ISSN 2715-7458
E-ISSN 2715-7466

Journal of Mathematical Pedagogy
Volume 6, No. 2, July 2025, pp. 72-93

Students' Abductive Reasoning in Solving Quadratic Pattern Generalization Problem Based on the Initial Mathematical Ability

Isti'anatun Nabilah[1], Ali Shodikin[2]

[1,2] Universitas Negeri Surabaya, Kampus Ketintang Surabaya 60231, Indonesia
Email: istianatunnabilah.21074@mhs.unesa.ac.id

Abstract

The purpose of this study was to describe students' abductive reasoning in solving quadratic pattern generalization problem based on initial mathematical ability. Abductive reasoning is a process of drawing conclusions based on certain facts where the conclusion is still an assumption that can be revised based on new information. This type of research is descriptive research with a qualitative approach. The subjects of this study were 6 junior high school students based on the category of students' initial mathematical ability, namely 2 students with high initial mathematical ability, 2 students with moderate initial mathematical ability, and 2 students with low initial mathematical ability. The data collection technique was carried out through an initial mathematical ability test to determine the research subjects, quadratic pattern generalization problem to identify students' abductive reasoning processes, and interviews. Data analysis was carried out based on the process and indicators of abductive reasoning. The results of the study showed that in the process of (1) realizing the existence of abductive problems, all subjects had never solved quadratic pattern generalization problem so that the problem was perceived as unfamiliar or unusual because it was something new that was obtained, they also found differences in quadratic pattern generalization problem with number pattern problems that had been encountered before, they do this process as an initial process of abductive reasoning; (2) identifying solutions, all subjects explained the discrepancy between the information obtained from the facts of observations and previous knowledge, namely students with high and moderate initial mathematical abilities explained the difference in the given quadratic pattern generalization problem only presenting the 4^{th} pattern, while the number pattern questions that have been encountered usually contain patterns 1 to 4 in sequence, students with low initial mathematical abilities explained the difference lies in the process of working on it; all subjects mentioned alternative solution guesses that might help to solve the problem, namely making conjectures of different pattern shapes and determined the fixed difference in the number of black circles in each pattern; students with high initial mathematical abilities grouped the black circles into three shapes, using the quadratic formula, the first and second level arithmetic sequence formulas, and obtained two alternative solutions; students with moderate initial mathematical abilities used the first level arithmetic sequence formula and obtained one alternative solution; while students with low initial mathematical abilities did not get an alternative solution; (3) choosing the best solution, students with high and medium initial mathematical ability choose a certain solution from the alternative solutions provided and explain the reasons for choosing this solution as the best solution, while students with low initial mathematical ability do not; students with high initial mathematical ability choose one formula from two formulas obtained because this formula is easier; students with low initial mathematical ability only get one formula and choose this formula because it is easy to use (4) assimilating the chosen solution, students with high and medium initial mathematical ability use the chosen solution to solve the problem, while students with low initial mathematical ability do not; the solution used by students with high initial mathematical ability produces the correct answer, while the solution used by students with medium initial mathematical ability can be the right solution if it is adjusted to the estimated pattern shape that has been made.

Keywords: abductive reasoning, initial mathematical ability, pattern generalization, quadratic pattern generalization problem.

Abstrak

Tujuan penelitian ini adalah mendeskripsikan penalaran abduktif siswa dalam menyelesaikan soal generalisasi pola kuadrat ditinjau dari kemampuan awal matematis. Penalaran abduktif merupakan suatu proses penarikan kesimpulan yang didasarkan pada fakta tertentu dimana kesimpulan tersebut masih berupa dugaan yang dapat direvisi berdasarkan informasi baru. Jenis penelitian ini adalah penelitian deskriptif dengan pendekatan kualitatif. Subjek penelitian ini 6 siswa SMP berdasarkan kategori kemampuan awal matematis siswa, yaitu 2 siswa dengan kemampuan awal matematis tinggi, 2 siswa dengan kemampuan awal matematis sedang, dan 2 siswa dengan kemampuan awal matematis rendah. Teknik pengambilan data dilakukan melalui tes kemampuan

awal matematis untuk menentukan subjek penelitian, soal generalisasi pola kuadrat untuk mengidentifikasi proses penalaran abduktif siswa, dan wawancara. Analisis data dilakukan berdasarkan proses dan indikator penalaran abduktif. Hasil penelitian menunjukkan bahwa pada proses (1) menyadari keberadaan masalah abduktif, semua subjek belum pernah menyelesaikan soal generalisasi pola kuadrat dan soal tersebut dianggap asing atau tidak biasa karena merupakan suatu hal baru yang didapatkan, mereka juga menemukan perbedaan soal generalisasi pola kuadrat dengan soal pola bilangan yang pernah ditemui sebelumnya, mereka melakukan proses ini sebagai proses awal penalaran abduktif; (2) mengidentifikasi solusi, semua subjek menjelaskan ketidaksesuaian informasi yang diperoleh dari fakta-fakta hasil pengamatan dengan pengetahuan sebelumnya yaitu siswa dengan kemampuan awal matematis tinggi dan sedang menjelaskan perbedaan soal generalisasi pola kuadrat yang diberikan hanya menyajikan pola ke-4 sedangkan soal pola bilangan yang pernah ditemui biasanya memuat pola ke-1 hingga pola ke-4 secara berurutan, siswa dengan kemampuan awal matematis rendah menjelaskan perbedaannya terletak pada proses pengerjaannya; semua subjek menyebutkan dugaan-dugaan solusi alternatif yang mungkin dapat membantu untuk menyelesaikan permasalahan yaitu membuat dugaan bentuk pola yang berbeda-beda serta menentukan selisih tetap dari banyaknya bulatan hitam pada setiap pola; siswa dengan kemampuan awal matematis tinggi mengelompokkan bulatan hitam menjadi tiga bentuk, menggunakan rumus kuadrat, rumus barisan aritmatika tingkat satu dan dua, serta mendapatkan dua solusi alternatif penyelesaian; siswa dengan kemampuan awal matematis sedang menggunakan rumus barisan aritmatika tingkat satu dan mendapatkan satu solusi alternatif penyelesaian; sedangkan siswa dengan kemampuan awal matematis rendah tidak mendapatkan solusi alternatif penyelesaian; (3) memilih solusi terbaik, siswa dengan kemampuan awal matematis tinggi dan sedang memilih solusi tertentu dari alternatif solusi yang disediakan dan menjelaskan alasan memilih solusi tersebut sebagai solusi terbaik sedangkan siswa dengan kemampuan awal matematis rendah tidak; siswa dengan kemampuan awal matematis tinggi memilih satu rumus dari dua rumus yang didapatkan karena rumus tersebut lebih mudah; siswa dengan kemampuan awal matematis rendah hanya mendapatkan satu rumus dan memilih rumus tersebut karena mudah digunakan (4) mengasimilasi solusi yang dipilih, siswa dengan kemampuan awal matematis tinggi dan sedang menggunakan solusi yang dipilih untuk menyelesaikan permasalahan sedangkan siswa dengan kemampuan awal matematis rendah tidak; solusi yang digunakan siswa dengan kemampuan awal matematis tinggi menghasilkan jawaban yang benar sedangkan solusi yang digunakan siswa dengan kemampuan awal matematis sedang dapat menjadi solusi yang tepat jika disesuaikan dengan dugaan bentuk pola yang telah dibuat.

Kata kunci: penalaran abduktif, kemampuan awal matematis, generalisasi pola, soal generalisasi pola kuadrat.

How to Cite: Nabilah, I. & Shodikin, A. (2025). Students' Abductive Reasoning in Solving Quadratic Pattern Generalization Problem Based on The Initial Mathematical Ability. *Journal of Mathematical Pedagogy, 6*(2), 72-93.

Introduction

Reasoning is one of the student's abilities that can be developed through mathematics learning. The goal of mathematics learning is for students to have skills or proficiency in developing reasoning, communication skills, and solving problems faced in everyday life (Kemendikbud, 2016). In addition, one of the goals of mathematics subjects is that students are able to use reasoning on patterns and properties, carry out mathematical manipulations in making generalizations, compile evidence, or explain mathematical ideas and questions (Kemendikbud, 2022). These goals apply to learning materials in mathematics subjects at every level of education. Reasoning is set by the National Council of Teachers of Mathematics (NCTM) as one of the process standards that students must have as well as skills or proficiencies that must be developed at all levels (Nuñez-Gutierrez & Cabañas-Sánchez, 2022). Students can more easily understand concepts by using their reasoning in making guesses based on their own experiences, so that student learning outcomes improve (Setiadi et al., 2012). This shows that reasoning is always one of the important goals of mathematics learning at all levels and has an important influence on student learning outcomes.

Based on the decision-making process, reasoning is classified into three types, namely deductive reasoning, inductive reasoning, and abductive reasoning (Hjelte et al., 2020). In this classification, the point lies in how students provide reasons for certain points of view, solutions, or conclusions. However, on the one hand, until now abductive reasoning is still foreign to the general public, not

equivalent to deductive reasoning and inductive reasoning (Woosuk, 2017). This is because in abductive reasoning the conclusion obtained is still a temporary conjecture. The conjecture can be true or false depending on the new facts observed. Reasonable conjectures in abductive reasoning are obtained from limited evidence (Ramesh & Howlett, 2025). Abductive reasoning develops plausible explanations or inferences for observed phenomena when there is incomplete or conflicting evidence.

