provide a detailed lens through which to examine how AI supports the cognitive processes involved in mathematical proof.

This study began with an initial test consisting of proof problems with the induction method. The problem in initial test consisting is "prove that $2^{2n} - 1$ divisible by 3". The results of the initial test were used as the basis for sampling. The sampling used was maximum variation sampling. The selection of subjects was based on the variation of answers from all students, then students were selected whose answer variations represented and had good communication skills. The chosen students have to prove main test that contain problems which are "prove that $5^{2n} - 1$ divisible by 8 for every n belong to \mathbb{N} ". However, this study involves ten voluntary students that join the analysis real course. Based on sampling, this study chose two students that require the conditions. The first student prove through modulo concept (SM), the second one prove through divisibility definition concept (SD).

The data analysis technique in this study followed the four stages proposed by Cohen et al. (2007), which include: (1) organizing meaningful units of data, (2) grouping or categorizing similar patterns, (3) constructing descriptive narratives to represent the findings, and (4) interpreting the results to draw conclusions. To ensure the validity of the study, triangulation of data collection methods was applied, utilizing both think-aloud protocols and semi-structured interviews. In addition to methodological triangulation, subject triangulation was also employed to enhance the credibility and depth of the findings.

Result and Discussion

Subject with modulo approach (SM)

$5^{2n} - 1$ terbagi oleh 8, $\forall n \in \mathbb{N}$
 • Untuk $n = 1$
 $5^{2(1)} - 1 = 25 - 1 = 24$ (habis dibagi 8)
 • Misal $n = k$
 $5^{2k} - 1$ habis dibagi 8
 • Buktikan untuk $n = k + 1$
 $5^{2(k+1)} - 1 = 5^{2k+2} - 1$
 Karena $5^2 \equiv 1 \pmod{8}$:
 $5^{2k+2} = 5^{2k} \cdot 5^2 \equiv 5^{2k} \cdot 1 \equiv 5^{2k} \pmod{8}$
 maka
 $5^{2k+2} - 1 \equiv 5^{2k} - 1 \equiv 0 \pmod{8}$

Figure 1 Subject SM proof

Based on the results, it can be seen that the subject completed the concept of modulo. Furthermore, the proof process will be identified based on the framework that has been prepared.

Understand problem (Construction)

The subject has understood what will be proven and what will be known in the proof. This is reinforced by the results and the following interview excerpt.

- Researcher* : what was done first?
Subject SM : what will be proven there is that 8 will be able to divide $5^{2n} - 1$
Researcher : what numbes is n?
Subject SM : n is any natural number

Based on this, it can be seen that the subject can arrange what is known, namely a number in the form of $5^{2n} - 1$ for n natural numbers. Then what will be proven is that the number 8 can divide the number. In addition, through the narrative during the think-aloud, the subject will carry out proof for all natural numbers so that the number is divisible by 8. The process of understanding the problem in this case does not involve too much AI, although later on the AI prompt given the subject asks to obtain what will be proven and what is known.

Making a plan (AI Assisted evaluation and revision)

Based on **Figure 1**, the subject carried out the mathematical induction principle process starting from P(1) to P(k+1). However, during the think-aloud and interview process, it was found that the subject was confused in determining the next step when he had reached the assumption that P(k) was true. Then the subject gave a prompt to the AI, in this case the program used was ChatGPT. The prompt used was "solve the following problem along with the steps using mathematical induction". The ChatGPT solution directed the idea of proof with a modulo approach, the results of which can be seen in **Figure 2**.

