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Mapping High School Students' Metacognition Based On Mathematics Skills

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Abstract

Problem solving and metacognitive are two things that are interrelated. Problem solving is synonymous with cognitive processes whereas metacognition is synonymous with controlling and regulating cognitive processes. This study aims to describe the metacognition of students with different mathematical abilities. The subjects in this study consisted of one student with high, medium, and low math abilities. Data analysis was carried out based on metacognition indicators and problem solving. It was found that, there were differences in the three subjects in solving problems. The three subjects were able to fulfill the three components of metacognition, namely awareness, regulation, and evaluation. From this study, it is suggested to conduct a deeper assessment of the metacognition process of students with high, medium, and low math abilities to see their characteristics.
Keywords: Metacognition, Problem Solving, Mathematics Skills

Abstrak

Pemecahan masalah dan metakognitif adalah dua hal yang saling barhubungan. Pemecahan masalah identik dengan proses kognitif sedangkan metakognisi identik dengan mengontrol dan mengatur proses kognitif. Penelitian ini bertujuan untuk mendeskripsikan metakognisi siswa berkemampuan matematika berbeda. Subjek dalam penelitian ini terdiri dari satu siswa berkemampuan matematika tinggi, sedang, dan rendah. Analisis data yang dilakukan berdasarkan indikator metakognisi dan pemecahan masalah. Ditemukan bahwa, terdapat perbedaan pada ketiga subjek dalam memecahkan masalah. Ketiga subjek mampu memenuhi tiga komponen metakognisi, yaitu awareness, regulation, dan evaluation. Dari penelitian ini, disarankan untuk melakukan penilaian yang lebih dalam terhadap proses metakognisi siswa berkemampuan matematika tinggi, sedang, dan rendah untuk melihat karakteristiknya.
Kata kunci: Metakognisi, Pemecahan Masalah, Kemampuan Matematika.

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Introduction

Problem-solving ability is one of the factors forming a person to become a superior human being. According to Karatas & Baki (2013), problem-solving is the work that a person does in finding solutions to the problems at hand. Problem-solving can help students develop their knowledge through the problems given to them (Bostic, Pape, & Jacobbe, 2016). Through problem-solving, students will try to identify mathematical ideas from a problem and express them in algebraic form (Guerra & Lim, 2014). This process may make students read the problem over and over again, and try to find the right strategy to solve it. This can make students trained to use their knowledge and skills when facing a problem.

When students carry out the problem-solving process, they need 3 requirements to complete them well, including (1) awareness of their abilities; (2) being able to control the ability and the selection of the right strategy; and (3) can evaluate the results of their thinking (Al-Shabibi, 2018). These three requirements are related to metacognitive skills, namely awareness, regulation, and evaluation (Wilson & Clarke, 2002).

Problem-solving and Metacognition are two related things. Problem-solving is needed in carrying out a task or finding a solution to a problem, while metacognition is needed to understand how the task can be carried out. Metacognition can be interpreted as a person's awareness of controlling his mind (Hastuti, et al, 2016). These metacognitive skills can help a person identify the problem at hand, help someone to understand what the meaning is in the problem, and help them to find a solution (Kuzle, 2019). This skill is attached to someone when they are solving a problem.

Many researchers have conducted research related to metacognition and problem-solving. Among them are Desoete (2001), Lioe (2003), Wilson & Clarke (2004), Lesh (2007), Panauorra (2009), Kuzle (2011), Molenar (2011), Karan & Irizary (2011), Magiera & Zawojewski (2011), In 'am (2012, Zarimah & Tajudin (2013). The research that has been carried out generally examines the metacognition process of related subjects. However, this has not brought up the metacognitive characteristics of students in solving problems. In this regard, then The researcher examines in-depth researches to analyze and describe the characteristics of students' mathematical metacognition in solving linear programming problems. The characteristics of students' metacognition are described based on the process of awareness, regulation, and evaluation which are the components of metacognition.

The attention to the metacognitive process experienced by students in problem-solving is the optimal way to improve mathematical abilities. One way to see the activities of cognition-metacognition activities can be seen through the problem-solving steps that are passed by students. The process of solving mathematical problems experienced by students is very unique and different, the factor that causes this to exist is a person's varying mathematical abilities. According to Soemarmo (2010), mathematical ability is a skill in implementing and solving problems. In this study, the mathematical ability in question is the cognitive ability of students in working on math problems. Those abilities are grouped into 3, namely someone who has the low ability, someone who has the medium ability, and someone who has the high ability. The students selected in this study were based on heterogeneous student groups in terms of their mathematical abilities. This is done so that this research is expected to bring up a picture of cognition - metacognition that is following the mathematical abilities of each student.