Abductive reasoning as a process of drawing conclusions where the conclusions can be revised based on new information (Velazquez-Quesada et al. 2013). The conclusion in abductive reasoning can be updated because it is still a temporary conjecture. Examples of the use of abductive reasoning in everyday life include in medical diagnosis where a doctor observes symptoms in a patient, the doctor makes guesses about possible causes based on his knowledge of the causal relationship between the disease and symptoms. Another example is when you wake up in the morning and find a wet lawn, from this observation it can be explained by assuming that it rained last night or the sprinkler was on (Aliseda, 2006). Facts such as patient symptoms and wet lawns in some references are understood as surprising facts because these facts are observed information, but the cause of these facts cannot be ascertained so that the most reasonable explanation of what caused them is needed.

The logical formulation of this abductive reasoning is

The surprising fact, C, is observed;

But if A were true, C would be a matter of course;

Hence, there is reason to suspect that A is true; (Aliseda, 2006). The logical formulation of abductive reasoning can also be represented as follows.

$$\begin{array}{l} C \\ \underline{A \rightarrow C} \\ \therefore \ A^* \end{array}$$

The representation of the logical formulation of abductive reasoning means that premise 1 is C, namely the results or facts observed. Premise 2 is $A \rightarrow C$ (read: if A then C), yaitu as a rule whose truth is known, where A is the antecedent and C is the consequent. While A^* is a conclusion obtained from observed rules and facts. Although in the science of mathematical logic this kind of conclusion-drawing process is not valid. However, this formulation can be used as a reason to guess the cause of an event from observed facts.

In the context of mathematics learning, one of the roles of abductive reasoning is to describe the process of solving mathematical problems (Reid, 2018). Through abductive reasoning, students can make guesses and find connections between one piece of information and another so that a solution is obtained to solve mathematical problems. Open-ended problem solving can give rise to abductive reasoning and the abductive reasoning process can appear when determining problem-solving strategies (Shodikin et al., 2021). Abductive reasoning also plays a role in the proof process, namely to explain surprising facts and can build a new rule (Pedemonte & Reid, 2011). This is in accordance with the opinion of Shodikin et al. (2021), which states that abductive reasoning can be used to find the most appropriate strategy in solving mathematical problems which can ultimately build new schemes in students' cognitive processes. The construction of this new scheme helps students develop their knowledge in the learning process. In abductive reasoning, students are required to utilize all available information to produce the best explanation and enable students to be more successful in solving problems (Wackerly, 2021). This is what makes abductive reasoning one of the reasoning skills that students must have.

Abductive reasoning is also a key factor in the generalization process (Park & Lee, 2016). Generalization is the process of determining a more general rule. Students' errors in using generalization can cause students to have difficulty in finding mathematical concepts properly. Pattern generalization contributes to the development of abilities related to problem solving by emphasizing

the analysis of specific cases, organizing data systematically, guessing, and generalizing (Barbosa et al., 2012). Pattern generalization is the process of finding similarities in each term in a pattern (Setiawan et al., 2020). Of the various types of patterns, the pattern that is the focus of this study is the quadratic pattern. A quadratic pattern is a pattern that has a difference between its terms that is not constant (Somasundram et al, 2019). Quadratic patterns can also be called nonlinear patterns. The fixed difference in the quadratic pattern has a square value so that it cannot be found in the difference in the first-order sequence. In this case, students are required to utilize abductive reasoning in the process of determining the value of the fixed difference to obtain a general formula. Therefore, abductive reasoning is very necessary in the process of solving quadratic pattern generalization problem.

Basically, each student has different initial mathematical abilities. Initial ability is a prerequisite that students must have before entering the next higher learning (Haeruman et al., 2017). Students' initial mathematical abilities are classified into three categories, namely high initial mathematical abilities, moderate initial mathematical abilities, and low initial mathematical abilities. This classification is based on the initial mathematical ability test scores obtained by students. Initial mathematical ability is an important thing that must be considered in the mathematics learning process because initial mathematical ability affects student learning outcomes and directs students to new information (Salam et al., 2019). New information obtained by students based on their initial mathematical abilities will affect the differences in strategies used by students in solving problems. In this case, it is supported by the results of the study by Salam et al. (2023), which shows that differences in students' initial abilities affect students' reasoning. Therefore, to be able to use their reasoning well, students need high initial mathematical abilities in order to be able to analyze new information using previously possessed abilities. This shows that the level of students' initial mathematical abilities influences the differences in students' abductive reasoning processes.

In a study by Furqoni & Subekti (2022) which aims to describe students' abductive reasoning abilities in terms of learning styles, the results of the study showed that there were differences in students' abductive reasoning abilities with visual, auditory and kinesthetic learning styles where students with kinesthetic learning styles tended to find only one solution strategy. Likewise, the results of the study by Shodikin et al. (2021) showed that students who use abductive reasoning can determine problem-solving strategies but do not always produce new schemes. This shows that abductive reasoning plays an important role in the process of drawing conclusions and enables students to be more successful in solving problems.

The relationship between abductive reasoning assessed through quadratic pattern generalization problem and the influence of initial mathematical ability on abductive reasoning raises questions about the abductive reasoning process of students with different initial mathematical abilities in solving quadratic pattern generalization problem. Research on abductive reasoning is also still limited so that researchers are interested in conducting research on students' abductive reasoning in solving quadratic pattern generalization problem based on initial mathematical ability. Based on the background that has been described and several relevant research results, this study aims to describe students' abductive reasoning in solving quadratic pattern generalization problem based on students' initial mathematical ability.

Method

This type of research is descriptive research with a qualitative approach. This research is used to describe the results of the research with the aim of providing a description, explanation, and validation of something being researched (Ramadhan, 2021). This is because descriptive research with a qualitative approach is the purpose of this study, namely to describe the abductive reasoning of

students who have high initial mathematical abilities, moderate initial mathematical abilities, and low initial mathematical abilities in solving quadratic pattern generalization problem. The main data in this study were obtained from the results of the initial mathematical ability test, the results of solving quadratic pattern generalization problem, and the results of interviews. This interview aims to obtain more detailed information about students' abductive reasoning which is still not clearly visible in the written data of the results sheet for working on quadratic pattern generalization problem.

The main instrument in this study is the researcher himself. The researcher is directly involved in determining the focus of the study, selecting research subjects, collecting data, analyzing data, interpreting data, and making conclusions. The researcher also acts as an observer and interviewer for the research subjects and reporter of the results of his research, so that only the researcher knows the relationship between the realities in the field that cannot be delegated to others. In addition, supporting instruments are needed to obtain research data, namely the initial mathematical ability test, quadratic pattern generalization problem, and interview guidelines. The initial mathematical ability test is an instrument used to determine the level of students' initial mathematical ability. The scores obtained by students are used as the basis for selecting research subjects. The questions used in the initial mathematical ability test are consulted with the supervisor first. The initial mathematical ability test in this study uses algebraic form material, linear equations and inequalities of one variable, and comparisons which are prerequisite materials for pattern generalization. The quadratic pattern generalization problem are used to identify and describe students' abductive reasoning in solving quadratic pattern generalization problem based on six indicators of abductive reasoning. The abductive reasoning questions consist of one descriptive question. The interview guideline is used to obtain more detailed information about students' abductive reasoning that is still not clearly visible in the written data of the results sheet for completing the quadratic pattern generalization problem. The interview guideline is prepared by paying attention to the abductive reasoning indicators in order to obtain a picture of the abductive reasoning indicators that have been fulfilled by students in completing the quadratic pattern generalization problem. The interviews conducted are open and semi-structured, so that the questions in the interview guideline are the main questions. This aims to allow researchers to develop questions in accordance with the answers of the research subjects related to completing the quadratic pattern generalization problem and the ongoing interview, but still in a controlled conversation flow.

This study used two subjects in each category of students' initial mathematical ability. This was done in the hope of obtaining a better picture of students' abductive reasoning in solving quadratic pattern generalization problems through analysis activities on the similarities and differences in the abductive reasoning process in each research subject. So, the method of drawing conclusions used in this study is if the data produced by both subjects is the same, then the data is used in the conclusion. Meanwhile, if the data produced by the two subjects is different, then it is taken based on the tendencies of the two research subjects. In addition, the reason for selecting 6 subjects with different levels of initial mathematical ability, namely high, medium, and low and the same gender is to maintain the validity of the data and the credibility of the data so that students' abductive reasoning in solving the described quadratic pattern generalization problems is only influenced by the students' initial mathematical ability.

The subjects in this study were junior high school students in grade IX. In this study, one class was selected to be given an initial mathematical ability test to make it easier for researchers to group students who have high initial mathematical abilities ($75 <$ test scores ≤ 100), moderate initial mathematical abilities ($25 \leq$ test scores ≤ 75), and low initial mathematical abilities ($0 <$ test scores < 25). Meanwhile, if the results of the implementation of the initial mathematical ability test in the previously determined class have not found the desired subjects, then the initial mathematical ability test can be carried out in another grade IX.

Interview data analysis was conducted through three stages, namely data reduction, data presentation, and drawing conclusions. At the data reduction stage, the activities carried out by the researcher were the selection, focusing, abstraction, simplification, and transformation of data that appeared in field notes or written transcriptions. At the data presentation stage, the researcher presented the data that had been obtained from the data reduction stage. At the conclusion drawing stage, the researcher would draw conclusions based on the results of the analysis of the initial mathematical ability test data, quadratic pattern generalization problem, and interviews so that a description of the abductive reasoning of students with high initial mathematical abilities, moderate initial mathematical abilities, and low initial mathematical abilities in solving quadratic pattern generalization problem would be obtained.

In order to identify students' abductive reasoning, this study uses abductive reasoning indicators according to Shodikin et al. (2021) which is a description of the abductive reasoning process according to Velazques-Quesada et al. (2013) which is presented in Table 1.