	Translation:	
$5^{2n} - 1$ habis dibagi 8 $\forall n \in \mathbb{N}$		$5^{2n} - 1$ is divisible by 8 $\forall n \in \mathbb{N}$
Artinya:		
$5^{2n} \equiv 1 \pmod{8}$	That means:	$5^{2n} \equiv 1 \pmod{8}$
Periksa pola:	Check the pattern:	
<ul style="list-style-type: none"> • $5^2 = 25 \equiv 1 \pmod{8}$ • $5^4 = (5^2)^2 = 1^2 = 1 \pmod{8}$ • Jadi, $5^{2n} \equiv 1 \pmod{8}$ 	<ul style="list-style-type: none"> • $5^2 = 25 \equiv 1 \pmod{8}$ • $5^4 = (5^2)^2 = 1^2 = 1 \pmod{8}$ • So, $5^{2n} \equiv 1 \pmod{8}$ 	
$5^{2n} - 1 \equiv 1 - 1 = 0 \pmod{8}$		$5^{2n} - 1 \equiv 1 - 1 = 0 \pmod{8}$
✅ Maka terbukti bahwa selalu habis dibagi 8.		✅ Thus, it is proven that it is always divisible by 8.

Figure 2 Subject SM Prompt through ChatGPT

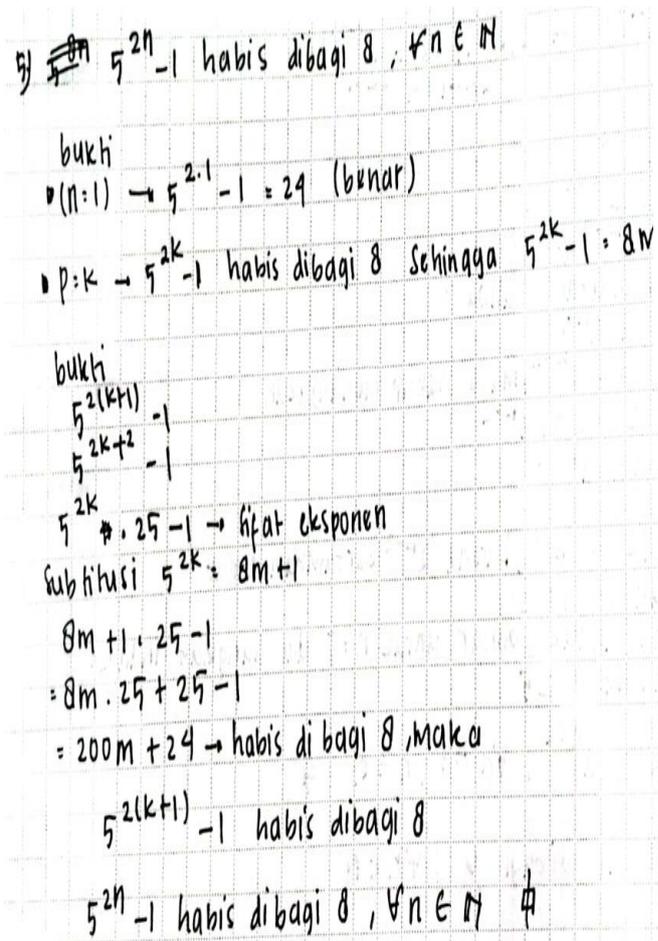
Based on the ChatGPT results, the subject finds an idea to continue the proof. The idea that emerges will be used as the subject's plan.

Carry out plan (AI Assisted evaluation and revision)

Based on Figure 1, part P(k+1) can be seen that the subject does not directly use the results of the AI prompt. The AI prompt uses a modulo 8 congruence pattern while the subject uses the concept of modulo congruence to be used as a connecting premise between P(k) and P(k+1). The subject finds that $5^{2(k+1)}$ is congruent to 5^{2k} in modulo 8, then the subject uses the assumption that P(k) is divisible by 8 as a guarantee that it also applies to P(k+1) because both are congruent in modulo 8. Therefore, the subject uses AI to plan the proof, which in this case uses the modulo concept, to be applied with the principle of mathematical induction.

Subjects with divisibility-definition approach (SD)

Results of the subject SD proof can be seen in **Figure 3**.