Metacognitive activity is also explained by the opinion of Magiera & Zawojewski (2011) and Wilson Clarke (2004). According to Magiera & Zawojewski (2011) awareness activities include statements made about oneself or other people's mathematical thoughts, this shows thoughts about (1) What one knows, including one's knowledge to complete a particular task, one's relevant knowledge about mathematics, and one's problem-solving strategies; (2) Where a person is in the process of solving the problem; (3) What one needs to do in problem-solving, what one has done in problem-solving, or what one can do to solve the problem. The regulation activity includes statements made about oneself or the mathematical thinking of others, indicating (1) planning strategies; (2) Goal setting; (3) Selection of problem-solving strategies. And the evaluation activity includes judgments made about oneself or the mathematical thinking of others, which shows (1) the effectiveness and limitations of thinking; (2) The effectiveness of the chosen strategy; (3) Assessment of results; (4) Assessment of the difficulty of the problem; (5) Assessment of progress, ability or understanding.

According to Wilson and Clark (2004) awareness activities include activities about individual awareness, where a person is in the phase of the problem-solving process from the cognitive content he has and knowledge of personal learning or problem-solving strategies which includes their knowledge of (1) what needs to be done in learning or problem-solving?; (2) What has been done in learning or problem-solving?; (3) What should be done in learning or problem-solving?. Meanwhile, regulation activity occurs when a person uses his metacognitive skills to direct his knowledge and thoughts. While evaluation is an activity that refers to an assessment made by someone regarding the process, capacity, and limitations of thinking such as solving a problem. For example, a person can make judgments about the effectiveness of his thinking or the effectiveness of his chosen strategy

Cognition regulation presented by Magiera & Zawojewski (2011) and Walson & Clark (2004) has the same characteristics and is interconnected. Both of these theories refer to how the cognitive process is carried out by students to solve a problem. In this study, researchers focused on the theory presented by Wison & Clark (2004), namely Awareness, regulation, and evaluation.

Problem solving and metacognition are two things that are interconnected. Problem-solving is identical to a person's cognitive process to get a solution to the problem at hand. While metacognition is identical in one's process of controlling and regulating cognitive processes. These two things are a series of activities that cannot be separated. These two things do have different content and function, but their form and meaning have the same characteristics. In this study, researchers developed indicators to see students' metacognition based on Polya (1993) and Wilson & Clark (2004).

Table 1. Student Metacognition Indicators in Solving Linear Program Problems

Problem-solving steps	**Metacognition Indicators in problem solving**
Understanding the problem	1. Awareness. Thinking about what will be done to be able to understand the problem. 2. Regulation. The effective way to understand the problem. 3. Evaluation. Re-examine the ways used in understanding the problem.
Devise a plan	1. Awareness. Thinking about what will be done to be able to understand the problem. 2. Regulation. The effective way to understand the problem. 3. Evaluation. Re-examine the ways used in understanding the problem.
Do the plan	1. Awareness. Thinking about using his plan to solve problems. 2. Regulations. Implement Steps that are carried out according to plan
Looking back	1. Awareness. Think to examine all the steps taken. 2. Regulation. Check all the steps taken. 3. Evaluating. Check whether the steps are correct.

Method

This research is qualitative. The subjects used in this study were 3 high school students who had received linear programming material. The subjects of this study consisted of 1 student with high math ability, 1 student with moderate math ability, and 1 student with low math ability. The research method is structured in steps (1) the researcher gives linear programming questions to students in the form of an application of a function to get the maximum and/or minimum value. The problem used has been validated by 2 mathematicians and a math teacher. While working on the questions, the subjects did think aloud and recorded. (2) Researchers observed students during think-aloud to find out the appearance of indicators and/or descriptors of awareness, evaluation, and regulation. (3) The researcher corrected the output of student problem solving based on the students' work and adjusted it according to the answer key that had previously been made. Based on this, the results of the subject's work are classified into high-ability students, medium-ability students, and low-ability students. (4) Give verbal reports after students complete the questions. It consists of 14 items to measure the process of metacognition that occurs in students. This is done as a refinement of previous research that has been carried out by Biryukov (2001), Azsoy & Ataman (2009), Panaoura (2010), and Meriam & Idrus (2010). (5) Interview process. This is based on the results of the work of the subject who has solved the problem, think aloud recordings, and verbal reports. This is done to further explore the characteristics of the process of awareness, evaluation, and regulation. This was done after the students did think-aloud. The interview guidelines that have been prepared have been adapted to the indicators of awareness, evaluation, and regulation. (6) Transcribing the subject's think-aloud recordings and interview results. The aim is to obtain data about the characteristics of the metacognitive process during solving linear programming problems related to differences in student abilities. (7) Data Reduction. Data reduction is done by compiling an abstract in the form of a summary of core data, processes, and statements made by research subjects in solving linear programming problems (8) Data analysis. Analyzing the metacognitive process according to the indicators of awareness, evaluation, and regulation. (9) Data validation. The technique used in data validation is a triangulation technique, namely a data collection technique. Subjects convey ideas that come to mind in the process of thinking using sentences or words or spoken ones (interviews) in the process of solving problems. The data was obtained in the form of spoken words and written words (student answers). After that, interviews were conducted to obtain clearer data about students' problem-solving processes and students' metacognition.