Table 1. Abductive reasoning process and indicators

Abductive reasoning process	Abductive reasoning indicators	Code
Recognizing the existence of an abductive problem	Acknowledging the incompatibility of information obtained with prior knowledge (there are doubts, surprises, oddities).	A1
Identifying solutions	Able to explain the mismatch of information obtained with the prior knowledge (there are doubts, surprises, oddities).	B1
	Mentioning the alleged alternative solutions that can explain plausible solutions and which may be taken to answer the problem based on experience.	B2
Selecting the best solutions	Choosing a particular solution from the provided alternatives solutions.	C1
	Explaining the reason why choosing that solution as the best solution.	C2
Assimilating those chosen	Implementing the chosen solution to overcome the problem.	D1

Result and Discussion

The research subjects used in this study were students of a junior high school in Gresik Regency, grade IX in the 2024/2025 academic year. A total of 26 students took the initial mathematical ability test. Based on the test results, two students with the highest scores from each category of high, medium, and low initial mathematical ability were selected who were of the same gender and had good communication skills as research subjects. The female gender was chosen as the subject in this study based on the advice of the mathematics teacher in class IX-F, that female students in the class are better at communicating than male students. Good communication is needed for the smooth running of the research process. The determination of these subjects was based on the benchmark assessment criteria adopted from Budiyono (2015). Based on these criteria, the data on the selected research subjects are presented in Table 2.

Table 2. Research subject data

Student Name Initials	Gender	Initial Mathematical Ability	Score	Subject Code
NNA	Female	High	90	H1
TAZ	Female		84	H2
SS	Female	Medium	68	M1
NAN	Female		68	M2
CZZ	Female	Low	24	L1
QANF	Female		24	L2

The quadratic pattern generalization problems were given to six research subjects. After the research subjects worked on the quadratic pattern generalization problems, interviews were conducted with each subject in turn. The results of solving the quadratic pattern generalization problems and interviews were used to explore information and students' abductive reasoning processes in solving the quadratic pattern generalization problems that were not written on the answer sheet.

The quadratic pattern generalization problems given to the subjects in this study was about generalization of patterns that only presented the 4th pattern image, then the subjects were asked to determine the general formula for determining the number of black circles in the n^{th} pattern and determining the number of black circles in the 10^{th} pattern using the general formula that had been obtained previously.

The following are the results of solving the quadratic pattern generalization problems by research subjects.

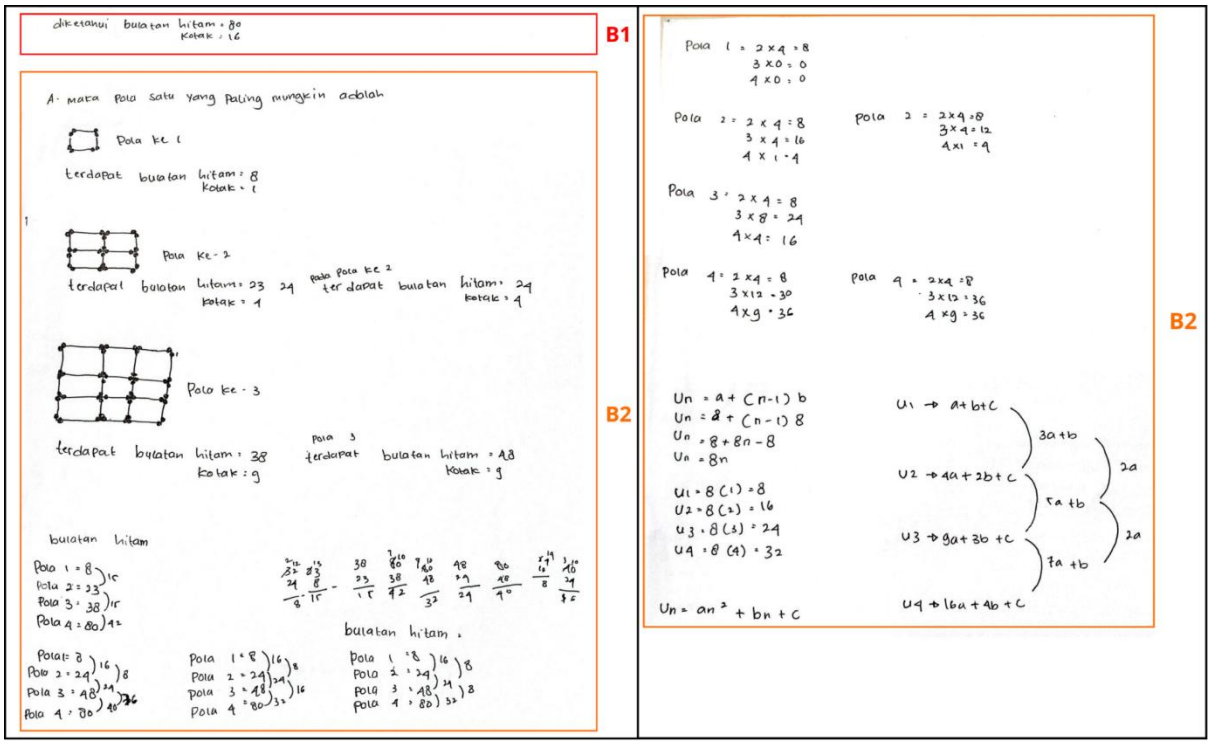

Figure 1. H1 work results (a)

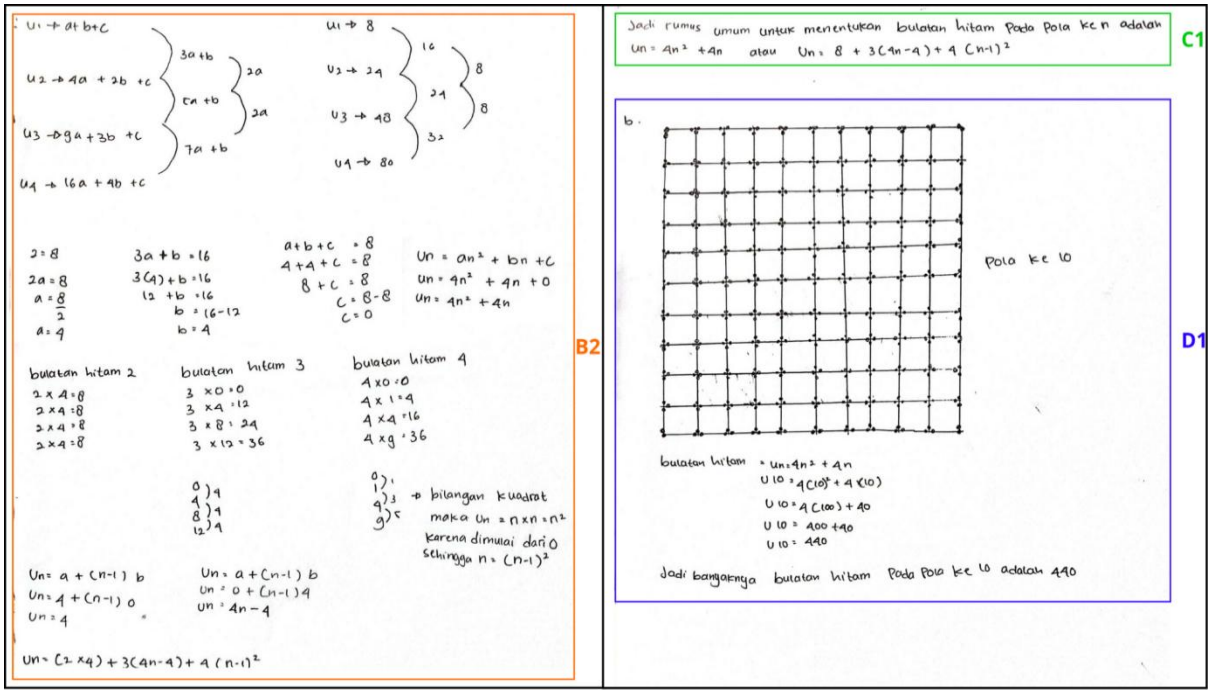

Figure 2. H1 work results (b)

Table 3. Transcript of interview with H1

H1 interview result		
R	:	"Have you ever solved a problem like this before?"
H1	:	"Never."
R	:	"How is this question different from the number pattern questions you have encountered before?"
H1	:	"If there is a pattern, there is a picture of the pattern, usually there is pattern 1, pattern 2, pattern 3, pattern 4 in sequence, then asked to determine the 5th pattern. But in this question there is only the 4th pattern. So at first it was a little confusing. But when it was done, it was a little easier."
R	:	"In the answer you wrote. How did you get that information?"
H1	:	"In the question asked for black circles, so I counted the black circles first."
R	:	"How many formulas did you get?"
H1	:	"There are three, but one formula is not suitable, so there are two formulas that can be used."
R	:	"From the two formulas you got. What formula do you use to determine the number of black dots in the 10th pattern?"
H1	:	"Formula $U_n = 4n^2 + 4n$."
R	:	"Why did you choose that formula?"
H1	:	"I saw it was easier, sis, so I just used that formula."