Translation:

Given:

$5^{2n} - 1$ is divisible by 8, for all $n \in \mathbb{N}$

Proof:

- Base Case:
For $n = 1$:
 $5^{2 \cdot 1} - 1 = 25 - 1 = 24 \rightarrow$ (correct)

- Inductive Step:
Assume for $n = k$:
 $5^{2k} - 1$ is divisible by 8, so
 $5^{2k} - 1 = 8m$

To Prove:

$5^{2(k+1)} - 1$ is divisible by 8

- $5^{2(k+1)} - 1$
- $= 5^{2k+2} - 1 \rightarrow$ (exponent property)
- $= 5^{2k} \cdot 25 - 1$

Substitute:

From the hypothesis, $5^{2k} = 8m + 1$

- $(8m + 1) \cdot 25 - 1$
- $= 8m \cdot 25 + 25 - 1$
- $= 200m + 24 \rightarrow$ divisible by 8

Therefore,

$5^{2(k+1)} - 1$ is divisible by 8

Figure 3 SD Subject Result

Hence, the results will be discussed according to the framework that has been arranged .

Understand problem (AI Assisted evaluation and revision)

Based on **Figure 3**, it is not yet clear whether the subject understands the given problem. Furthermore, through think-aloud and impact interviews, it is known that the subject has been utilizing AI from the start, in this case ChatGPT. The prompt given to AI is asking to work on the problem. The prompt results that identify AI assistance in helping the subject understand the problem can be seen in **Figure 4**.

Langkah 1: Basis Induksi	Translation:
Cek untuk $n = 1$:	Step 1: Base Case of Induction
$5^{2(1)} - 1 = 5^2 - 1 = 25 - 1 = 24$	Check for $n = 1$:
Karena 24 habis dibagi 8 ($24 \div 8 = 3$), maka basis induksi benar .	$5^{2(1)} - 1 = 5^2 - 1 = 25 - 1 = 24$
	Since 24 is divisible by 8 ($24 \div 8 = 3$), the base case of induction is true.

Figure 4 SD subject AI prompt result

Based on these results, the subject understands that what will be proven in the problem is that when substituting natural numbers into the form, the result is definitely divisible by 8. This is shown by a quote during the think-aloud which states that "every time I enter a number starting from 1, 2, and so on, it must be shown to be divisible by 8". This statement is reinforced by the following interview quote.

Researcher: After you get the results, what can be concluded?

SD Subject: So, for example, I substitute 1, 2, or 3 into $5^{2n} - 1$, then the result will be divisible by 8.

Researcher: That means only 1, 2, or 3, right?

SD Subject: No sir, but so on.

Researcher: That means what set is it called?

Elementary School Subject: Oh yes, the set of natural numbers, sir.

Based on the quote, it can be concluded that through the help of ChatGPT, the subject can know that for any natural number, $5^{2n} - 1$, it will be shown that the form will be divisible by 8.

Making a plan (AI Assisted evaluation and revision)

The planning process carried out by the elementary school subject is seen in **Figure 3**. The subject uses the definition of divisibility which will later be associated with the assumption $P(k)$ to prove the statement $k+1$. However, the plan is guided by ChatGPT which is also a continuation of the previous prompt.

Langkah 2: Hipotesis Induksi	Translation:
Misalkan untuk $n = k$, pernyataan berlaku, yaitu:	Step 2: Inductive Hypothesis
$5^{2k} - 1$ habis dibagi 8	Suppose for $n = k$, the statement holds, namely:
Artinya, ada suatu bilangan bulat m sehingga:	$5^{2k} - 1$ is divisible by 8
$5^{2k} - 1 = 8m$	This means, there exists an integer m such that:
	$5^{2k} - 1 = 8m$

Figure 5 SD Subject Make a Plan through ChatGPT

An interview quote states that by using this premise, we will obtain $5^{2k} = 8m + 1$ which will be substituted into $P(k+1)$. Based on this excerpt, it can be concluded that the plan that will be carried out by the elementary school subject is to use the definition of divisibility and the assumption premise to be substituted into $P(k+1)$.