Efforts to describe a person's problem-solving abilities and metacognitive skills are not easy, but in the problem-solving process, Wilson & Clark (2004) developed a method for assessing metacognition by using verbal reports in the form of self-reports when students solve problems, so that students will reveal what's on his mind. The self-report above is presented using an action card asking questions about metacognition experienced by the subject during problem-solving. To facilitate the metacognitive assessment, the researcher recorded the activities carried out by the subject during the problem-solving process.

Result

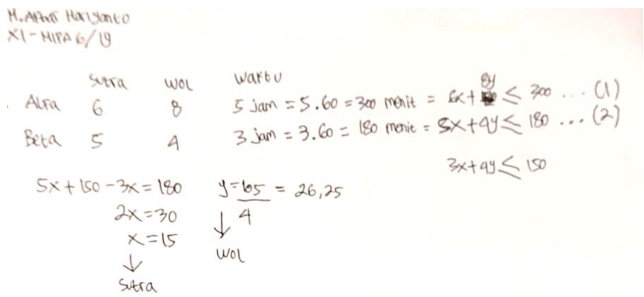
Metacognition of High Mathematics Ability Students in Problem Solving Linear Programs

Based on data exposure and metacognitive data trangulation of subjects with high mathematical abilities, the following will describe the research findings obtained.

Table 2. Triangulation of Metacognitive Data through Think Aloud Method and Interview of Subject (KT) in TPM

Aspect	Code TA/W	Description
Awareness	Conclution	KT recall their knowledge to identify problems. KT mentions the problem in the question. KT knows a plan in general. KT knows the plan to be carried out. KT think and check between the results obtained and the questions asked.
Regulation	Conclution	KT draws up a plan to solve the problem. KT ensures that the sentences used to represent the questions do not fall out of the original intent of the questions. KT chooses an algorithm that can be used to solve the problem. KT regulates or describes the actions to be taken. KT decided to determine the final result. KT checks or changes the representation/model of the correction rule if there are errors in the results obtained.
Evaluation	Conclution	KT checked his work again. KT states if the information on the questions obtained is sufficient to solve the problems. KT analyzes the correspondence between the information provided and the representations made. KT analyzes if the results of the evaluation carried out can correct the mistakes that have been made

KT can restate the appropriate statement on the question in the form of a table. KT can analyze the relationship between the statements related to the questions and problems given. This is indicated by the representation made to solve the problem. In addition, KT mentions things that are known to make it easier for him to understand the problem. In the aspect of regulation, KT makes a mathematical model from the table that has been made to facilitate it in solving problems. The mathematical model is 6x+8y=300….. (1) , 5x+4y=180….. (2). KT writes 5x+150=3x+180. KT gets this equation by substituting 4y=150-3x at 5x+4y=180. It appears that the subject finds the number of units of silk fabric that can be produced is 15 units and the number of units of wool fabric that can be produced is 26 units.



After that, KT seeks the maximum profit that can be realized by the company. The process of finding the maximum profit by multiplying the profit on silk, 30,000, with the number of units of silk that can be produced (15) and adding the result of the multiplication operation between the profit on wool, 40,000, and the number of units wool fabric that can be produced (26). Then the result is 1,490,000. Next, KT looks for the remaining time on each machine. In determining the remaining time, KT chooses 0.25 (in minutes). Subjects get a value of 0.25 from the difference between the y point (26.25) and the number of units of wool that can be produced (26). KT assumes that the number of units of wool that can be perfectly produced is 26 and that the number of units of wool that has not been completed to be produced is 26.25. Then the subject determines the remaining time through 0.25 minutes multiplied by 8 and gets an answer of 2 minutes for the beta engine.