The results of solving the questions and interviews showed that the subjects could fulfill the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, H1 had never solved the given quadratic pattern generalization problem and found differences between the questions and the number pattern questions that had been encountered before. In the process of identifying solutions, H1 mentioned the differences between the given quadratic pattern generalization problem and the number pattern questions, namely that the given quadratic pattern generalization problem only present the 4th pattern, while the number pattern questions usually contain

patterns 1 to 4. H1 wrote down the information known from the quadratic pattern generalization problem and explained it with logical reasons. This is in line with the results of the study by Cahyanti et al. (2019) which explained that students with high mathematical abilities can collect information by recording information on the answer sheet and providing logical reasons for what was done. The information obtained can help H1 in compiling estimates of the 1st to 3rd pattern forms which are equipped with information on the number of black circles in each pattern. Then H1 determines the fixed difference of the number of black circles in each pattern and substitutes the fixed difference into the formula $U_n = a + (n-1)b$ to obtain the general formula $U_n = 8n$. However, H1 feels that the formula is not quite right because after trying to apply it, it shows results that are not appropriate. H1 realizes that the difference is still obtained at the second level, so H1 substitutes the fixed difference into the two-level arithmetic sequence formula $U_n = an^2 + bn + c$ and gets the general formula $U_n = 4n^2 + 4n$. However, H1 is not sure about the formula that has been produced because he has only applied the second-order arithmetic sequence formula a few times. This doubt prompted H1 to use another step to get a general formula for determining the number of black circles. H1 groups the black circles into 3 forms, namely black circles containing 2, black circles containing 3, and black circles containing 4. H1 also determines and substitutes the fixed difference of the black circles containing 2 and 3 into the formula $U_n = a + (n-1)b$ so that the formula $U_n = 2(4)$ and $U_n = 3(4n-4)$ are obtained and uses the formula $U_n = (n-1)^2$ for the black circles containing 4 because the fixed difference is patterned as a square number starting from zero and the formula $U_n = 4(n-1)^2$ is obtained. Next, H1 adds up the three formulas and obtains the general formula $U_n = 2(4) + 3(4n-4) + 4(n-1)^2$ which is simplified to $U_n = 8 + 3(4n-4) + 4(n-1)^2$. This is in accordance with the research results of Shodikin et al. (2021) which stated that dissatisfaction with the results obtained in the first experiment caused an imbalance that led to the formation of a new scheme by combining two contexts. In the process of choosing the best solution, H1 chose the formula $U_n = 4n^2 + 4n$ from the two formulas obtained because the formula was easier than the other formulas obtained. In the process of assimilating the selected solution, H1 used the formula $U_n = 4n^2 + 4n$ to determine the number of black circles in the 10th pattern and produced the correct answer, namely 440 black circles. Based on this description, H1 meets all the indicators of abductive reasoning and gets two alternative solutions to solve the quadratic pattern generalization problem.

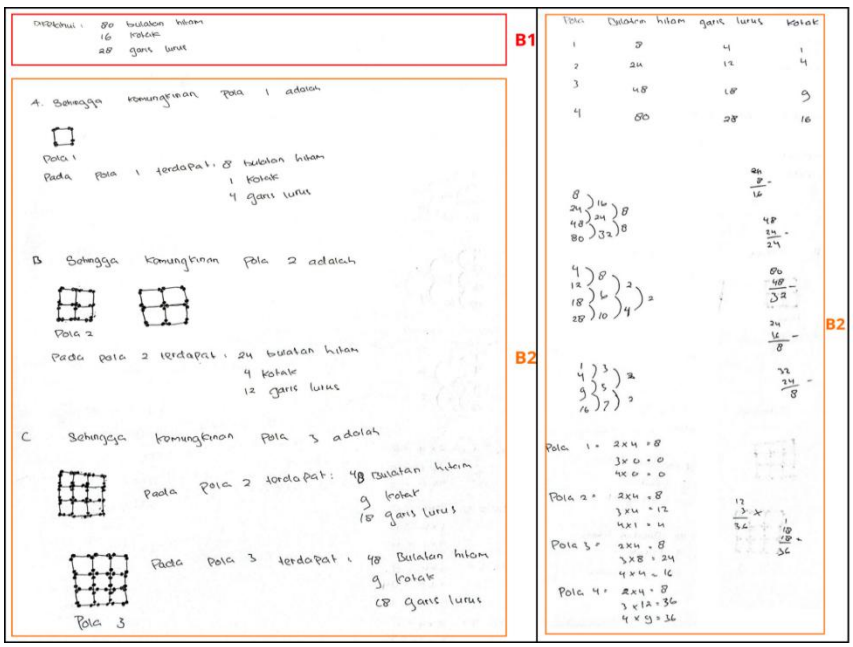

Figure 3. H2 work results (a)

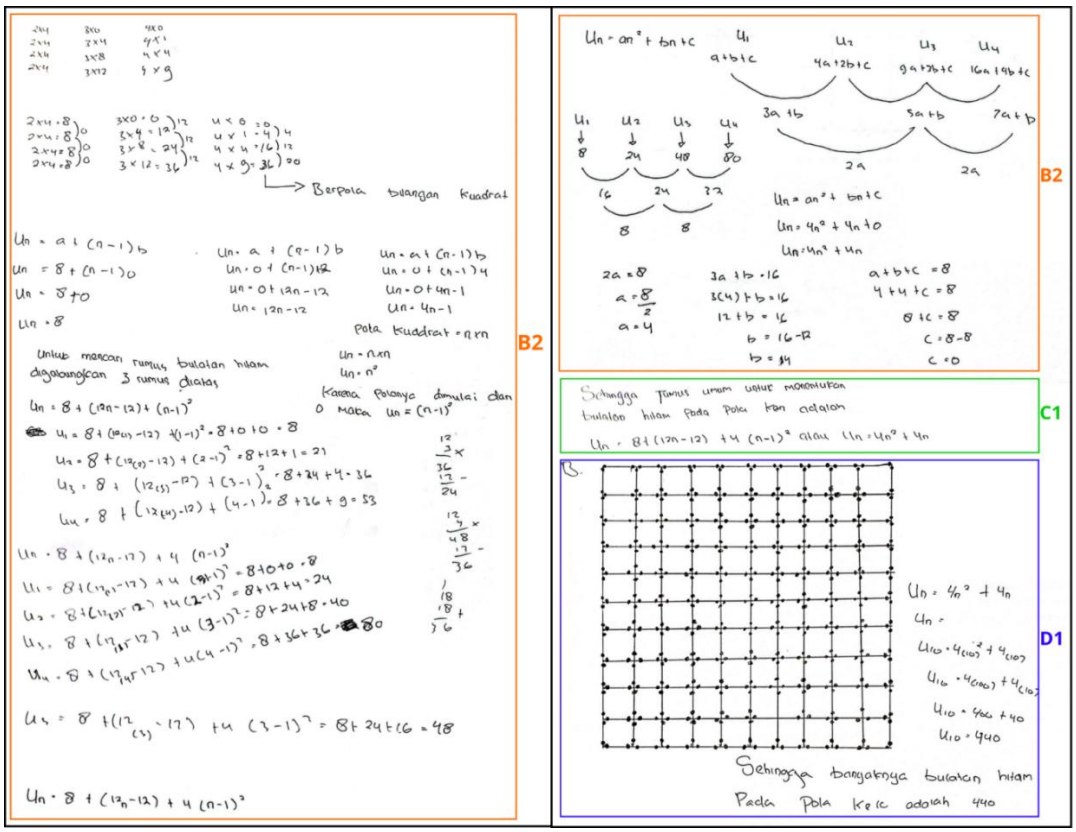

Figure 4. H2 work results (b)

Table 4. Transcript of interview with H2

H2 interview result		
R	:	"Have you ever solved a problem like this before?"
H2	:	"Never."
R	:	"What oddities did you find in the question?"
H2	:	"This question is strange, sis, because suddenly there is only the 4th pattern. I wonder what the 1st pattern looks like."
R	:	"In the answer you wrote. How did you get that information?"
H2	:	"I counted, Sis. In this 4th pattern there are 80 black circles, 16 squares, and 28 straight lines."
R	:	"How many formulas did you get?"
H2	:	"I got 2 formulas, Sis".
R	:	"From the two formulas you got. What formula do you use to determine the number of black dots in the 10th pattern?"
H2	:	"Formula $U_n = 4n^2 + 4n$."
R	:	"Why did you choose that formula?"
H2	:	"Because I think the formula is simpler and easier."

The results of solving the problems and interviews showed that the subjects could fulfill the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, H2 had never solved the given quadratic pattern generalization problem and found something strange in the problem. In the process of identifying the solution, H2 mentioned the strangeness of the quadratic pattern generalization problem, namely that it only presented the 4th pattern so that H2 thought about

the shape of the 1st pattern. Furthermore, H2 wrote down the information known from the quadratic pattern generalization problem. The information obtained can help H2 in compiling an estimate of the shape of the 1st to 3rd patterns which is equipped with information on the number of black circles in each pattern. H2 made an estimate of the shape of the 1st to 3rd patterns which was the same as that made by H1. Then H2 creates a data table of the number of black circles in each pattern and groups the black circles into three types, namely black circles containing 2, containing 3, and containing 4. H2 determines the fixed difference of each type of black circle in each pattern and substitutes the fixed difference into the formula $U_n = a + (n-1)b$ to obtain the formula $U_n = 8$ for black circles containing 2, the formula $U_n = 12n - 12$ for black circles containing 3, and the formula $U_n = 4n - 1$ for black circles containing 4. However, H2 realized that the formula for the black spheres containing 4 did not produce results that matched the number of black spheres containing 4 in each pattern because H2 did not find a fixed difference in the first row. Then H2 found a pattern from the number of black spheres containing 4, namely a square pattern, so H2 used the formula $U_n = (n-1)^2$ which was multiplied by 4 and obtained the formula $U_n = 4(n-1)^2$. H2 added the three formulas and obtained the general formula $U_n = 8 + (12n - 12) + 4(n-1)^2$. After doing these steps, H2 realized that there was another U_n formula, namely $U_n = an^2 + bn + c$. H2 tried to use the formula to make sure the results were the same or different and H2 got a different general formula, namely $U_n = 4n^2 + 4n$. In the process of choosing the best solution, H2 chose the formula $U_n = 4n^2 + 4n$ from the two formulas obtained because the formula was easier than the other formulas obtained. In the process of assimilating the selected solution, H2 uses the formula $U_n = 4n^2 + 4n$ to determine the number of black circles in the 10th pattern and produces the correct answer, which is 440 black circles. Based on this description, H2 fulfills all indicators of abductive reasoning and gets two alternative solutions to solve the quadratic pattern generalization problem. This is in accordance with the results of research by Hasanah et al. (2019) which states that students with high mathematical abilities can solve problems based on the plans that have been made.