Carry out plan (AI Assisted evaluation and revision)

The implementation of the plan carried out by the SD subject can be seen in **Figure 3**. The AI assistance carried out by the subject was actually already visible since the planning. The results of the AI-assisted planning were then implemented by the subject and can be seen in **Figure 3** as well. The important part at this stage is when the subject decides to substitute $5^{2k} = 8m + 1$ in $P(k+1)$. This was done by the subject by considering the direction from the AI which can be seen in **Figure 6**.

Gunakan sifat eksponen:	Translation:
$5^{2k+2} = (5^{2k} \cdot 5^2)$	Use the exponent rule:
Sehingga:	$5^{2k+2} = (5^{2k} \cdot 5^2)$
$5^{2(k+1)} - 1 = 5^{2k} \cdot 25 - 1$	Thus:
Gunakan hipotesis induksi $5^{2k} = 8m + 1$, maka:	$5^{2(k+1)} - 1 = 5^{2k} \cdot 25 - 1$
$(8m + 1) \cdot 25 - 1$	Use the inductive hypothesis $5^{2k} = 8m + 1$, then:
$= 8m \cdot 25 + 25 - 1$	$(8m + 1) \cdot 25 - 1$
$= 200m + 24$	$= 8m \cdot 25 + 25 - 1$
	$= 200m + 24$

Figure 6 SD Subject Carry out the plan thorough ChatGPT

The results are then used as assistance for elementary school subjects in compiling proofs. Through the substitution results, it is obtained that $200m+24$. The conclusion obtained is because 200 is divisible by 8 and so is 24 divisible by 8. Therefore, the linear combination of the two is also divisible by 8.

Based on the stages that have been explained, AI has shown a significant impact in solving proof problems. This supports the research of (Yoon et al., 2024) that through the help of AI, students can make decisions regarding the steps of proof. This is similar to the findings of the study which stated that in implementing the plan, students consider what concept will be used. The concept was obtained from the results of the AI prompt that had been carried out,

In addition, this study found that in compiling what is known and what is being asked, it is not necessary to compile it in the form of implications. This is slightly different from the opinion of (Siswono et al., 2024) who stated that every proof problem can be more easily stated in implications to understand the problem. This is because through implications it will appear what is known and what is asked through antecedents and consequences. Polya (2004) stated several stages, one of which is understanding the problem which can be made easier by writing what is known and what will be asked. Polya (2004) also stated that one of the stages is re-checking which in this study did not appear directly but was integrated at each stage. This is because at each stage AI can help provide direct reflection without having to reach the final stage. This is in line with the research of Uripno et al. (2024) which showed results that AI can help in reflecting student errors in solving problems.

Conclusion

Based on the results that have been presented, it can be concluded that several stages that students go through include understanding the problem, making a plan, implementing the plan, and re-checking. The stage of understanding the problem is carried out with the help of AI and without the help of AI. At this stage, the subject is translating what is known and what will be proven. AI plays a role in helping to show what should be known and what is asked. The stage of making a plan is carried out with the help of AI. AI plays a role in providing ideas and concepts that will be used to be implemented at the proof stage with the principle of mathematical induction. The stage of implementing the plan is carried out with the help of AI. This process is a follow-up to the previous stage. The ideas built in the previous stage are arranged at this stage by paying attention to the key concepts that are highlighted to be involved in the algebraic manipulation process of part $P(k+1)$. The re-checking stage that is carried out is not directly visible. However, AI helps direct the subject in carrying out integrated checks in each stage. Further study are needed to develop learning instrument that integrated with AI. This study have described the obstacles and students process in proofing assisted by AI. So, this study drive other study in implement AI to enhance students proofing skills especially proofing by principle of mathematical induction/

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