In determining the answer to the first problem point c, the subject is still confused about what to do. In this case, the subject is not aware of the error that has been made that to find the remaining time on each machine, what needs to be done is to make substitutions in equation 1 and equation 2. However, the subject assumes that the remaining time on each machine can be found through the result of multiplying the remainder of 0.25 minutes multiplied by 8 (y-coefficient). In the evaluation aspect, in the first problem, KT revealed that KT rechecked the results of his work. The checks made by KT are on the correctness of the mathematical model that he has made and when the subject is looking for profits that can be realized by the factory. The subject also revealed that by making corrections to the answers he had made, KT felt confident and satisfied with his answers. KT revealed that it had difficulty determining the number of units of wool fabric it could produce. This is because the number or result of the calculation is not round, which is 26.25. In addition, the subject also realized that the subject had difficulty finding the remaining time on each machine. This is not following Erbas & Okur (2012) that students who can show awareness and evaluation aspects can get the right answer.

Metacognition of Moderate Mathematics Ability Students in Problem Solving Linear Programs

Based on data exposure and metacognitive data trangulation of subjects with moderate mathematical abilities, the following will describe the research findings obtained.

Table 3. Triangulation of Metacognitive Data through Think Aloud Method and Interview of Subject (KS) in TPM

Aspect	Code TA/W	Description
Awareness	Conclution	KS mentioned the existing statement on the problem at hand. KS mentions the problem or question in the question. KS knows a plan in general. KS knows the plan to be carried out. KS thinks of improvement if it finds any errors. KS thought about whether there might be a different way to solve the problem. KS thinks and checks between the results obtained and the questions asked.
Regulation	Conclution	KS selects the appropriate representation/model. KS selects or sorts out relevant information. KS regulates or describes the actions to be taken. KS chooses an algorithm that can be used to solve the problem. KS took the decision to determine the final result KS took the decision to carry out a different plan to determine the final result. KS checks if there is another rule/way to get a different answer.
Evaluation	Conclution	KS analyzes the suitability of the information provided with the representation that has been made. KS analyzes whether the chosen algorithm can be used to solve the problem. KS analyzes whether there are alternative answers to solve the questions.

In the awareness aspect, KS wrote down the information on the answer sheet according to the understanding that KS had. KS wrote down two equations to produce 1 unit of silk, $A = 6\alpha + 5\beta$ and to produce 1 unit of silk, $B = 8\alpha + 4\beta$. KS stated that A is the number of silk fabrics that can be produced. KS revealed that to produce one unit of silk cloth it takes 6 minutes on an alpha machine and to produce one unit of silk it takes 5 minutes on a beta machine. Then to produce one unit of wool it takes 8 minutes on an alpha machine and to produce one unit of wool it takes 4 minutes on a beta machine.

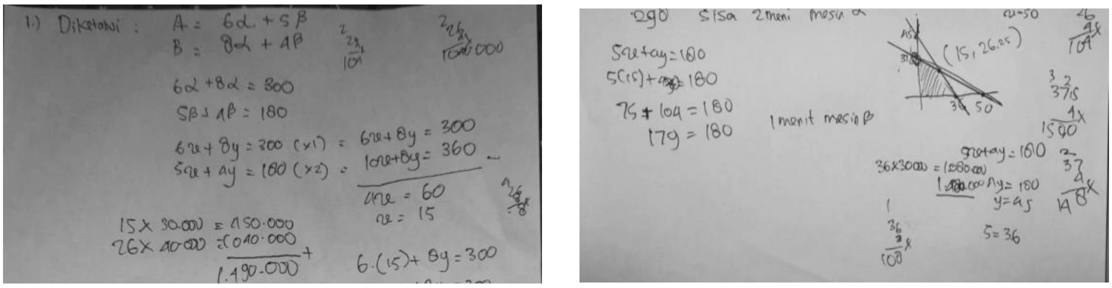


Figure 2. KS Work in Solving Problems

In the aspect of regulation, KS thinks of using the elimination-substitution method to find a solution to the 2 equations that have been made, namely 6x + 8y = 300 and 5x + 4y = 180. After that, find a solution to the problem you are looking for by using the substitution method to get the optimal value of the equation f(x,y)=30.000x + 40.000y. KS was not aware of the equation rules he had made. KS just thinks quickly to solve it without exploring the meaning of the equation. This can be seen in Figure 4.3.c. After that, KS continued to find the remaining time on each machine by substitution. KS also revealed that there is another way to get a solution to this problem, which is to use a coordinate system. KS explained the steps he took to get this solution, namely by first finding the equation for the alpha machine: 6x + 8y = 300, the second looking for the equation for the beta machine: 5x + 4y = 180. After that, describe the two equations in the Cartesian coordinate system and look for the set of solutions according to the information in the problem. Then find the optimum point of this problem by finding the point of intersection between the two equations.