The results of solving the quadratic pattern generalization problem by the two students with high initial mathematical ability showed the correct answer because the general formula produced by the two students contained the quadratic row formula. The two students each produced two formulas, where there is one same formula and one different formula. The different formulas can both be used as a solution to the solution of quadratic pattern generalization problem. This is in accordance with the results of Chua & Hoyles (2009), which states that in using various ways to see patterns in the task of generalizing quadratic patterns can produce various kinds of rules (general formulas) equivalent to the same pattern. The fulfillment of abductive reasoning indicators by H1 and H2 is presented in Table 5.

Table 5. Comparison of Fulfillment of Abductive Reasoning Indicators H1 and H2

Indicator codes	H1	H2
A1	Found the incompleteness of the pattern presented in the problem, found the difference between the quadratic pattern generalization problem and the number pattern problem that had been encountered before and had never solved the quadratic pattern generalization problem	Found an unusual problem and found the incompleteness of the pattern presented in the problem and had never solved the problem of generalizing the quadratic pattern.
B1	Explained the difference between the quadratic pattern generalization problem given and the number pattern problems that have been encountered before, which	Explained the peculiarity found in the problem where there was only a picture of the 4th pattern

Indicator codes	H1	H2
	lies in the presentation of the pattern	
B2	Conjectured a square-shaped pattern, determine the fixed difference of the number of black circles in each pattern, group the black circles into 3 shapes, and use the linear and quadratic series formula to get the general formula $U_n = 4n^2 + 4n$ and $U_n = 8 + 3(4n - 4) + 4(n - 1)^2$	Conjectured a quadratic pattern, create a table containing black sphere data, divide the black spheres into 3 types, determine the fixed difference of the number of black spheres in each pattern, and use the quadratic and linear sequence formula to get the general formula $U_n = 8 + (12n - 12) + 4(n - 1)^2$ and $U_n = 4n^2 + 4n$
C1	Choosed the formula $U_n = 4n^2 + 4n$ as the best solution	Choosed the formula $U_n = 4n^2 + 4n$ as the best solution
C2	Explained that the formula $U_n = 4n^2 + 4n$ is easier than the other formulas obtained	Explained that the formula $U_n = 4n^2 + 4n$ is simpler than the other formulas obtained
D1	Used the formula $U_n = 4n^2 + 4n$ to determine the number of black circles in the 10th pattern	Used the formula $U_n = 4n^2 + 4n$ to determine the number of black circles in the 10th pattern

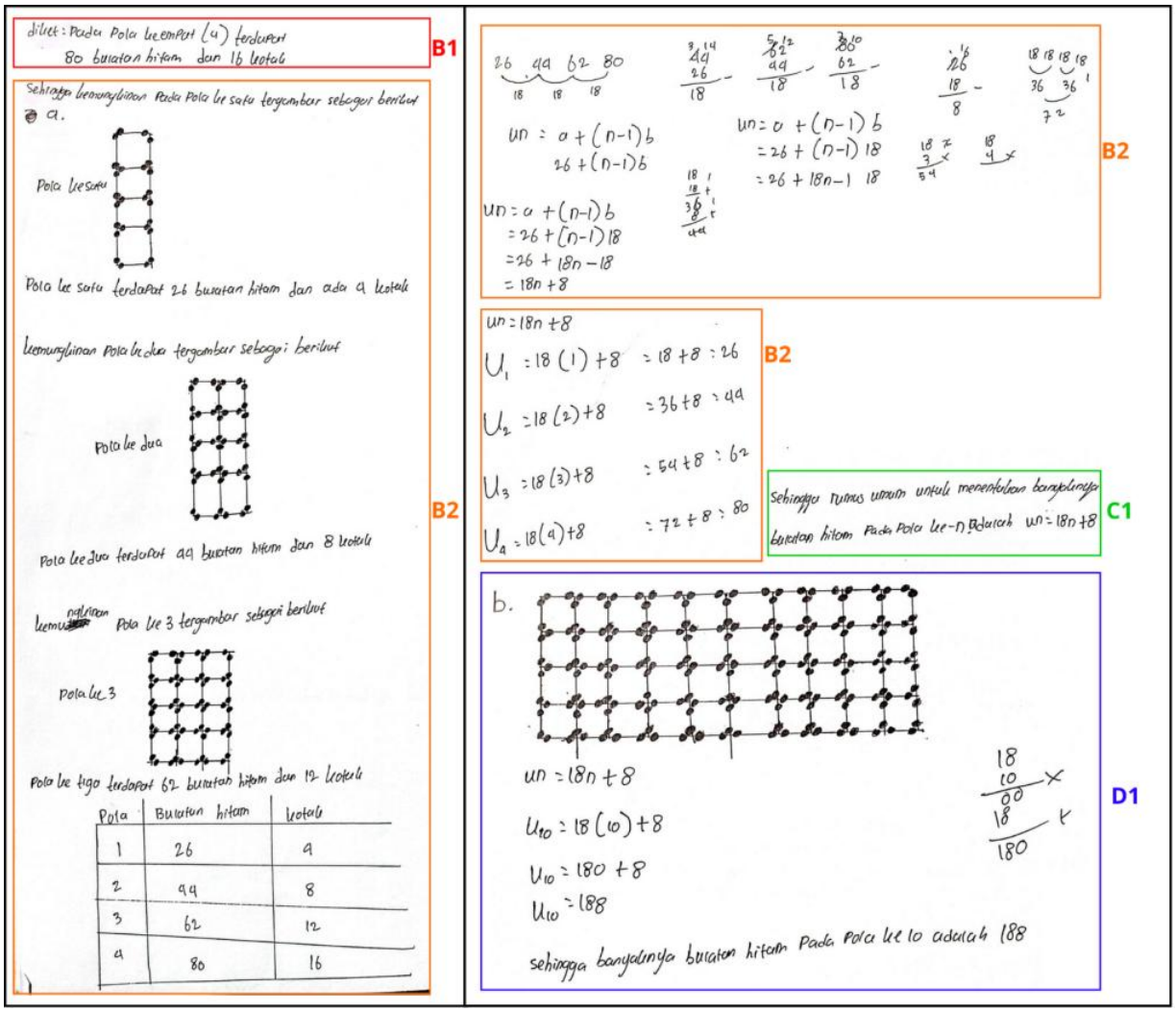

Figure 5. M1 work results

Table 6. Transcript of interview with M1

M1 interview result		
R	:	"Have you ever solved a problem like this before?"
M1	:	"Never."
R	:	"How is this question different from the number pattern questions you have encountered before?"
M1	:	"I have worked on number pattern problems but did not use the U_n formula because the pattern was complete."
R	:	"Then what do you use?"
M1	:	"I forgot, sis, the main difference is there. Same as before, sis, I've seen questions that start from pattern 1. The difference is that the questions you gave start from pattern 4."
R	:	"How many formulas did you get?"
M1	:	"There is one, Sis."
R	:	"What is the formula?"
M1	:	"$U_n = 18n + 8$"
R	:	"From one formula that you got. Do you use the formula to determine the number of black circles in the 10th pattern?"
M1	:	"Yes, Sis."
R	:	"Why did you choose that formula?"
M1	:	"Because I only got that one formula, sis, and the formula is easy to use."

The results of solving the questions and interviews showed that the subjects could fulfill the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, M1 had never solved the given quadratic pattern generalization questions and found differences between the questions and the number pattern questions that had been encountered before. In the process of identifying solutions, M1 mentioned the differences between the given quadratic pattern generalization questions and the number pattern questions, namely that the given quadratic pattern generalization questions only present the 4th pattern, while the number pattern questions usually start from the 1st pattern. M1 wrote down the information known from the quadratic pattern generalization questions. This is in accordance with the results of the study by Listanti et al. (2020) which explained that students with early mathematical abilities were recording information obtained from the questions on the answer sheet. The information obtained can help M1 in compiling estimates of the 1st to 3rd pattern shapes which are equipped with information on the number of black circles in each pattern. M1 made estimates of the pattern shapes that were different from those made by M1. Then M1 creates a data table of the number of black circles, determines the fixed difference of the number of black circles in each pattern, and substitutes the fixed difference into the formula $U_n = a + (n-1)b$ to obtain the general formula $U_n = 18n + 8$. M1 tries to apply the formula and gets the same result as the number of black circles in each pattern. In the process of choosing the best solution, M1 chooses the formula $U_n = 18n + 8$ because the formula is easy to use and only gets one formula. In the process of assimilating the selected solution, M1 uses the formula $U_n = 18n + 8$ to determine the number of black circles in the 10th pattern and produces an answer of 188 black circles. Based on this description, M1 meets all indicators of abductive reasoning and gets one alternative solution.