KS carried out the plan according to the steps he had previously prepared. KS does elimination on 2 equations, namely 6x + 8y = 300 and 5x + 4y = 180 and finds x = 15. Then KS does substitution and finds y = 26.25. KS revealed that the realized production was 15 silk fabrics and 26 wool fabrics. KS revealed that 26.25 was rounded up to 26, this is because to produce wool cloth it must be an integer. After that, KS performed a multiplication operation and made a profit of 1,040,000 for silk and 450,000 for wool. So that KS gets the answer that the profit that can be realized is 1,490,000. KS gets the answer for the time remaining on the alpha engine is 2 minutes and the time remaining on the beta engine is 1 minute. This is following Mokos and Kafoussi (2013) who revealed that there are special requirements attached to students when choosing a problem-solving strategy. This depends on the level of students' understanding of the problems at hand.

In the evaluation aspect, in the first problem of TPM, KS revealed that there was no other answer choice. This is because KS cannot find alternative answers even though they have found the set of solutions, but KS is not aware of this. KS also stated that apart from the substitution and elimination process, there is another way to solve this problem, namely using a coordinate system. KS also believes that the correct answers to problem number 1 are x = 15 and y = 26, with the maximum profit being 1,490,000.

Metacognition of Low Mathematics Ability Students in Problem Solving Linear Programs

Based on data exposure and metacognitive data trangulation of subjects with low mathematical abilities, the following will describe the research findings obtained.

Table 4. Triangulation of Metacognitive Data through Think Aloud Method and Interview of Subject (KR) in TPM

Aspect	**Code TA/W**	**Description**
Awareness	Conclution	KR recalls the knowledge possessed to identify problems. KR mentions the existing statement on the problem at hand. KR mentioned the problem in the problem. KR thought of an algorithm that might work. KR thinks of improvement

		if it finds fault. KR rethinks the representation or model that fits the problem conditions.
Regulation	Conclution	KR selects the appropriate representation/model. KR arranges or describes the actions to be taken. KR chooses an algorithm that can be used to solve the problem. KR uses or changes the plans outlined. KR took the decision to determine the final result. KR checks or changes the representation / model of the correction rule if there are errors in the results obtained.
Evaluation	Conclution	KR analyzes whether the information obtained is sufficient to solve the problem. KR analyzes the suitability of the information provided with the representation that has been made. KR analyzes whether the chosen algorithm can be used to solve the problem. KR if the results of the evaluation carried out can correct the mistakes that have been made. KR analyzes the results the solution is correct

In the awareness aspect, KR began to present the information contained in the questions in the form of tables after reading the problems. KR revealed that the information contained in problem number 1 was sufficient to solve the problem. KR revealed that the information in the question was still a lot of words and was messy, so KR needed to re-present it to make it easier to understand. KR also revealed that he was confused about the share of time available on each machine. The confusion was caused by the time unit in the question description, namely in hours, so KR changed the time unit in minutes to be adjusted to other information. The information presented in the table revealed by KR that the time required to produce 1 unit of silk fabric is 6 minutes on the machine and 5 minutes on the machine. The time required to produce 1 unit of wool is 8 minutes on the machine and 4 minutes on the machine. In addition, KR wrote that the profit that can be realized by the factory to produce 1 unit of silk fabric is 30,000/unit and the profit that can be realized by the factory to produce 1 unit of woolen fabric is 40,000/unit. In addition, KR also wrote that the available time on the machine is 5 hours or 300 minutes and the time available on the machine is 3 hours or 180 minutes. KR also revealed that the information contained in problem 1 was sufficient to solve the problem. In addition, KR also revealed that the purpose of this problem is to find the number of each product that can be produced, the profit, and the remaining time.