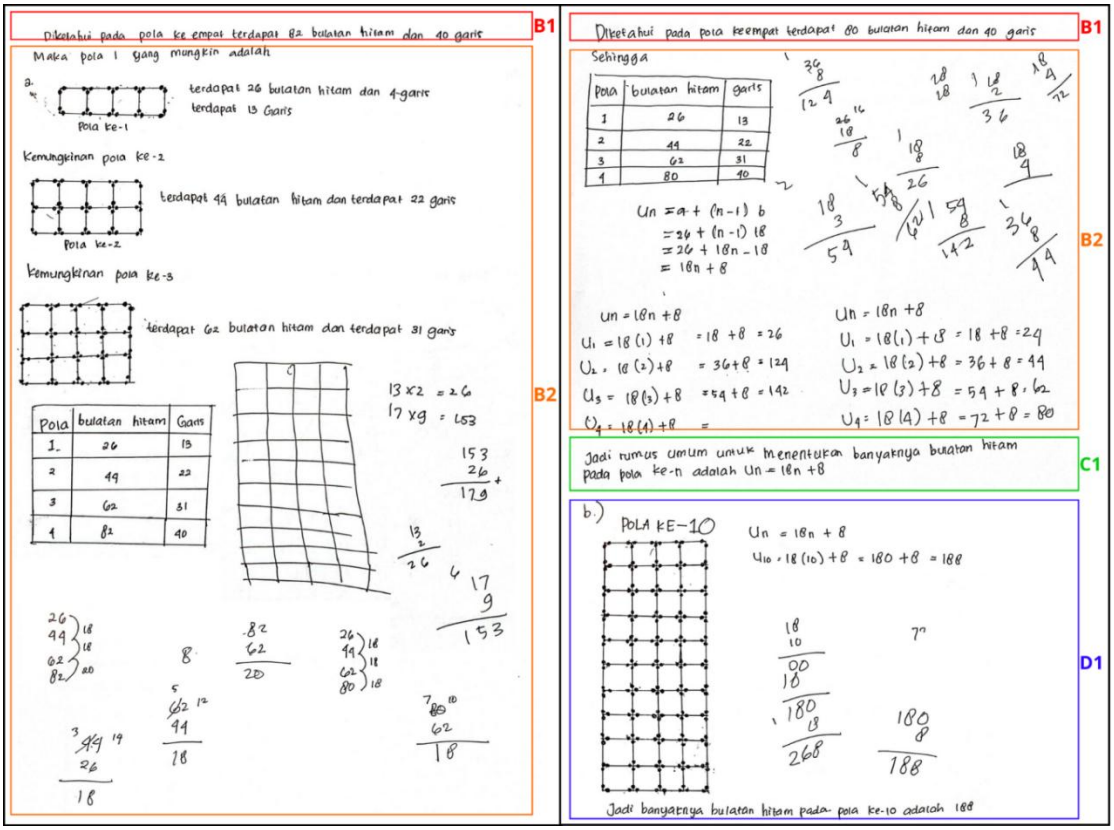

Figure 6. M2 work results

Table 7. Transcript of interview with M2

M2 interview result		
R	:	"Have you ever solved a problem like this before?"
M2	:	"Never."
R	:	"How is this question different from the number pattern questions you have encountered before?
M2	:	"The pattern is definitely different."
R	:	"What is odd or inconsistency in the question I gave?"
M2	:	"Suddenly the 4th pattern, sis, usually the pattern starts from the 1st pattern."
R	:	"How many formulas did you get?"
M2	:	"There is one, Sis."
R	:	"So, what's the formula?"
M2	:	"$U_n = 18n + 8$"
R	:	"From one formula that you got. Do you use the formula to determine the number of black circles in the 10th pattern?"
M2	:	"Yes, Sis."
R	:	"Why did you choose that formula?"
M2	:	"Because it's easy to understand, sis, and when I tried it earlier, the results were the same. So, I used that formula."

The results of solving the problems and interviews showed that the subjects could fulfill the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, M2 had never solved the given quadratic pattern generalization problem and found the oddity and difference between the quadratic pattern generalization problem and the number pattern problem. In

the process of identifying the solution, M2 mentioned the difference between the quadratic pattern generalization problem and the number pattern problem, namely in the pattern presented and the oddity of the quadratic pattern generalization problem, namely that it only presents the 4th pattern, while the number pattern problem presents a pattern starting from the 1st pattern. M2 wrote down the information known from the quadratic pattern generalization problem. The information obtained can help M2 in compiling an estimate of the shape of the 1st to 3rd patterns which is equipped with information on the number of black circles in each pattern. The alleged shape of the pattern made by M2 is the same as that made by M1, which is a linear pattern but in different directions. This is in accordance with the results of research by Chua & Hoyles (2015) which explains that students are more accustomed to handling linear patterns than non-linear (quadratic) patterns because non-linear patterns are less frequently displayed in math textbooks. Then M2 made a data table of the number of black circles. This step of making a table can be called a numerical strategy. M2 did this step because it previously made a conjecture of a linear patterned shape. This is in accordance with the results of research by Chua & Hoyles (2015) which explains that in solving linear pattern tasks students prefer numerical strategies. M2 also determined the fixed difference from the number of black circles in each pattern and substituted the fixed difference into the formula $U_n = a + (n-1)b$ to obtain the general formula $U_n = 18n + 8$. M2 tries to apply the formula and gets the same result as the number of black circles in each pattern. In the process of choosing the best solution, M2 chooses the formula $U_n = 18n + 8$ because the formula is easy to understand and only gets one formula. In the process of assimilating the chosen solution, M2 uses the formula $U_n = 18n + 8$ to determine the number of black circles in the 10th pattern and produces an answer of 188 black circles. Based on this description, M2 meets all indicators of abductive reasoning and gets one alternative solution.

The general formula obtained by students with moderate initial mathematical abilities does not contain a quadratic form so that the formula is not the right solution to solve the generalization problem of quadratic patterns but can be the right solution if adjusted to the estimated pattern shape made by students with moderate initial mathematical abilities. This is in accordance with the research results of Shodikin et al. (2021), which explains that the use of certain types of abductive reasoning does not necessarily produce the correct answer and the answer to the application of abductive reasoning in solving problems is open.

The fulfillment of abductive reasoning indicators by M1 and M2 is presented in Table 8.

Table 8. Comparison of Fulfillment of Abductive Reasoning Indicators M1 and M2

Indicator codes	M1	M2
A1	Found the incompleteness of the pattern presented in the problem and found the difference between the quadratic pattern generalization problem and the number pattern problem that had been encountered before and had never solved the quadratic pattern generalization problem	Found an unusual problem, found the incompleteness of the pattern presented in the problem and found the difference between the quadratic pattern generalization problem and the number pattern problem that had been encountered before and had never solved the quadratic pattern generalization problem
B1	Explained that the previously encountered number pattern problem contains patterns starting from the 1st pattern, while the given quadratic pattern generalization problem contains patterns starting from the 4th pattern	Explained that the pattern presented in the generalization problem of quadratic patterns starts from the 4th pattern, while in number pattern problems it usually starts from the 1st pattern

Indicator codes	M1	M2
B2	Conjectured the linear pattern, made a data table of the number of black circles, determine the fixed difference of the number of black circles in each pattern, and used the linear sequence formula to get the general formula $U_n = 18n + 8$	Conjectured the linear pattern, made a data table of the number of black circles, determined the fixed difference of the number of black circles in each pattern, and used the linear sequence formula to get the general formula $U_n = 18n + 8$
C1	Choosed the formula $U_n = 18n + 8$ as the best solution	Choosed the formula $U_n = 18n + 8$ as the best solution
C2	Explained that the formula $U_n = 18n + 8$ was easy to use and was the only formula that was obtained	Explained that the formula $U_n = 18n + 8$ was easy to understand and when experimenting with the application of the formula in the 1st pattern to the 4th pattern, the results are obtained accordingly
D1	Uses the formula $U_n = 18n + 8$ to determine the number of black circles in the 10th pattern	Used the formula $U_n = 18n + 8$ to determine the number of black circles in the 10th pattern

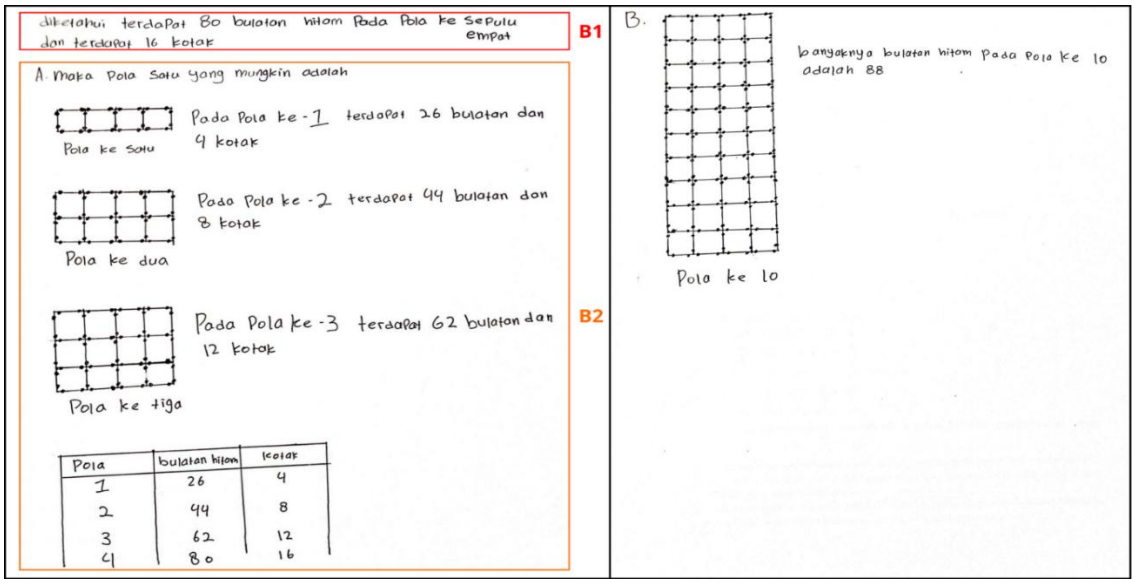

Figure 7. L1 work results

Table 9. Transcript of interview with L1

L1 interview result		
R	:	"Have you ever solved a problem like this before?"
L1	:	"Never."
R	:	"What surprising things did you find in this question?"
L1	:	"When I saw the black circles, I couldn't guess what picture this was."
R	:	"In the question, you are asked to determine the formula, where is the formula?"
L1	:	"I can't find the formula, Sis."
R	:	"How do you determine the number of black dots in the 10th pattern?"
L1	:	"I drew it first, then I counted the black dots in the picture, Sis."
R	:	"Apart from counting directly from the image, can you determine the number of black dots in the 10th pattern?"