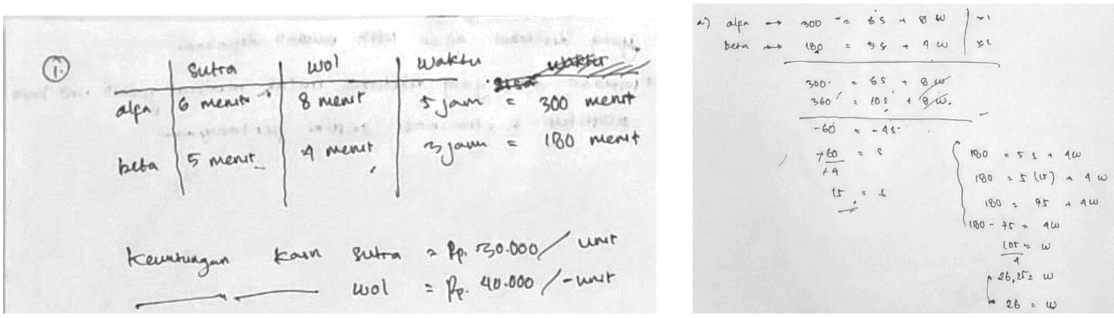


Figure 3. The Work of Subjects with Low Mathematics Ability in Solving Problems

In terms of regulation, KR experienced an error in understanding the problem. KR missed an important point in the problem, namely the constraint function. KR did not provide complete information regarding the function of the constraint. The available information states that there is an inequality function (maximum constraint), but the subject only writes the equation. This is because the subject is not familiar with the problem of linear programming in the form of a story so KR does not realize his mistake. After that, KR presented the plan that had been prepared to solve the problem, namely, first KR made a table based on the information contained in the problem so that it was easy to understand, then wrote down what was known such as the time available on each machine. After that, the subject wrote the equation according to the information in the table he had made, namely 6S+8W=300…(1) and 5S+4W=300…(2) (RR3). After that, KR uses the method of substitution and elimination to get an answer that fits the problem.

KR gets the answer s = 15 and w = 26, meaning that the number of silks that must be produced so that the maximum profit is 15 units and the number of woolen fabrics that must be produced so that the maximum profit is 26 units. In the problem of point b, KR performs a multiplication operation on each profit from each production that can be realized. KR writes 30,000 x 15 = 450,000 for silk and 40,000 x 26 = 1,040,000 for wool. KR states that the time remaining on the machine is 2 minutes and the time remaining on the machine is 1 minute. KR gets this result from the substitution process S = 15 and W = 26 in the two equations that have been made. The subject revealed that to get the answer he had found, the subject did trial-and-error in the process. This is following Erbas & Okur (2012) that students who can show awareness and evaluation aspects can give the correct answer.

In the evaluation aspect, the subject of KR revealed that he rechecked the work that had been done. KR checks on the part of the equation that has been made, the substitution and elimination methods used, and checks the accuracy of the answer. KR realized the mistake he had made in section W = 26.25. Since W is the number of wools that can be produced on each machine, W is an integer member. In the process of re-examining the problems of points a and b, KR revealed that KR was unsure of the results of the substitution and elimination methods he used. KR then rethought his answer and then decided to use the S = 15 and W = 26 which he had previously determined. In addition, KR also checks the units of time used at the stage of understanding the problem. KR revealed that there was a writing error when presenting the table based on the information contained in the problem, but KR was able to solve it. Although the answer written by KR is not complete, KR is satisfied with his work. KR also checked other rules/methods to get a different answer, but KR kept using the method he had used.

Discussion

Table 5. Metacognition Activity.

Metacognition Activities		**Subject**		
		High	**Moderate**	**Low**
Understand the Problems	*Awareness*	2	2	3
	Regulation	1	2	1
	Evaluation	1	1	1
Plan	*Awareness*	2	1	1
	Regulation	1	2	2
	Evaluation	1	1	1
Do the Plan	*Awareness*	1	2	1
	Regulation	2	2	2
Evaluation	*Awareness*	1	2	1
	Regulation	1	1	1
	Evaluation	2	1	2

Based on the research results presented in the table above, it appears that the three students with different abilities were able to show the activity of the metacognitive component. However, some differences affect metacognitive activity. TPM. KT does not show the dominant metacognitive aspect in solving linear programming problems. KT fulfills the three metacognitive components while solving linear programming problems, namely awareness, regulation, and evaluation. This is in line with the research conducted by Putri (2015 and Sudia (2015) that subjects with high mathematical ability can fulfill the three metacognitive components. The first step taken by KT in solving linear programming problems is reading the given task. After that, KT shows awareness aspect when understanding the information and problems encountered. This is in line with Wilson & Clarke (2004) which revealed that 68 out of 90 students started problem-solving with the awareness aspect.

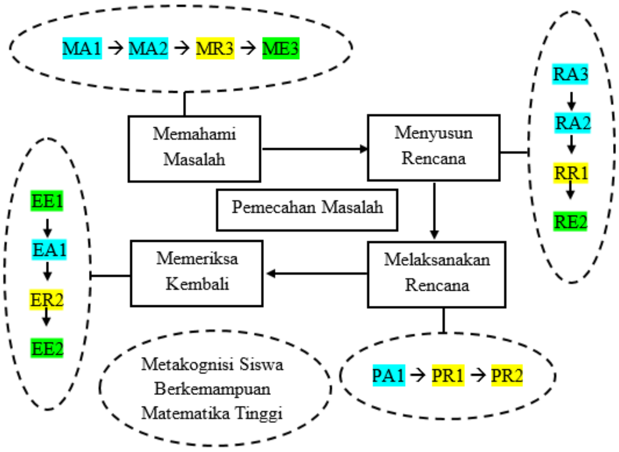


Figure 4. KT metacognition in problem-solving

In the aspect of awareness, KT tries to recall the knowledge it has to identify TPM problems. KT can present the information contained in the problem in the form of a table, mention the things that are known to make it easier for them to understand the problem, and make a representation of the information in the problem in the form of symbols. In the regulation aspect, on the problem, KT makes a mathematical model from the table that has been made to make it easier for him to solve the problem and think about using the graphical method he has done to solve the problem. This is following Mokos and Kafoussi (2013) who revealed that there are special requirements attached to students when choosing a problem-solving strategy. This depends on the level of students' understanding of the problems at hand. In the evaluation aspect, in the first problem, KT revealed that KT rechecked the results of his work. The checks made by KT are on the correctness of the mathematical model that he has made and when the subject is looking for a solution.

KS does not show the dominant metacognitive aspect in solving linear programming problems. KS fulfills the three metacognitive components while solving linear programming problems, namely awareness, regulation, and evaluation. This is in line with the research conducted by Putri (2015) that subjects with moderate mathematical abilities can fulfill the three metacognitive components. However, this contradicts Sudia (2015) which states that KS only fulfills two metacognitive components. The first step taken by KS in solving linear programming problems is reading the given task. After that, KS showed the awareness aspect when understanding the information and problems encountered. This is in line with Wilson & Clarke (2004) which revealed that 68 out of 90 students started problem-solving with the awareness aspect.

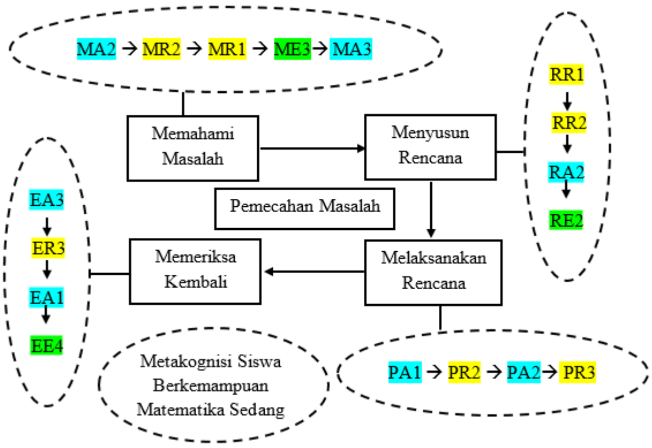


Figure 5. KS Metacognition in Problem Solving

In the awareness aspect, in the first problem of TPM, KS wrote down the information on the answer sheet according to the understanding that KS had, saying that it was easier to understand the problem using a table. In terms of regulation, in the first TPM question, KS thought of using the elimination - substitution method to find a solution. KS carries out the arrangement of plans according to the steps he has made. In the evaluation aspect, in the first problem of TPM, KS revealed that there was no other answer choice. This is because KS cannot find alternative answers even though they have found the set of solutions, but KS is not aware of this. KS also stated that apart from the substitution and elimination process, there is another way to solve this problem, namely using a coordinate system.