L1 interview result		
L1	:	"No, sis, because you must calculate it using pictures."
R	:	"Do you know the formula for U_n ?"
L1	:	"What is U_n the same as, Sis?"
R	:	"$U_n = a + (n-1)b$"
L1	:	"I know, Sis."
R	:	"Why not use that?"
L1	:	"I don't know, Sis, if that formula can be used in this question."

The results of solving the problems and interviews showed that the subjects could not meet the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, L1 had never solved the given quadratic pattern generalization problem and found something surprising in the problem. In the process of identifying the solution, L1 mentioned the surprising thing in the problem was not being able to guess the 4th pattern image presented. L1 wrote down the information known from the quadratic pattern generalization problem. The information obtained can help L1 in compiling an estimate of the shape of the 1st to 3rd patterns which is equipped with information on the number of black circles in each pattern. L1 made an estimate of the same pattern shape as that made by M2. Then L1 made a data table of the number of black circles in each pattern. However, L1 could not continue working on the quadratic pattern generalization problem so that he did not get a general formula to determine the number of black circles in the nth pattern. This is in accordance with the results of the study by Pebrianti et al. (2023), which explains that students with low initial mathematical abilities are unable to work on the problems given. L1 does not know if the formula $U_n = a + (n-1)b$ can be used to solve the problem. L1 does not carry out the process of selecting the best solution and does not carry out the process of assimilating the selected solution, because in the previous process L1 could not determine the general formula. In determining the number of black circles in the 10th pattern, L1 gets the answer 88 black circles. L1 gets the answer by drawing the shape of the 10th pattern first and counting the black circles through the image. However, the answer obtained by L1 is not quite right because it does not match the number of black circles in the 10th pattern image that has been made.

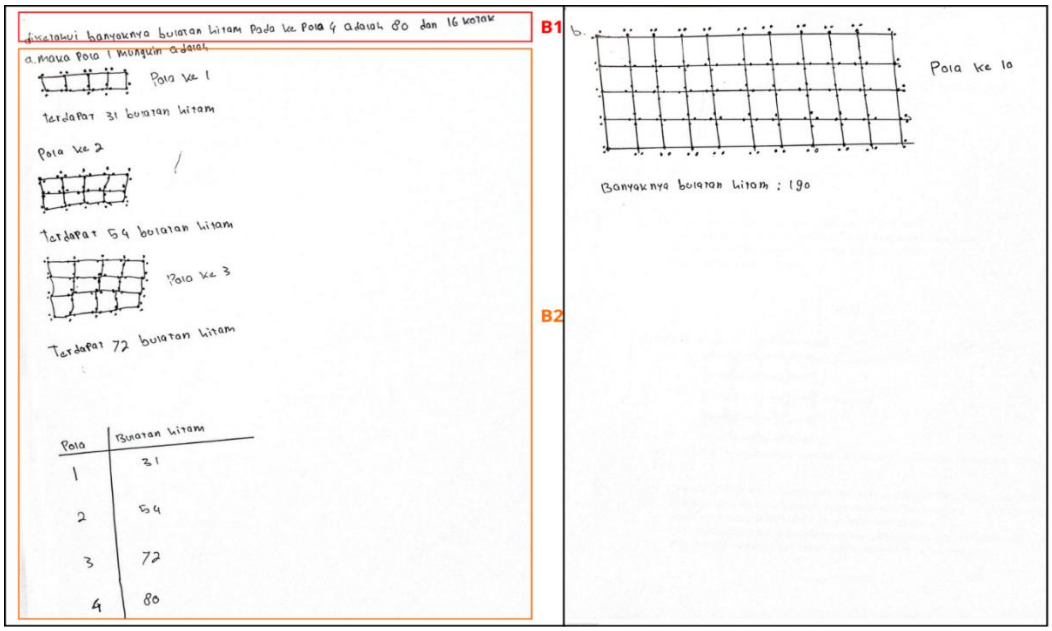

Figure 8. L2 work results

Table 10. Transcript of interview with L2

L2 interview result		
R	:	"Have you ever solved a problem like this before?"
L2	:	"Never."
R	:	"How is this question different from the number pattern questions you have encountered before?"
L2	:	"It seems like the process is working. I was confused when I was working."
R	:	"Then in the question there is an order to determine the formula, did you forget to write the formula or what?"
L2	:	"Don't forget, Sis. I can't determine the formula."
R	:	"How do you determine the number of black dots in the 10^{th} pattern?"
L2	:	"I count one by one, sis. So, I draw the 10^{th} pattern first, I adjust it to the previous pattern. From the 10^{th} pattern image, I count the black circles."
R	:	"Is drawing the pattern first the only way?"
L2	:	"Yes Sis. Just this one."
R	:	"Do you know the formula for U_n ?"
L2	:	"What is the formula for U_n, Sis?"
R	:	"That is $U_n = a + (n-1)b$."
L2	:	"I know, Sis."
R	:	"Why not use that?"
L2	:	"I'm still confused about that formula."

The results of solving the problems and interviews showed that the subjects could not meet the indicators of abductive reasoning. In the process of realizing the existence of abductive problems, L2 had never solved the given quadratic pattern generalization problem and found differences between the problem and the number pattern problem that had been encountered before. In the process of identifying solutions, L2 mentioned the difference between the quadratic pattern generalization problem and the number pattern problem, namely in the process of working on it. Then L2 wrote down the information known from the quadratic pattern generalization problem. The information obtained can help L2 in compiling an estimate of the shape of the 1st to 3rd patterns which are equipped with information on the number of black circles in each pattern. L2 made an estimate of the same pattern shape as that made by L1, but the estimate of the shape made by L2 was inconsistent. The alleged shape of the pattern made by L2 became the most different pattern from all subjects. This is in accordance with the results of Chua & Hoyles (2014) who explained that students have different interpretations of the pattern structure. Furthermore, L2 made a data table of the number of black circles in each pattern. However, L2 could not continue working on the quadratic pattern generalization problem so that he did not get a general formula to determine the number of black circles in the nth pattern. L2 still did not understand the application of the formula $U_n = a + (n-1)b$. L2 did not carry out the process of selecting the best solution and did not carry out the process of assimilating the selected solution, because in the previous process L2 could not determine the general formula. This is in accordance with the research results of Shodikin et al. (2021) which explains that students who carry out abductive reasoning do not always produce new schemes. In determining the number of black circles in the 10th pattern, L2 got the answer 190 black circles. L2 got this answer by drawing the shape of the 10th pattern first and counting the black circles through the picture. However, the answer obtained by L2 was not quite right because it did not match the number of black circles in the 10th pattern image that had been made.

The results of solving the generalization problem of square patterns by the two students with

low initial mathematical abilities showed wrong answers because the number of black circles in the 10th pattern that had been obtained did not match the image that had been made and did not match the calculation results if calculated using the general formula to determine the number of black circles in the nth pattern. This is in accordance with the research results of Hidayah et al. (2020), which stated that in the abductive reasoning process of the factual error type, students use facts outside the problem to solve it, but the facts are wrong. The fulfillment of abductive reasoning indicators by L1 and L2 is presented in Table 11.

Table 11. Comparison of Fulfillment of Abductive Reasoning Indicators L1 and L2

Indicator codes	L1	L2
A1	find unusual pattern shapes and have never solved generalization problems of quadratic patterns	found unusual problems in the problem and had never solved generalization problems of quadratic patterns
B1	Explained that it could not guess the shape of the pattern presented in the problem	explained the difference between the generalization problem of quadratic patterns and the number pattern problems that have been encountered before, which lies in the process of working on it
B2	conjecture a linear pattern, make a data table of the number of black circles, and could not continue the next step so that he did not get the general formula because he did not know if the linear line formula could be used to solve the generalization problem of quadratic patterns	conjecture a linear pattern, make a data table of the number of black circles, but the table contains incorrect data because the alleged shape of the pattern made is inconsistent, and cannot continue the next step so that it does not get a general formula because it is still confused with the application of the linear line formula

Conclusion

Students with high initial mathematical ability have never solved generalization problems of quadratic patterns, explain the differences between generalization problems of quadratic patterns and number pattern problems that they have encountered before, make predictions about the shape of the pattern, determine the constant difference in the number of black circles in each pattern, group the black circles into three shapes, use the quadratic formula, use the first and second order arithmetic sequence formulas, get two alternative solutions, choose a particular solution from the two alternative solutions obtained, explain the reasons for choosing the solution, and use the chosen solution to solve the problem. The alternative solution chosen by students with high initial mathematical ability produces the correct answer.