KR does not show the dominant metacognitive aspect in solving linear programming problems. KR fulfills the three metacognitive components while solving linear programming problems, namely awareness, regulation, and evaluation. This is not in line with the research conducted by Putri (2015) that subjects who have low mathematical abilities cannot fulfill the three metacognitive components. This is also contrary to Sudia (2015) which states that students with low mathematical abilities only fulfill one metacognitive component. The first step taken by KR in solving linear programming problems is reading the given task. After that, KR showed the awareness aspect when understanding the information and problems encountered. This is in line with Wilson & Clarke (2004) which revealed that 68 out of 90 students started problem solving with the awareness aspect.

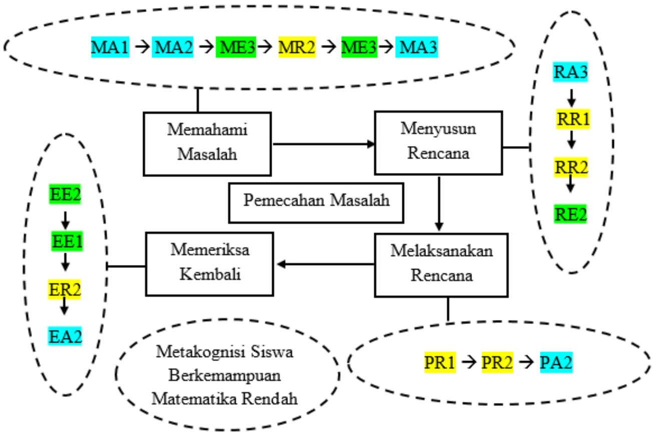


Figure 6. KR Metacognition in Problem Solving

In the aspect of awareness, in the first problem of TPM, KR began to present the information contained in the question in table form after reading problem number 1. KR revealed that the information on the problem at hand was sufficient to be used in solving the problem. KR stated that the information contained in the question was still a lot of words and was messy, so KR needed to re-present it so that it was easier to understand. In terms of regulation, in the first TPM question, KR had an error in understanding problem number 1. KR missed an important point in the given problem, namely the constraint function. KR did not provide complete information regarding the function of the constraint. Information on the problem states that there is an inequality function (maximum constraint), but the subject only writes the equation. This is because the subject is not familiar with the problem of linear programming in the form of a story, so that KR does not realize his mistake. After that, KR presented the plan that had been prepared to solve the problem, namely, first, KR made a table based on the available information so that it was easy to understand, then wrote down what was known, and wrote down the equations that matched the information in the table he had made. In the evaluation aspect, in the first problem of TPM, KR revealed that he rechecked the work that had been done. KR checks on the part of the equation that has been made, the substitution and elimination methods used, and checks the accuracy of the answer.

Conclusion

A student's ability to find solutions to solve mathematical problems can be identified through the metacognitive activities experienced. If the problem has a difficult level, it will make students more curious and want to solve it. Students who have high mathematical abilities can solve problems quickly and inaccurately. Students who have the high mathematical ability can fulfill 6 of 11 indicators of awareness, 5 of 11 indicators of regulation, and 3 of 9 indicators of evaluation. Students with moderate mathematical ability can solve problems quickly, but not accurately. Students who have the moderate mathematical ability can fulfill 7 of 11 indicators of awareness, 7 of 11 indicators of regulation, and 3 of 9 indicators of evaluation. Phenomena like this make metacognitive skills need to be taught to them so that they are accustomed to being aware of their way of thinking. Students with low math skills can solve problems carefully and slowly. Students with low mathematical ability can fulfill 6 of 11 indicators of awareness, 6 of 11 indicators of regulation, and 5 of 9 indicators of evaluation. Although students with low math abilities can carry out more evaluation activities, the results given are not better than other students. To improve metacognition skills, students need awareness of the steps of thinking. Awareness of students in thinking is needed to solve a problem. This makes metacognitive skills need to be taught to them so that they are accustomed to being aware of their way of thinking

Suggestion

The results of this study are to describe the metacognitive description of students who have high, medium, and low mathematical abilities when looking for solutions to solve problems that can be used as a reference in knowing the mathematics learning that has been accepted by students, especially the difficulties students have with competencies that must be mastered by students. . Learning in a class with heterogeneous students requires a student center approach so that it can attract students' interest in learning mathematics. However, important points in learning are not solely related to the learning environment, but metacognitive skills need to be taught using problem designs that are around students and not out of the topic of learning.

Education observers are encouraged to consider the differences in individual abilities that can affect student learning and problem solving, for example, students' mathematical abilities. This shows that students' mathematical abilities are also influenced by several factors, therefore elaborating the three kinds of students' abilities in one class requires a special way to form comfortable learning for students.

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