Students with moderate initial mathematical ability have never solved generalization problems of quadratic patterns, explain the differences between generalization problems of quadratic patterns and number pattern problems that have been encountered before, make predictions about the shape of the pattern, determine the constant difference in the number of black circles in each pattern, use the first-order arithmetic sequence formula, get one alternative solution, choose a particular solution from one alternative solution obtained, explain the reasons for choosing the solution, and use the chosen solution to solve the problem. The alternative solution chosen by students with moderate initial mathematical ability can be the right solution if it is adjusted to the prediction of the shape of the pattern that has been made.

Students with low initial mathematical ability have never solved generalization problems of quadratic patterns, explained the differences between generalization problems of quadratic patterns and number pattern problems that they have encountered before, and made predictions about the

shape of the pattern. Students with low initial mathematical ability did not get alternative solutions so they could not carry out the process of choosing the best solution and assimilating the chosen solution, but students with low initial mathematical ability solved generalization problems of quadratic patterns in other ways but did not produce the correct answer.

When mentioning alternative solution conjectures that might help to solve the problem, there are some students who make linear or non-quadratic pattern conjectures and produce linear formulas. This is due to the lack of working instructions on the generalization of quadratic pattern question sheet so it is advisable to examine more deeply the working instructions contained in the question instrument.

Acknowledgement

The researcher would like to thank all parties who have helped in writing this article. Especially to the Coordinator of Mathematics Education Study Program and the Dean of the Faculty of Mathematics and Natural Sciences, Universitas Negeri Surabaya, the principal, mathematics teachers, and students of SMPN 11 Gresik who have helped carry out this research.

References

Aliseda, A. (2006). *Abductive Reasoning. Logical Investigations into Discovery and Explanation.* Synthese Library, Vol. 330. Berlin: Springer.

Barbosa, A., Vale, I., & Palhares, P. (2012). Pattern Tasks: Thinking Processes Used By 6th Grade Student. *Revista Latinoamericana de Investigación en Matemática Educativa,*15(3), 273-293.

Budiyono. (2015). *Pengantar Penilaian Hasil Belajar*. Surakarta: UPT UNS Press.

Cahyanti, R. D. dan Rahaju, E. B. (2019). Penalaran Siswa Dalam Menyelesaikan Masalah Matematika Ditinjau Dari Kemampuan Matematika. *Jurnal Ilmiah Pendidikan Matematika,* 8(3). 2301-9085. https://doi.org/10.26740/mathedunesa.v8n3.p611-617

Chua, B.L., & Hoyles, C. (2009). Generalisation and Perceptual Agility: How did Teachers Fare in a Quadratic Generalising Problem. *Proceedings of the British Society for Research into Learning Mathematics*, 29(2).

Chua, B. L., & Hoyles, C. (2014). Generalisation of Linear Figural Patterns in Secondary School Mathematics. *The Mathematics Educator*, 15(2), 1-30. http://math.nie.edu.sg/ame/matheduc/journal/v15_2/n 1.aspx

Chua, B.L., & Hoyles, C. (2015). Seeing Through Students' Eyes: The Best-Help Strategy for Pattern Generalisation. *Proceedings of the 12th International Congress on Mathematical Education: Intellectual and Attitudinal Challenges,* 2012-2021.

Furqoni, M. R. & Subekti, F. E. (2022). Kemampuan Penalaran Abduktif Siswa Ditinjau dari Gaya Belajar. *Jurnal Cendekia: Jurnal Pendidikan Matematika,* 6(3), 2713-2724. https://doi.org/10.31004/cendekia.v6i3.1439

Haeruman, L.D., Rahayu W., & Ambarwati, L. (2017). Pengaruh Model Discovery Learning Terhadapt Peningkatan Kemampuan Berpikir Kritis Matematis Dan Self-Confidence Ditinjau Dari Kemampuan Awal Matematis Siswa SMA di Bogor Timur. *Jurnal Penelitian dan Pembelajaran Matematika,* 10(2), 157-168. https://doi.org/10.30870/JPPM.V10I2.2040

Hasanah, S. I., Tafrilyanto, C. F., & Aini, Y. (2019). Mathematical Reasoning: The Characteristics of Students' Mathematical Abilities in Problem Solving. *Journal of Physics: Conference Series,* 1188(1), 012057. https://doi.org/10.1088/1742-6596/1188/1/012057

Hidayah, I.N., Sa'dijah, C., Subanji, & Sudirman. (2020). Characteristics of Students' Abductive Reasoning in Solving Algebra Problems. *Journal on Mathematics Education*, 11(3), 347-362. http://doi.org/10.22342/jme.11.3.11869.347-362

Hjelte, A., Schindler, M., & Nilsson, P. (2020). Kinds of Mathematical Reasoning Addressed in Empirical Research in Mathematics Education: A Systematic Review. *Education Sciences*, 10(10), 289. https://doi.org/10.3390/educsci10100289

Kemendikbud. Nomor 033/H/KR/2022 tentang Capaian Pembelajaran Pada Pendidikan Anak Usia Dini, Jenjang Pendidikan Dasar dan Jenjang Pendidikan Menengah Pada Kurikulum Merdeka. Diakses 01 Mei 2024.

Kementerian Pendidikan dan Kebudayaan. (2016). *Modul Matematika SMA Kelompok Kompetensi H Kurikulum Matematika 2 dan Pemanfaatan Media Pembelajaran.* Jakarta: Direktorat Jenderal Guru dan Tenaga Kependidikan.

Listanti, D. R., & Mampouw, H. L. (2020). Profil Pemecahan Masalah Geometri Oleh Siswa SMP Ditinjau dari Perbedaan Kemampuan Matematika. *Jurnal Cendekia: Jurnal Pendidikan Matematika,* 4(1), 365-379. https://doi.org/10.31004/cendekia.v4i1.224

Nuñez-Gutierrez, Karina. & Cabañas-Sánchez, Guadalupe. (2022). Types of Mathematical Reasoning Evidenced By A Middle Schol Teacher In Pattern Generalization. *Proceedings of the 44th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education,* 676-683. Middle Tennessee State University.

Park, J.H., & Lee, K.-H. (2016). How can Students Generalize the Chain Rule? The Roles of Abduction in Mathematical Modeling. *Eurasia Journal of Mathematics, Science & Technology Education,* 12(9), 2331-2352 https://doi.org/10.12973/eurasia.2016.1289a

Pebrianti, A., Usdiyana, D., Dedy, E., & Sudihartinih, E. (2023). Kesulitan Siswa Dalam Menyelesaikan Soal Pemecahan Masalah Ditinjau dari Kemampuan Awal Matematika Siswa. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, 12(3), 3530-3541. https://doi.org/10.24127/ajpm.v12i3.7400

Pedemonte, B., & Reid, D. (2011). The Role of Abduction in Proving Processes. *Educ Stud Math,* 76, 281–303. https://doi.org/10.1007/s10649-010-9275-0

Ramadhan, Muhammad. (2021). *Metode Penelitian.* Surabaya: Cipta Media Nusantara.

Ramesh, M., & Howlett, M. (2025). Educating for Uncertainty: Integrating Abductive Reasoning into The Public Policy Curriculum. *Policy Design and Practice*, 1–13. https://doi.org/10.1080/25741292.2025.2506262

Reid, D. A. (2018). Abductive Reasoning in Mathematics Education: Approaches to and Theorisations of a Complex Idea. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(9), em1584. https://doi.org/10.29333/ejmste/92552

Salam, M., Hasnawati, H., Andini, I.A.P., Suhar, S., & Lambertus, L. (2023). Analisis Kemampuan Penalaran Matematis Siswa Ditinjau Dari Kemampuan Awal. *AKSIOMA: Jurnal Program Studi Pendidikan Matematika,* 12(2), 2351-2362. https://doi.org/10.24127/ajpm.v12i2.7448

Salam, M., Ibrahim, N., & Sukardjo, M. (2019). Effects of Instructional Models and Spatial Intelligence on The Mathematics Learning Outcomes After Controlling for Students' Initial Competency. *International Journal of Instruction,* 12(3), 699–716. https://doi.org/10.29333/iji.2019.1 2342a

Setiadi, dkk. (2012). *Kemampuan Matematika Siswa SMP Indonesia Menurut Benchmark Internasional TIMSS 2011.* Jakarta: Kemdikbud.

Setiawan, Y. E., Purwanto, Parta, I. N., & Sisworo. (2020). Generalization Strategy of Linear Patterns from Field-Dependent Cognitive Style. *Journal on Mathematics Education,* 11(1), 77–94. https://doi.org/10.22342/jme.11.1.9134.77-94

Shodikin, A., Purwanto, Subanji, & Sudirman. (2021). Students' Thinking Process When Using Abductive Reasoning in Problem Solving. *Acta Scientiae,* 23(2), 58–87. https://doi.org/10.17648/ACTA.SCIENTIAE.6026

Somasundram, P.A., Akmar, S.N., & Eu, L.K. (2019). Pattern Generalisation by Year Five Pupils. *International Electronic Journal of Mathematics Education,* 14(2), 353-362. https://doi.org/10.29333/iejme/5719

Velazquez-Quesada, F. R., Soler-Toscano, F., & Nepomuceno-Fernandez, A. (2013). An Epistemic and Dynamic Approach to Abductive Reasoning: Abductive Problem and Abductive Soluion. *Journal of Applied Logic,* 11, 505-522. https://doi.org/10.1016/j.jal.2013.07.002

Wackerly, J. W. (2021). Abductive Reasoning in Organic Chemistry. *Journal of Chemical Education,* 98(9), 2746–2750. https://doi.org/10.1021/acs.jchemed.1c00295

Woosuk Park. (2017). *Abduction in Context: The Conjectural Dynamics of Scientific Reasoning.* Studies in Applied Philosophy, Epistemology and Rational Ethics, 32. https://doi.org/10.1007/978-3-319-48956-8_1